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Intra-nuclear cascade model

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Abstract. Basic assumptions of the semi-classical INC model are "revisited" in order to examine their applicability limits at low energies. The various implementations of the INC model in use in transport codes are described.

1. Introduction

The recent activities in production of Rare Isotope Beams and Spallation Sources led to revival of interest in reliable, predictive, simulation of collisions of hadron-nucleus and nucleus-nucleus in the energy range of few tens MeV to few GeV per particle to be embedded in transport codes (e.g. MCNPX, GEANT). Owing to the complexity of the quantum-mechanical many-body problems, the processes are often approximately described by Intra-Nuclear Cascade (INC) models followed by de-excitation (sometime two-step) models. INC reproduces successfully wide variety of experimental data of hadron induced reactions, using a small number of adjustable parameters, most with clear physical meaning. INC models have been embedded in the MCNPX transport code, filling the high-energy gap in existing experimental cross-section libraries, which are limited to incident energies of 150 MeV or, for some isotopes, 20MeV. For calculations of residua there is a need to use models already above 20MeV. Understanding of the limitations of INC at low energies is important for evaluation of reliability of transport calculations used in wide variety of applications.

The INC models treat the interaction of incoming projectile with the nucleus as a series of independent collisions using on-mass-shell free particle-nucleon cross sections. For energies <~2-3 Gev pion production and absorption modes are included in via the $\Delta_{3,3}$ resonance (pion-nucleon isobar) formation in nucleon-nucleus scattering

$$N_1 + N_2 \Leftrightarrow \Delta_{3,3} + N_3$$

$$\Delta_{3,3} \Leftrightarrow \pi + N.$$

The colliding particles are treated as classical point-like objects moving between collisions on well defined trajectories in the target potential well. The collision processes are treated as classical, energy and momentum conserving, scatterings. Collisions violating the Pauli Principle are not allowed – this is the single significant "quantum" property of the model.

2. Basic assumptions of INC

In order to determine the applicability of the INC models at low energies we shall "revisit" their basic assumptions [1-3]:

The reactions are "deep inelastic" - the energy transferred into internal energy of the target is large in comparison with the binding energy of nucleons in the target - the many body scattering can be approximately formulated in terms of on-shell single-particle scattering probabilities.

The "reduced" de Broglie wavelength, $\lambda\!\!\!^{-}$, is much smaller than the inter-nucleon distance, d. In language of quantum mechanics [4] - the wave-packets representing the particles have good enough definition of position, energy and momentum to be followed on classical trajectories.

$\lambda\!\!\!^{-}$ is much smaller than the mean-free-path between collisions, Λ - the scattered wave reaches approximately its asymptotic value before the next scattering, classical treatment of scattering becomes reasonable.

The radius of the target nucleus, R, is large with respect to Λ. There will be many scatterings inside the nucleus and the interference terms between different scattered waves will tend to cancel out.

Λ is larger than d, and the time between interactions, Δt, is much longer than the time of an interaction, T - the scattering from different nucleons in the nucleus can be assumed to be approximately independent of each other

Summarizing, we have:

$\lambda\!\!\!^{-} << d < \Lambda < R$ (4a)

$\Lambda/\beta c > T \approx 10^{-23}$ sec => $\Lambda/3\beta \approx> 1$ fm (4b)

Fig. 1 shows the behaviour of the relevant quantities for proton on ^{208}Pb reaction as a function of incident proton energy, E. Calculating $\lambda\!\!\!^{-}$ we took into account that proton entering the nucleus gains ≈40 MeV kinetic energy.

$$\lambda\!\!\!^{-} = \hbar / \sqrt{(E+40)^2 - m_p{}^2}\ , \qquad (5)$$

where m_p is the proton mass. The mean free path, Λ, was calculated using the *Isabel* INC code [5] (which includes the

Pauli Principle) for central collisions:

$$\Lambda = -2R / Ln(N_{transp} / N_{tot}), \qquad (6)$$

where R=6.63 fm, N_{transp} is the number of "transparencies" (events when the proton traversed the diameter of the nucleus without interacting) and N_{tot} is the total number of events. For comparison also shown is $1/\rho\sigma$, with ρ=.16 fm^{-3} being the central nucleon density and σ the average proton-nucleon cross section (in fm^2). This is an estimate of the mean-free-path without the Pauli Principle effect.

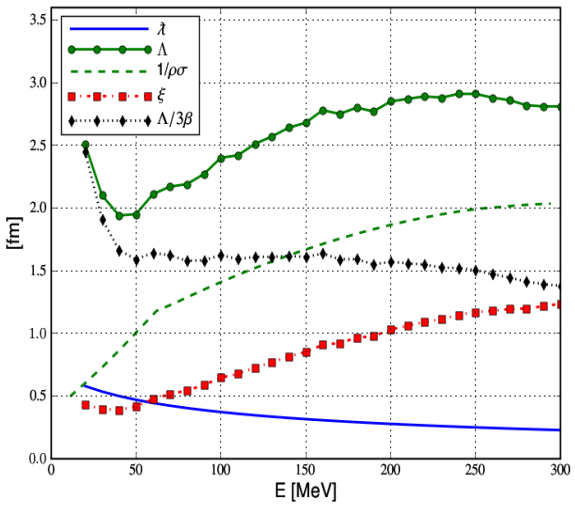

Fig. 1. Central collision proton on ^{208}Pb: $\hat{\lambda}$, Λ, $\xi=\Lambda/\hat{\lambda}/10$, $1/\rho\sigma$ and $\Lambda/3\beta$ as a function of incident proton energy.

The effect of Pauli Principle is very important. It is especially pronounced at E_{inc} < 40 MeV causing Λ to rise even though the nucleon-nucleon cross section is strongly increasing. In collisions of high energy particle with the Fermi Sea, the momentum transfer is small, and Pauli Principle limits the interaction to small fraction of the Fermi Sea close to its surface, thus increasing the mean free path.

From eq. (4a) with $d\approx$2 fm, requiring, very conservatively, that $d,\Lambda>5\hat{\lambda}$, INC is applicable only for E_{inc}>60 MeV. Requiring $d,\Lambda>10\hat{\lambda}$, moves the applicability of INC to E_{inc}>200 MeV. No additional restrictions follow from. (4b).

Most of the collisions are not central. *Isabel* calculations show that in the energies of few tens to few hundreds MeV about 60% of the collisions leading to inelastic reactions occur at impact parameters at which the nuclear density is less than a half of the central density. However, in the region of low nuclear density the degenerate Fermi gas potential is low, so the particle gains less kinetic energy entering the nucleus. 50 MeV proton has $\hat{\lambda}$=0.6 fm and can not be "localized" on nuclear periphery. On the other hand, proton of 250 MeV has $\hat{\lambda}$=0.25 fm and starts to be sensitive to the details of nuclear surface. Still, the lower density of the surface extends the applicability range of INC to lower energies.

It is clear that the applicability range of INC depends strongly on the specific reaction property to be calculated and the desired accuracy.

Thus, considering the total nucleon yields from nucleon induced reactions one may use INC starting from just a few tens of MeV. Using the original Serber's arguments [1], the incident particle will loose in each collision ≈10-20 MeV. Particle with energies of ≈<30 MeV will be absorbed in the target nucleus (their probability to interact with the target nucleons is large, and they will "thermalize") unless they are on far periphery of the nucleus. The "thermalized" by INC nucleus has the right excitation energy and momentum (since INC conserves energy and momentum) and may be properly treated by the de-excitation models. In the peripheral collisions the number of emitted particles is expected to be correct in the low energy regime, being determined by energy conservation and the ability of particles to escape the nucleus.

INC may be justified for low energies ($E_{inc}\approx$ >50 MeV) considering reactions which take place primarily on nuclear periphery (e.g. "quasi-elastic", "low multiplicity"). Here, however, the results may strongly depend on the target periphery modelling. We may expect discrepancies, especially when looking in forward direction, due to violation of assumption IV. For "quasi elastic" reactions we may expect distortion of forward angle cross sections due to interference with the elastic channel, which is not accounted for in INC.

Considering "violent" (high multiplicity, high excitation energy) events, which involve the inner parts of the target nucleus the reliability of INC is expected to degrade for energies below 100-200MeV, though it may be used, with caution, according to the original Serber's argument. [1].

Until now we have considered the energy limitations on the incident particle. However, an energetic projectile will give little momentum transfer to its Fermi Sea collision partners, and create low energy "participants". Inside the nucleus those should be "absorbed" contributing their energy to the excitation of the "remnant" target, on nuclear periphery they may some chance to escape. The target periphery is modelled in all the INC implementations, but each has a different way to deal with the low energy "participants" chosen considering agreement with the experimental data rather than from basic physical considerations.

High energy cluster (α, d, ^{3}He…) production is out of the scope of INC models. In order to calculate those "extra prescriptions" are used. In the "coalescence" model [12,13] the vicinity (configuration, momentum or phase space) of escaping particle is searched for potential particles to share its energy and form a cluster. An alternative "kick-out" process [2] assumes existence of "virtual" clusters in the nucleus which elastically scatter with the cascading particles and then, taking into account their survival probability, escape the nucleus.

3. Implementations of INC

The variety of INC implementations involves Monte Carlo (MC) sampling methods [14] and may be divided into three classes: "Space-Like MC" (e.g. refs. [2, 3, 6-8]), "Time-Like MC" (e.g. refs. [5, 9]) and "Nucleon Dynamics" (refs. [10, 11]). A brief description of the three approaches is given emphasizing the treatment of the low-energy "participants".

3.1. Space-Like and Time-Like MC

The target nucleus density distribution is represented by a continuous distribution, e.g. Fermi distribution:

$$\rho(r) = \rho_0 /[1+\exp(r-c)/a] \,, \quad (8)$$

A step–function distribution is used to approximate the nuclear charge distribution. The nucleus is divided into several (usually 8 or 16) concentric regions, each of constant density. The ratio of proton to neutron density is assumed to be Z/(A-Z) in all the regions.

The nucleon distribution in the target is assumed to be that of a degenerate Fermi gas in a potential well.

$$E_{F_i} = (\hbar^2/2m)(3\pi^2\rho_i)^{2/3}, \quad (9)$$

where the subscript i stands for either protons or neutrons, m is the nucleon mass, and ρ_i is the density of protons and neutrons, respectively.

Due to the variation of the Fermi energy, the nuclear potentials of the protons and neutrons differ in the various density regions. Conservation of energy and momentum requires that the kinetic energy of the particles and their direction change as they cross density region boundary (refraction). If the impact angle at the region boundary is greater that the critical angle the particle is reflected. The Coulomb interactions between the target nucleus and the incident or emitted charged particle should be considered.

Consider a single bombarding particle entering the target nucleus. The Lorentz invariant probability per unit path length of the particle to interact with the nucleons of the nucleus is:

$$\begin{aligned} Q &= \frac{1}{v_1}\int \sigma_{12} v_{12} \frac{\partial \rho_2}{\partial \vec{p}_2} d\vec{p}_2 \\ &\approx \frac{1}{v_1}\sum \sigma_{12} v_{12} \frac{\partial \rho_2}{\partial \vec{p}_2}\Delta\vec{p}_2 = \lambda^{-1} \end{aligned}, \quad (10)$$

where v_1 is the laboratory velocity of the bombarding particle, σ_{12} , v_{12} are the cross section and relative velocity of the incident particle and the particles with momentum p_2 and density ρ_2 . The mean free path of the bombarding particle is λ and the probability of the particle to interact at the distance between a and $a+da$ is given by:

$$dP_{\text{int}}(a) = e^{-Qa} Q da \quad (11)$$

The calculation may now proceed in two ways:

In "**Space-Like MC**" method (e.g. refs. [2, 3, 6-8]) the collision site is determined by a correlation between a random number and the probability to interact at a distance between a and $a+\delta a$ obtained by partial integration of Eq. (11). The collision partner is chosen from the distribution given by Eq. (10). The types of the particles after the collision ("participants") are chosen by isospin and branching ratio considerations, and their momenta chosen using relativistic kinematics. The collision is allowed only if the outgoing particles obey Pauli principle.

After the first interaction, one of the "participants" is followed and its possible next collision site is determined again by integration of Eq. (11). The process continues until the particle "dies" - leaves the target nucleus or its energy falls below certain "energy-cutoff". The other "participants" are then treated in the same way, one after another.

The process ends when all the "participants" die. The particles leaving the target contribute to the high energy part of the spectrum, whereas the particles which fall below "energy cutoff" and the holes in the Fermi sea form the excitation energy of the residual target.

The "energy cutoff" is generally chosen close to E_F+E_B for neutrons and $E_F+E_B+E_C$ for protons, E_F, E_B, E_C being respectively Fermi, binding and Coulomb barrier energies. The reason for the "energy cut-off" is that the low energy particles are not fulfilling eq. (4a) and should be taken care of by the de-excitation models.

In "**Time-Like MC**" (e.g. refs. [5, 9]) a convenient method of calculation is to divide the Fermi sphere of momentum into n parts of equal volume and calculate the mean cross section, $\sigma_{1,2i}$, and the mean velocity, $v_{1,2i}$, for each subvolume. If Q is calculated in the rest system of the nucleus, the momentum distribution is the "undistorted" Fermi gas distribution for each subvolume

$$\begin{aligned} \frac{\partial \rho_2}{\partial \vec{p}_2}\Delta\vec{p}_2 &= \frac{\rho}{n} \\ Q &= \frac{\rho}{n}\sum_{i=1}^{n}\frac{v_{1,2_i}}{v_1}\sigma_{1,2_i} \end{aligned} \quad (12)$$

The probability that a collision takes place in a time interval $\tau=a/v_1$ is calculated by integrating Eq. (11)

$$\begin{aligned} P_{\text{int}}(\tau) &= 1-\exp(-Q\tau v_1) \\ &= 1-\prod_{i=1}^{n}\exp\left(-\frac{\tau}{n}\rho v_{1,2_i}\sigma_{1,2_i}\right) \end{aligned} \quad (13)$$

Each term of the product of the right hand side of Eq. (12) is formally equivalent to the probability of no collision between the incident particle and a beam of particles of momentum p_i and density ρ occurring in a time interval τ/n. In other words, the probability of collision of a "participant" in the time interval a may be calculated by dividing the interval τ into n equal parts and calculating for each interval $\delta\tau=\tau/n$ the probability of collision between the cascade particle and a hypothetical nucleon gas having a density ρ and nucleon

momentum p_i. For each interval $\delta\tau$ a different momentum p_i is chosen out of the undistorted momentum distribution in a completely arbitrary sequence. A test is made to see if the collision occurs in that step by the comparison of a random number to the quantity

$$P_{\text{int}}(\delta\tau) = 1 - \exp(-\rho v_{1,2_i}\sigma_{1,2_i}\delta\tau) \approx \rho\sigma_{1,2_i}v_{1,2_i}\delta\tau \quad (14)$$

The initial $\delta\tau$ is chosen to be Λ/vn, v being the incident particle laboratory velocity, Λ is calculated for nucleon density in the centre of the nucleus using the total cross sections of the incoming particle with a stationary protons and neutrons and n=20-30.

Like in Space-Like MC the interaction between the incoming particle and a partner from the Fermi sea occurs the types of the particles after the collision, "participants", are chosen by isospin and branching ratio considerations, and their momenta chosen using relativistic kinematics. The collision is allowed only if the outgoing particles obey Pauli principle.

Then, the time interval is updated and the cascading for each of the "participants" continues in the next time interval.

Like in the Space-Like MC, cascade process stops when all the "participants" leave the target volume or fall below a certain "energy cut-off".

In "Space-Like MC" first the incoming particle is followed, then each of the particles with which the incoming particle had interacted, are calculated one after another. On the other hand, in "Time-Like MC" at first only the incoming particle is followed, but, after the first allowed interaction, both the incoming particle and its collision partner are followed in each time interval, and with each allowed interaction the number of particles to be followed in the subsequent interval increases. The advantage of the Time-Like MC method is that it makes it possible to take into account effects of correlation between two close cascade particles and to consider local changes in nuclear density (and, possibly, nuclear potential) due to previous interactions.

The Time-Like MC code *ISABEL* [5] takes into account the depletion of the density in the Fermi sea as the cascade develops. Since the detailed nature of the density rearrangement is unknown, two extreme prescriptions are applied:

Fast rearrangement. After each collision with a target partner, the density distribution ρ_i of the "partner type" in the target is instantaneously and uniformly reduced for the whole nucleus. In addition "distance restriction" is usually applied – any given particle is not allowed to interact within a distance smaller than some $r_{min,i}$ from its last interaction.

Slow rearrangement. After each collision a hole of radius rmin is punched in the density distribution configuration space around the position of the interaction. No more interactions are allowed in this hole. The holes may be either isospin dependant or isospin independent, i.e. we may punch them for protons and neutrons independently, with possibly different radia.

The depletion of the Fermi seas affects the Pauli blocking. Two options for dealing with Pauli blocking are included in *ISABEL* [5]:

Full Pauli blocking – After each interaction cascade nucleons are tested for Pauli principle violation. If cascade nucleon energy is lower than the target Fermi energy - the interaction is forbidden.

Partial Pauli blocking – After each interaction proton and neutron Fermi sea depletion factors (ratio of actual to original number of particles in the Fermi sea) are calculated. If the energy of a cascade proton (neutron) generated in an interaction is lower than the Fermi energy, a random number is compared to the corresponding depletion factor, and only if it is smaller – the reaction is forbidden. Intuitively, as we deplete the Fermi sea, cascade particle are allowed to fill the "empty" states below the Fermi energy.

3.2. Nucleon Dynamics

In the Nucleon Dynamics implementation of INC (refs. [10, 11]), the target nucleus consists of discrete nucleons distributed following Fermi density distribution and degenerate Fermi gas momentum distributions in a potential well. The incident particle and target constituents are moving on classical trajectories in the potential well and scatter whenever their relative distance is less than $\sqrt{(\sigma(E^{cm})/\pi)}$, $\sigma(E^{cm})$ being the free space cross section and E^{cm} their centre of mass energy. Pauli principle violation is not permitted. Particles may reflect of the potential well walls or reaching the surface with high enough energy escape the target.

In this approach there is a need to set a restriction on the interaction range or, equivalently, $E^{cm}{}_{min}$ - limiting value of E^{cm}, below which particles do not scatter. This ensures that low energy particles reaching (or created at) target periphery will be able to escape rather than interact with nuclear interior. $E^{cm}{}_{min}$ of ≈1925 MeV corresponding to relative lab energy of ≈100 MeV (range restriction of ≈1.3 fm) gives generally good results. It is in line with requirements III,V of chapter 2. The physical argument behind it is that the long range (or small momentum transfer) interactions are accounted for by the average nuclear potential.

The process stops at a time when properties of the reaction "stabilize" [10] or, alternatively, when all the "participants" left in the target are below certain "energy cut-off" (similar to that used in the Time-Like MC approach).

4. Examples

In the following examples the "Time-Like Basis Cascading" is represented by the code *Isabel* [5] and the "Nucleon Dynamics" by *INCL4.4* [15]. Both are appended with the ABLA de-excitation code [16].

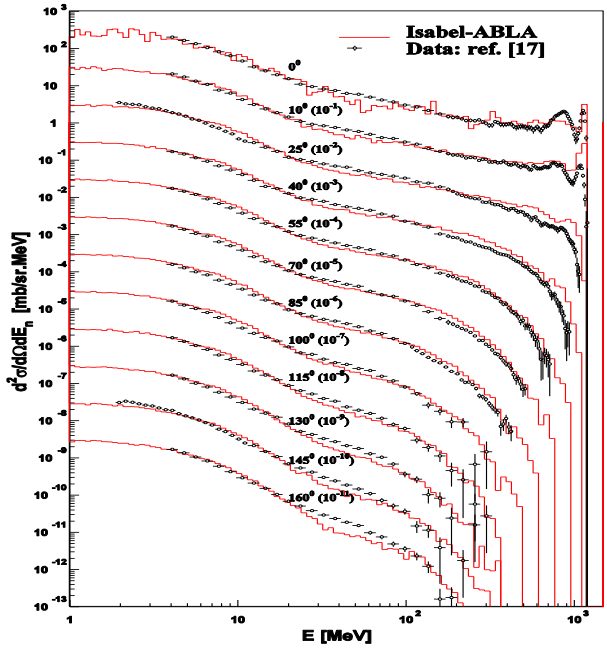

Fig. 2. Neutron double differential cross section for p(^{208}Pb,nX) reaction at 1.2 GeV.

Fig. 2 shows an example of results INC approach at high energies. Here INCL and *Isabel* give generally very similar results, and the double differential cross sections are well reproduced. As expected, the biggest discrepancy appears at energies of few tens MeV. This discrepancy grows toward backward angles, where the contribution of scattering of a particle of a massive "collective object" is contributing.

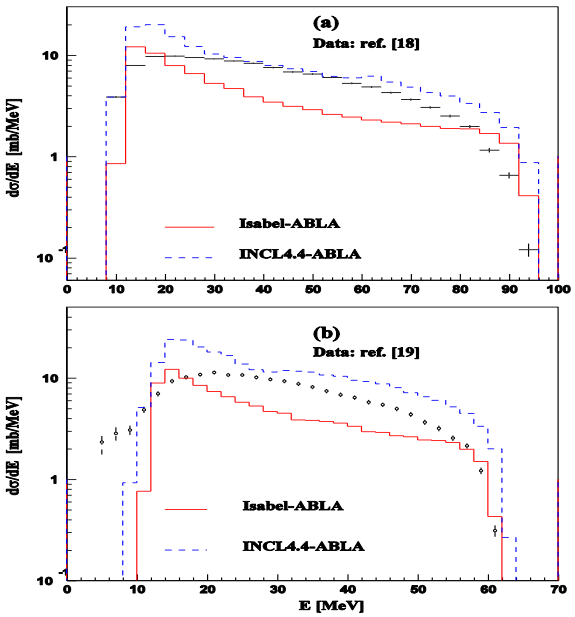

Fig. 3. Proton $d\sigma/dE$ for (a) n(^{208}Pb,pX) at 96 MeV, (b) n(^{209}Bi,pX) at 63 MeV.

Fig. 3 shows that for projectiles of incident energy in the range of ≈50-100 MeV the deviation between the models and experimental cross sections may reach factors of 2-5.

The calculations are very sensitive to details, such as the momentum distribution of target nucleons on the border of the target, implementation of the Pauli principle and energy cut-offs.

The difference between the models reflects the different method to treat the low energy interactions. At low energies INCL gives generally higher $d\sigma/dE$ than *Isabel*. This is probably due to the $E_{\min}^{cm}$ cut in INCL allowing the low energy particles to escape. These particles are captured in *Isabel*. Considering $d\sigma/d\Omega dE$ in Fig.4: at forward angles, sensitive to the peripheral scattering, *Isabel* is closer to experiment, INCL being too high probably due to the $E_{\min}^{cm}$ cut; at backward direction *Isabel* is underestimating the flux probably due to its extra absorption or lack of contribution of scattering of massive "collective object".

It should be mentioned that neither INCL nor *Isabel* include "refraction", which was "discredited" for high energy calculations [2,9], but may improve the results for low energies (or even at high energies when using an energy dependent potential [20]).

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