Nonlinear MHD effects on the Alfvén eigenmode evolution

Y. Todo (NIFS, Japan)

H. L. Berk, B. N. Breizman (Univ. Texas, USA)

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Outline

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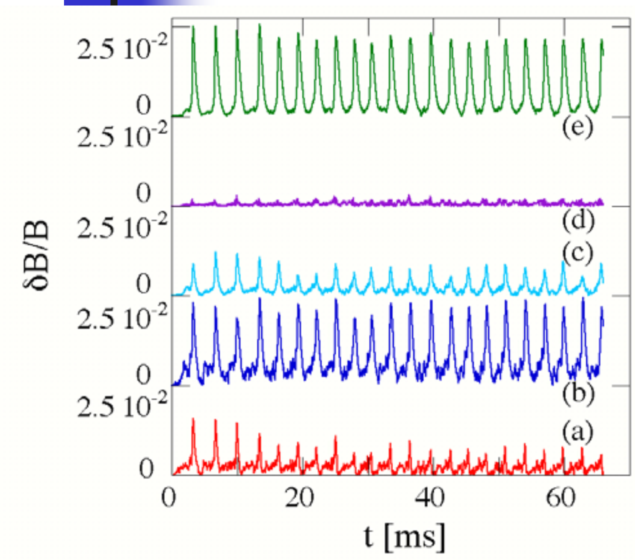
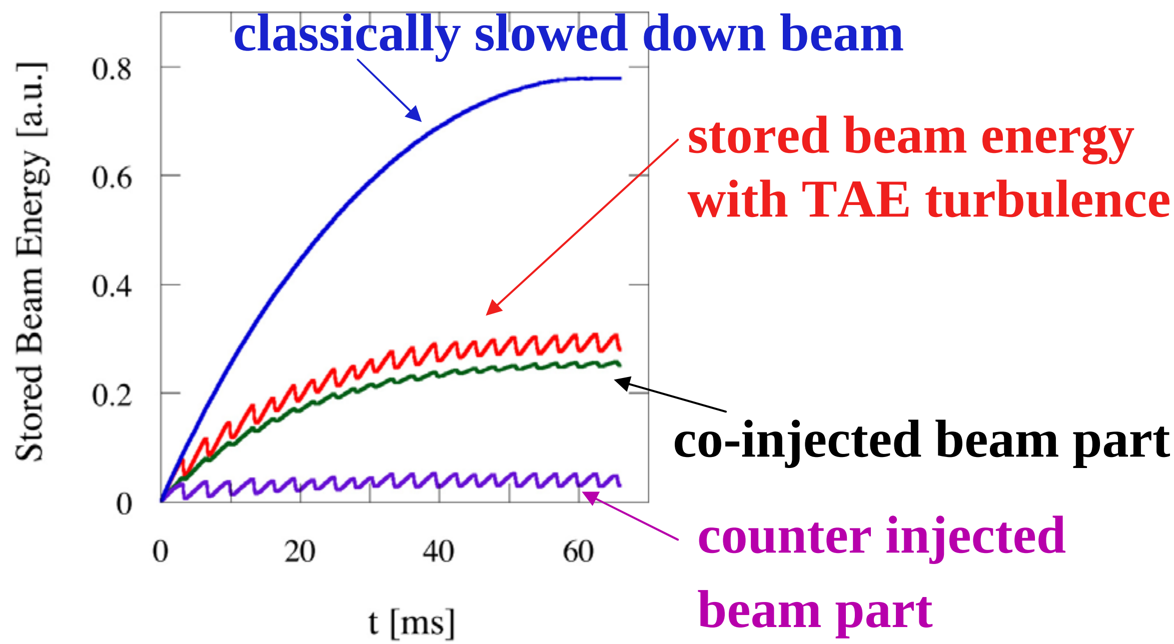
- Initial condition
- Comparisons between linear and nonlinear simulations
- Investigation on important nonlinear products
- Summary

Introduction





- We made the first numerical demonstration of TAE bursts with parameters quite similar to that of a TFTR experiment and closely reproduced many experimental characteristics [Y. Todo, H. L. Berk, and B. N. Breizman, Phys. Plasmas 10, 2888 (2003)].
- These include: **a) the synchronization** of multiple TAEs, **b) the modulation depth** of the drop in the stored beam energy, **c) the stored beam energy.**

Objective

- The saturation amplitude in the simulation is $\delta B/B \sim 2\times10^{-2}$ at the mode peak while that inferred from the experimental plasma displacement is $\delta B/B \sim 10^{-3}$.
- In the simulation, nonlinear MHD effects were not considered. The objective of this work is to investigate whether and which nonlinear MHD effects suppress the TAE saturation level.

Guiding center approximation for energetic particles

$$\mathbf{u} = \mathbf{v}_{//}^{*} + \mathbf{v}_{E} + \mathbf{v}_{B}$$

$$\mathbf{v}_{//}^{*} = \frac{v_{//}}{B^{*}}[\mathbf{B} + \rho_{//} B \nabla \times \mathbf{b}]$$

$$\mathbf{v}_{E} = \frac{1}{B^{*}}[\mathbf{E} \times \mathbf{b}]$$

$$\mathbf{v}_{B} = \frac{1}{q_h B^{*}}[-\mu \nabla B \times \mathbf{b}]$$

$$\rho_{//} = \frac{m_h v_{//}}{q_h B}$$

$$\mathbf{b} = \mathbf{B} / B$$

$$B^{*} = B(1 + \rho_{//} \mathbf{b} \cdot \nabla \times \mathbf{b})$$

$$m_h v_{//} \frac{dv_{//}}{dt} = \mathbf{v}_{//}^{*} \cdot [q_h \mathbf{E} - \mu \nabla B]$$

Linear and nonlinear MHD Equations

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho_{eq} \mathbf{v})$$

$$\rho_{eq} \frac{\partial}{\partial t} \mathbf{v} = -\nabla p + (\mathbf{j}_{eq} - \mathbf{j}'_{heq}) \times \delta \mathbf{B} + (\delta \mathbf{j} - \delta \mathbf{j}'_h) \times \mathbf{B}_{eq}$$

$$+ \nu \rho [\frac{4}{3} \nabla (\nabla \cdot \mathbf{v}) - \nabla \times \vec{\omega}]$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot (p_{eq} \mathbf{v}) - (\gamma - 1) p_{eq} \nabla \cdot \mathbf{v}$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}_{eq} + \eta (\mathbf{j} - \mathbf{j}_{eq})$$

$$\mathbf{j} = \frac{1}{\mu_0} \nabla \times \mathbf{B}$$

$$\vec{\omega} = \nabla \times \mathbf{v}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$

$$\rho \frac{\partial}{\partial t} \mathbf{v} = -\rho \vec{\omega} \times \mathbf{v} - \rho \nabla (\frac{v^2}{2}) - \nabla p + (\mathbf{j} - \mathbf{j}'_h) \times \mathbf{B}$$

$$+ \nu \rho [\frac{4}{3} \nabla (\nabla \cdot \mathbf{v}) - \nabla \times \vec{\omega}]$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot (p \mathbf{v}) - (\gamma - 1) p \nabla \cdot \mathbf{v}$$

$$+ (\gamma - 1) [\nu \rho \omega^2 + \frac{4}{3} \nu \rho (\nabla \cdot \mathbf{v})^2 + \eta \mathbf{j} \cdot (\mathbf{j} - \mathbf{j}_{eq})]$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta (\mathbf{j} - \mathbf{j}_{eq})$$

$$\mathbf{j} = \frac{1}{\mu_0} \nabla \times \mathbf{B}$$

$$\vec{\omega} = \nabla \times \mathbf{v}$$

The viscosity and resistivity are $\nu = 10^{-6} v_A R_0$ and $\eta = 10^{-6} \mu_0 v_A R_0$ in this work.

Energetic particle current density

Energetic ion current density without ExB drift:

$$\mathbf{j}'_h = \int q_h(\mathbf{v}^*_{//} + \mathbf{v}_B)\, f d^3v - \nabla \times \int \mu f \mathbf{b} d^3v$$

parallel + curvature drift + grad-B drift   magnetization current

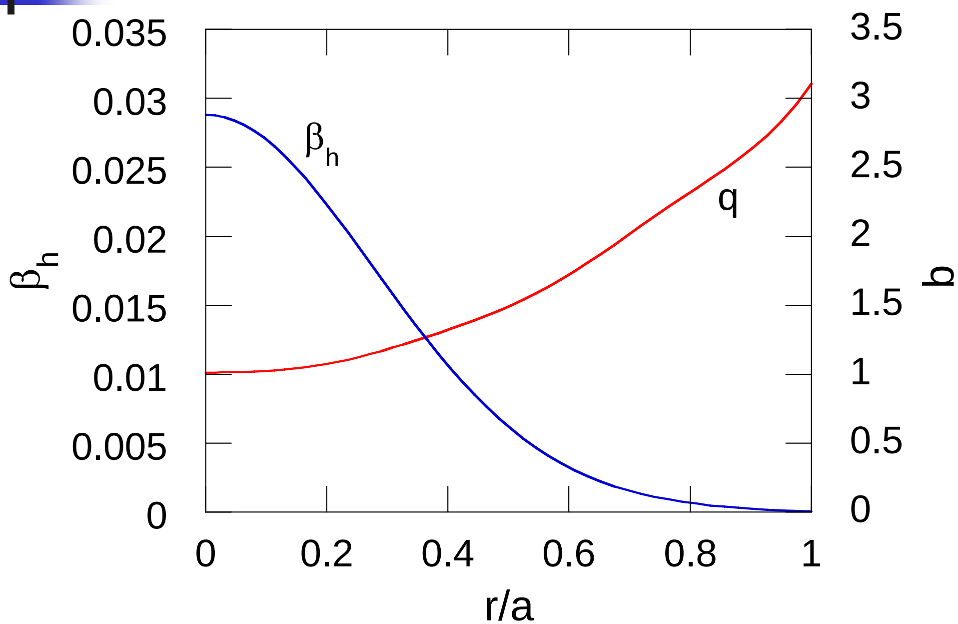
The energetic ion pressures are calculated using the δf particle weight:

$$P_{h//}(\mathbf{x}) = P_{h//0}(\mathbf{x}) + \sum_i^N m_h v_{//i}^2 w_i S(\mathbf{x} - \mathbf{x}_i) \ ,$$

$$P_{h\perp}(\mathbf{x}) = P_{h\perp 0}(\mathbf{x}) \frac{B(\mathbf{x})}{B_0(\mathbf{x})} + B(\mathbf{x}) \sum_i^N \mu_i w_i S(\mathbf{x} - \mathbf{x}_i) \ .$$

In the linear simulation, change in the n=0 harmonics of energetic particle pressure is removed.

Plasma profile



$$\beta_h \cong \beta_{h0} \exp[-(r/0.4a)^2]$$

$$q \cong 1.0 + 2.0(r/a)^2$$

$$a/(v_A \Omega_h) = 11.1$$

$$R_0/a = 3.2$$

$$f(v_{//}, P_\varphi) = g(v_{//}) h(P_\varphi)$$

$$g(v_{//}) = \frac{1}{|v_{//}|^3 + v_c^3} \frac{1}{2}\left[1 - \tanh\left(\frac{|v_{//}| - v_b}{\Delta v}\right)\right]$$

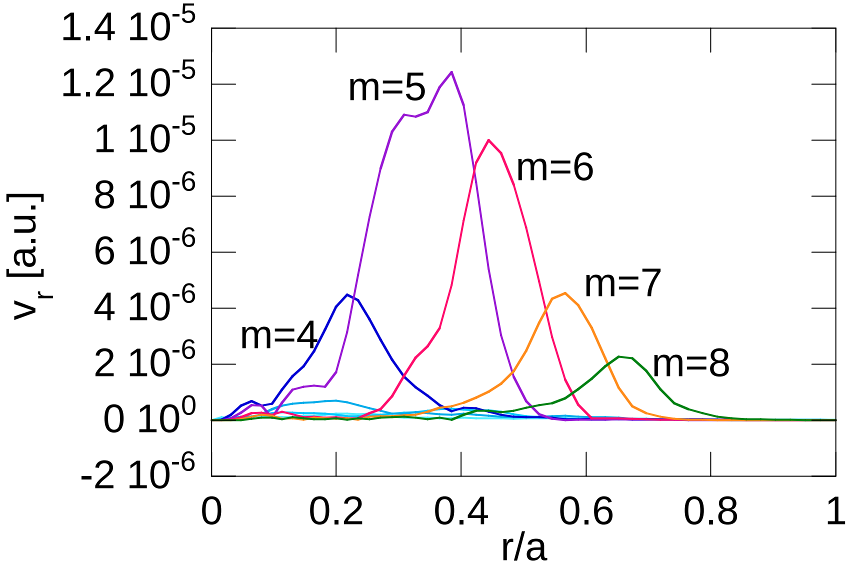
balanced injection

slowing down distribution

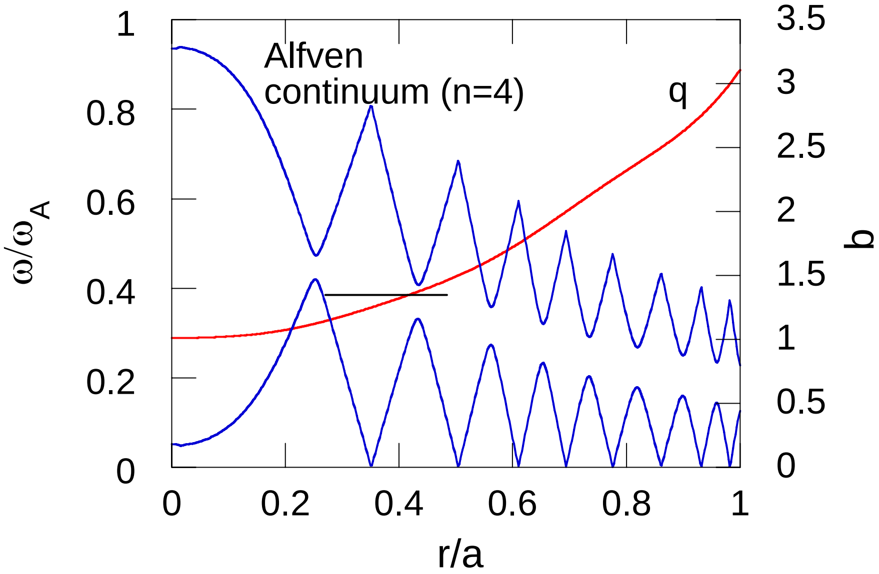
$$v_b = 1.2 v_A \;,\; v_c = 0.58 v_A \;,\; \Delta v = 0.1 v_A$$

$$\mu = 0$$

Most unstable TAE (n=4) and Alfven continuous spectra (n=4)

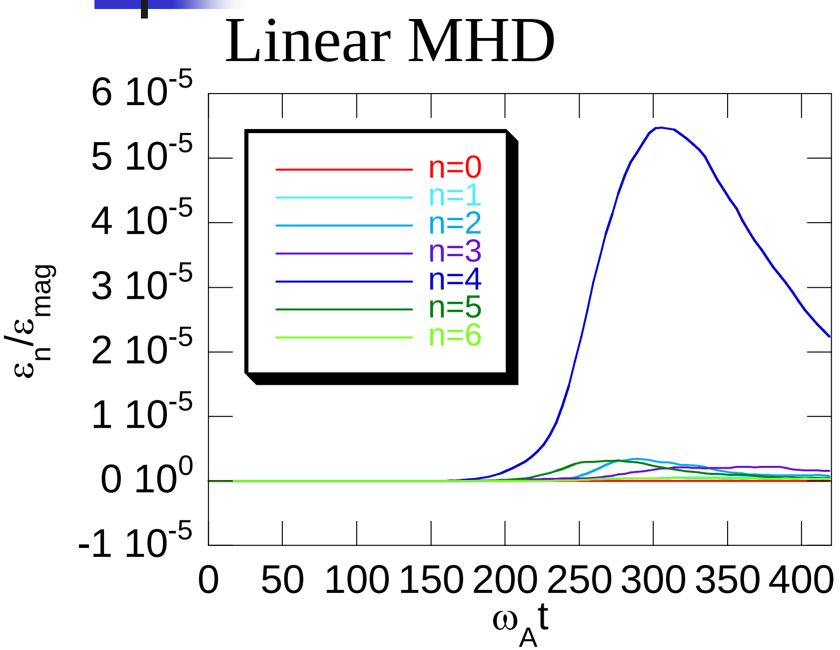


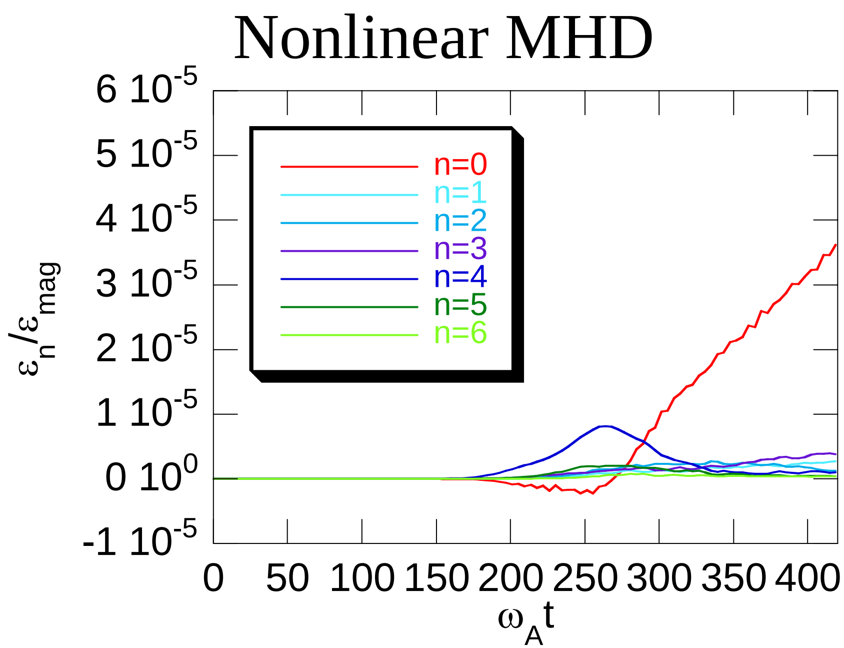
Spatial profile of the most unstable TAE with n=4.



Alfven continuous spectra with n=4, safety factor profile, and frequency and location of the most unstable n=4 TAE.

Comparison between linear and nonlinear MHD runs



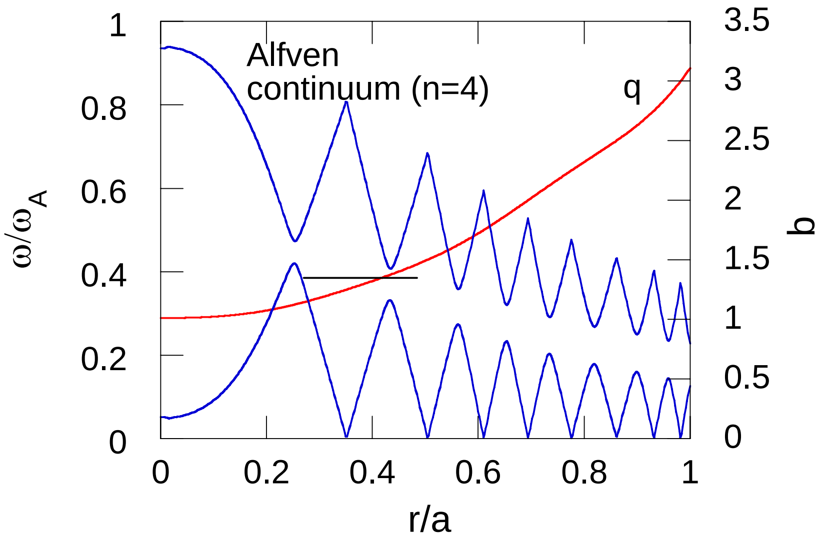


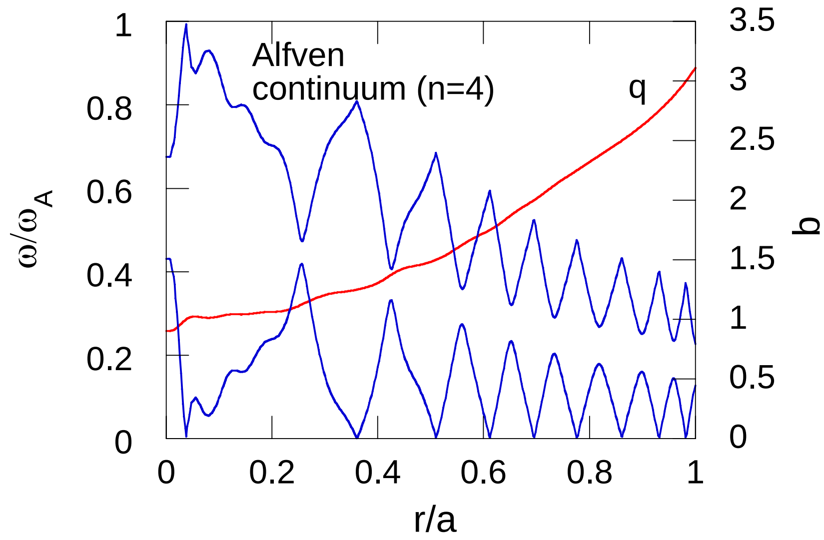
Energy evolution of each toroidal harmonic is plotted.

(1) The energy saturation level of the n=4 harmonics is dramatically suppressed to 15% in the nonlinear MHD run.

(2) The saturation levels are $\delta B_{n=4}/B \sim 2\times10^{-2}$ in the linear run and $\delta B_{n=4}/B \sim 8\times10^{-3}$ in the nonlinear run.

Comparison in q profile and Alfven continuous spectra with n=4 in the nonlinear run



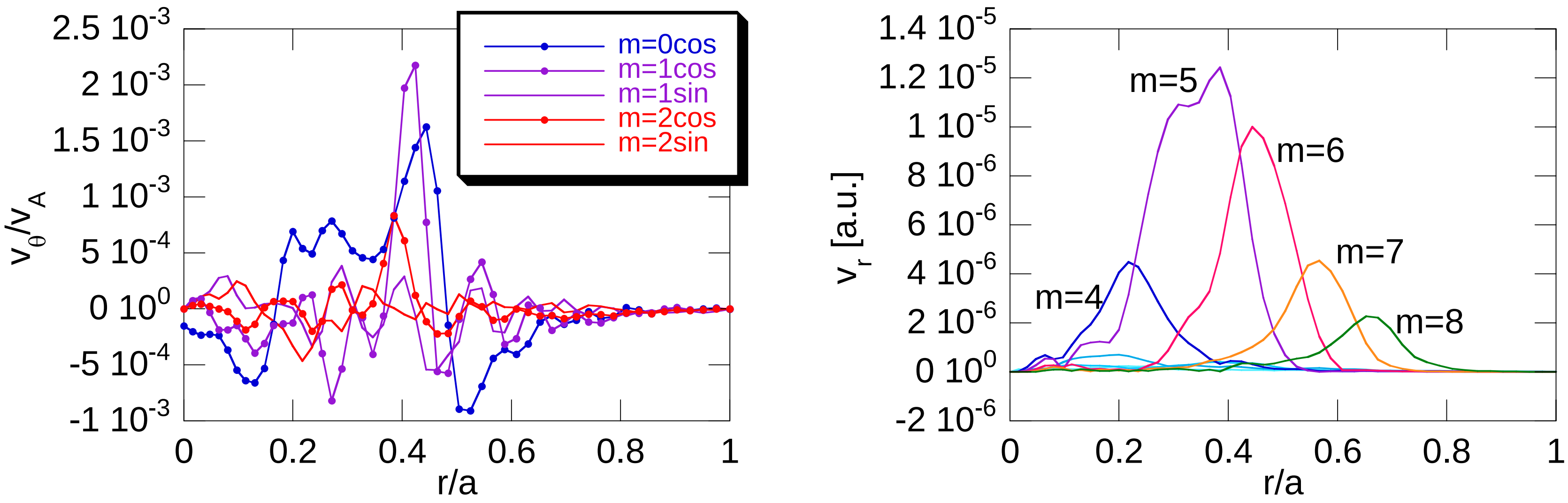


$\omega_A t=0$

$\omega_A t=280$: after the saturation

q-profile steepening at the TAE gap.

Generation of n=0 poloidal flow (zonal flow) in the nonlinear run

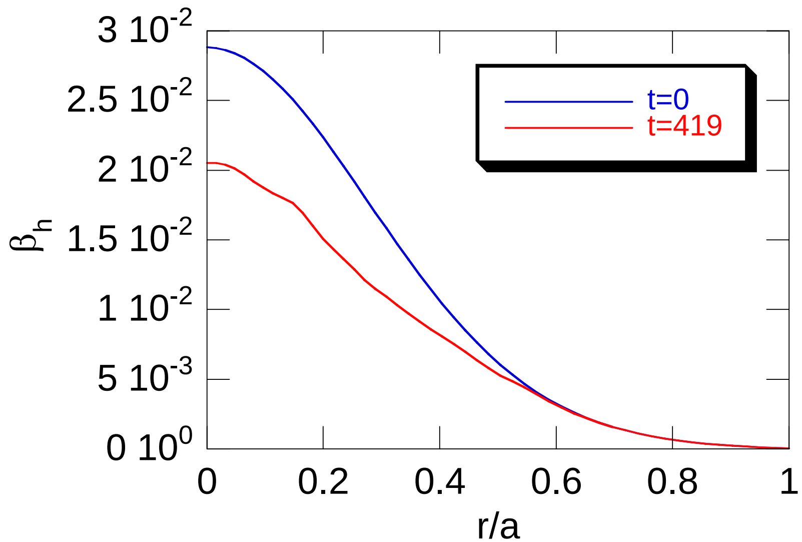


(1) Poloidal flow with m/n=0,1/0 is generated (left panel; similar to [Spong et al., Phys. Plasmas **1**, 1503 (1994)]).

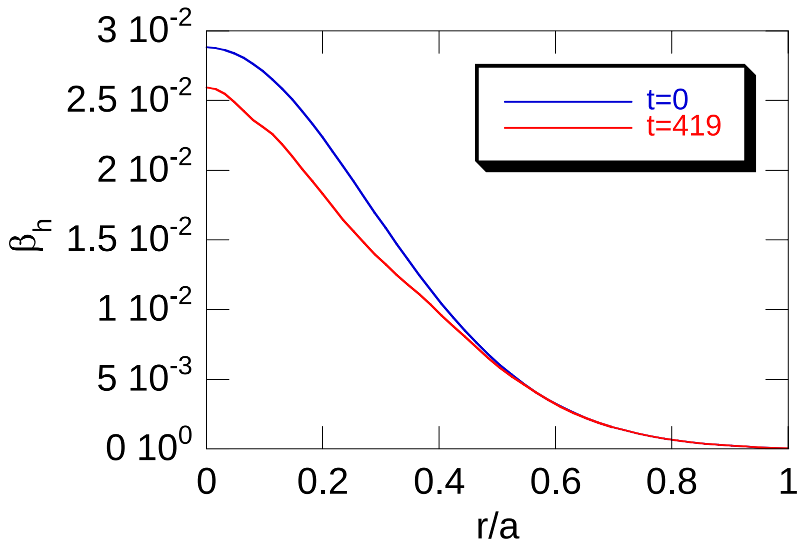
(2) The poloidal flow sharply peaks near the peak of the TAE (right panel).

Comparison in energetic ion transport between linear and nonlinear MHD runs

Linear MHD

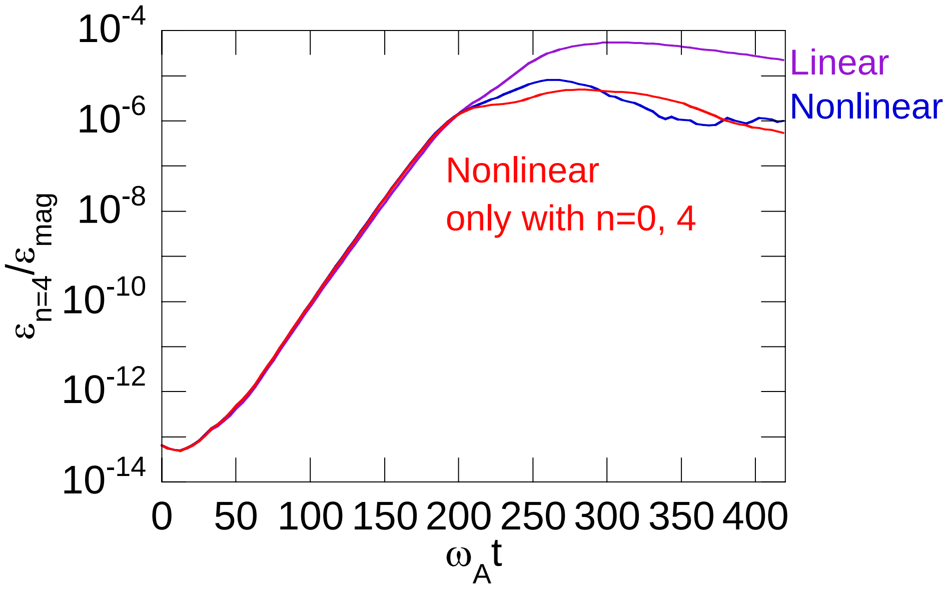


Nonlinear MHD



- ✓ Energetic ion beta profiles are plotted for t=0 and $\omega_A t$=419 for the linear and nonlinear runs. Particles are lost at r/a=0.6 for consistency with the δf scheme and for numerical stability.
- ✓ Energetic ion transport is also suppressed by the nonlinear MHD effects.

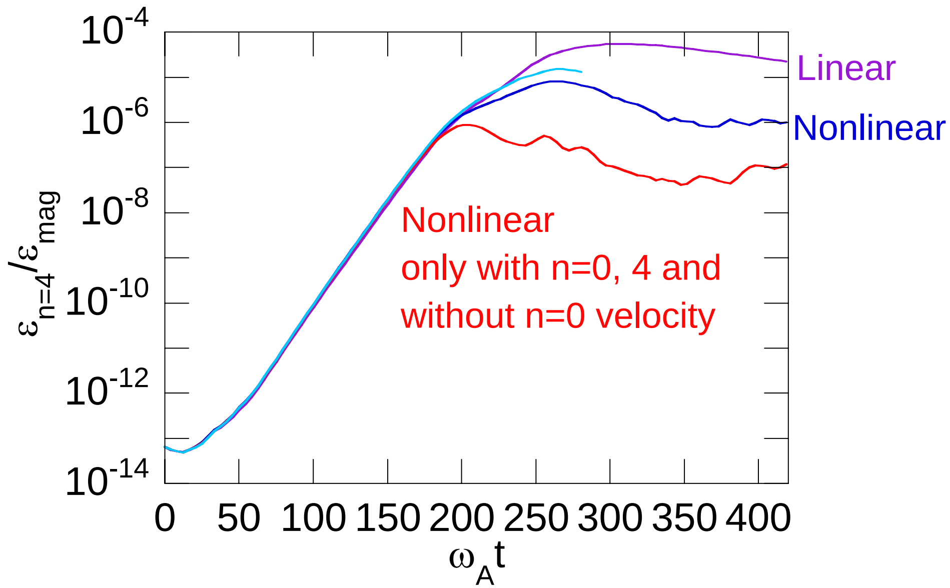
Comparison between the nonlinear run and a nonlinear run where only n=0 and 4 harmonics are retained



The saturation level is comparable to the standard nonlinear run.

These results indicate that the nonlinear suppression effects arise through the n=0 harmonics.

Nonlinear run where only n=0 and 4 harmonics are retained and n=0 velocity field is removed



The saturation level is much lower than the standard nonlinear run.

(1) These results indicate that the suppression effect arises through the change in the n=0 harmonics of the magnetic field ($\delta B_{n=0}$).

(2) The $\delta B_{n=0}$ is generated by $E_{n=0}=-v_{n=4} x B_{n=4}$ because there is no $v_{n=0}$.

(3) The n=0 velocity field relaxes $\delta B_{n=0}$ and the suppression effect.

Summary

- Linear and nonlinear MHD simulations are compared for toroidal Alfven eigenmode (TAE) evolution.
- A suppression effect on TAE saturation level was observed in the nonlinear MHD run. The saturation level at the mode peak was $\delta B_{n=4}/B \sim 8\mathrm{x}10^{-3}$ in the nonlinear run for the initial condition where $\delta B_{n=4}/B \sim 2\mathrm{x}10^{-2}$ in the linear run.
- The suppression effect arises through the change in the n=0 harmonics of the magnetic field ($\delta B_{n=0}$).
- The $\delta B_{n=0}$ is generated by $E_{n=0} = -v_{TAE} \mathrm{x} \delta B_{TAE}$.
- The n=0 velocity field relaxes $\delta B_{n=0}$ and also relaxes the suppression effect.

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