Max-Planck-Institut
für Plasmaphysik

Damping of Kinetic Shear Alfvén Modes in Tokamak Plasmas

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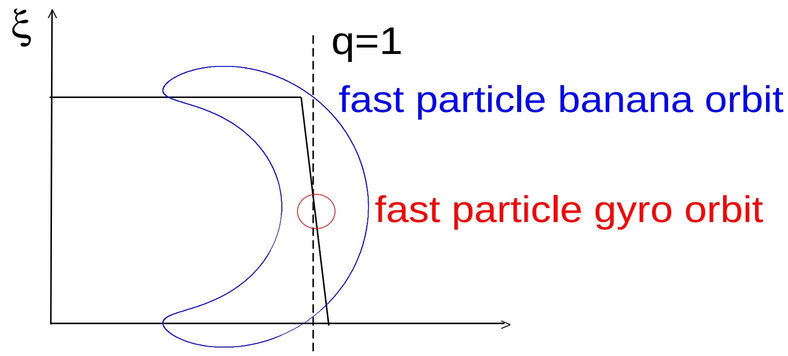
- Motivation & Model
- Numerical Tool: LIGKA
- TAE Damping Rates
- Cascade Modes
- Outlook

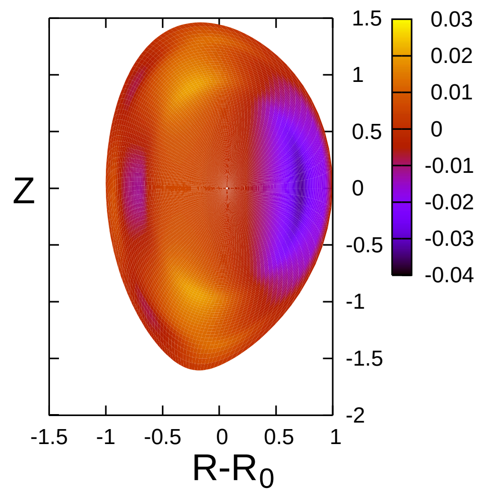
Problems under Investigation

- Multi-Scale-Length Physics:
  kinetic effects can modify large-scale MHD modes significantly when gyroradius/banana-orbit-width and mode-dimensions become comparable

- Energetic Particle Physics:
  large fast particle pressures in future fusion devices require self-consistent (non-perturbative, non-hybrid) treatment
  Related Physics Problem: growth and damping rates for fast-particle-driven TAE's (low-n, low-m)

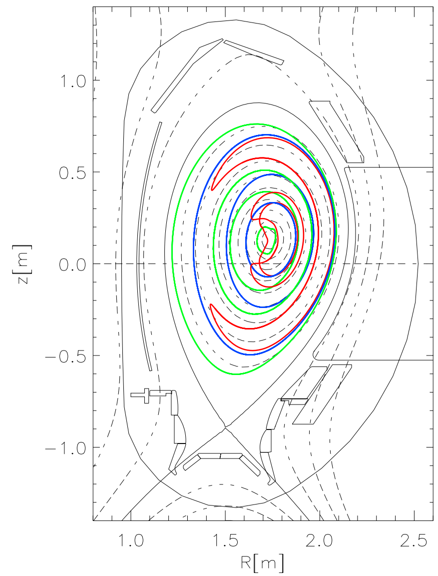
Eigenfunction: Internal Kink Mode

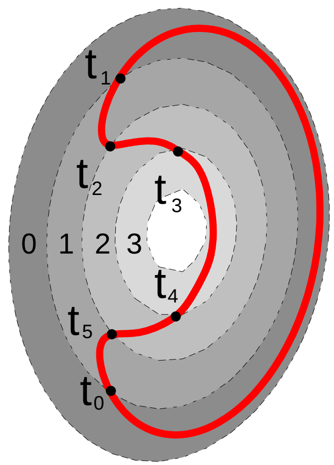




LIGKA: Linear Gyrokinetic Shear Alfvén Physics

- Linear: mode frequency, growth rate and mode structure
- Gyrokinetic: particle on gyroorbit feels perturbation due to the mode
- Non-hybrid: solving for Ampére's law (GKM) and quasi-neutrality equation simultaneously
- Non-perturbative: allowing for change in the eigenmode structure
- Accurate treatment of particle orbits: numerical integration over unperturbed particle orbits (HAGIS)
- Accurate treatment of non-zero banana orbit widths: radial coupling due to broad banana orbits
- Takes into account the Landau pole contributions for negative growth rates
- Realistic tokamak geometry (so far without X-point)





Linear Gyrokinetic Equations

gyrokinetic equation:

$$h = \frac{ie}{T} F_0 \sum_m \int_{-\infty}^{t} dt' e^{i[n(\varphi'-\varphi)-m(\theta'-\theta)-\omega(t'-t)]} e^{-im\theta} \cdot (\omega - \omega_*^T) J_0 \cdot [\phi_m(r') - (1 - \frac{\omega_d(\theta')}{\omega}) \psi_m(r')]$$

quasi-neutrality:

$$\sum_a e_a [\frac{e_a}{m_a} \nabla_\perp \frac{n_{a0}}{B^2} \nabla_\perp + \frac{3 e_a \varrho_a n_{a0}}{4 m_a^4} \nabla_\perp^4] \phi + \int J_0 f d^3\mathbf{v}] = 0$$

gyrokinetic moment equation:

$$-\frac{\partial}{\partial t} \left[\frac{c^2}{4\pi} \nabla \cdot (\frac{1}{v_A^2} \nabla_\perp \phi) \right] + \frac{c}{4\pi} (\mathbf{B} \cdot \nabla) \frac{\nabla \times \nabla \times \frac{c}{i\omega} (\nabla \psi)_\parallel}{B^2} + (\frac{c}{i\omega} \nabla (\nabla \psi)_\parallel \times \mathbf{b}) \cdot \nabla \frac{j_{0\parallel}}{B}$$

$$= - \sum_a \int d^3 v (e \mathbf{v}_d \cdot \nabla f)_a + \frac{3}{4} \frac{c}{v_A^2} \frac{e_a v_{th,a}^2}{\Omega_a^2} \nabla_\perp^4 \frac{\partial}{\partial t} \phi$$

Ideal MHD Limit and 'Reduced Kinetic Limit'

- Ideal MHD Limit: QN equation reduces to $\phi = \psi$
  system is reduced to shear Alfvén equation with singularity $\omega = k_{\|} v_A$
- 'Reduced Kinetic Limit':

$$\begin{aligned} \phi - \psi &= \hat{r}_{LT}^2 \nabla_\perp^2 \phi \\ \nabla_\perp \cdot \frac{\omega^2}{v_A^2} \nabla_\perp \phi + \frac{\partial}{\partial s} \nabla_\perp^2 \frac{\partial \psi}{\partial s} &= 0 \end{aligned}$$

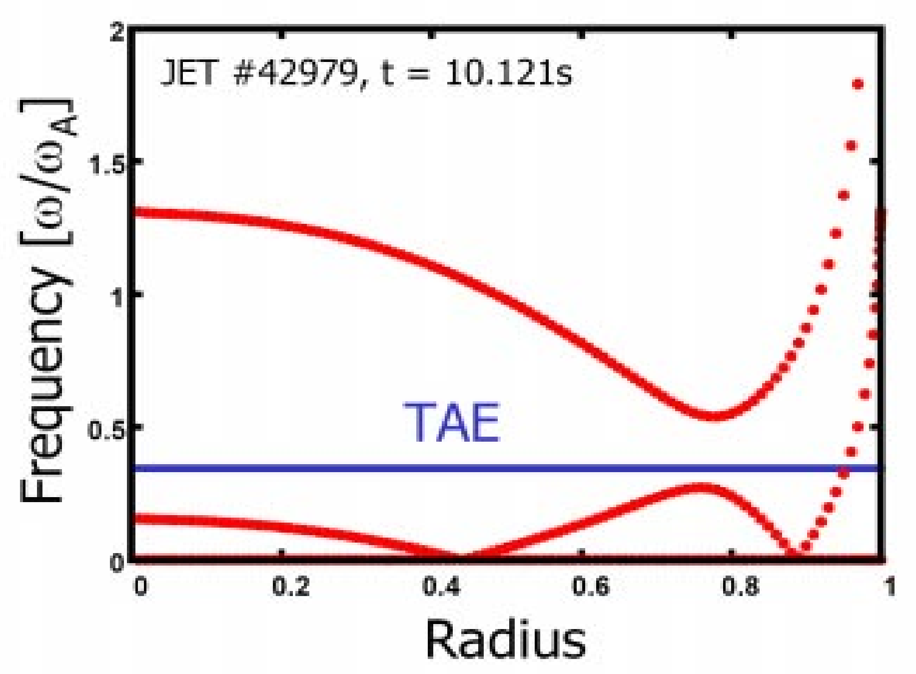
Can be combined into one single fourth order equation as used by Berk,Mett, Lindberg [Phys. Fluids B **5** (1993)] or Fu, Berk, Pletzer [PoP, **12** (2005)]:

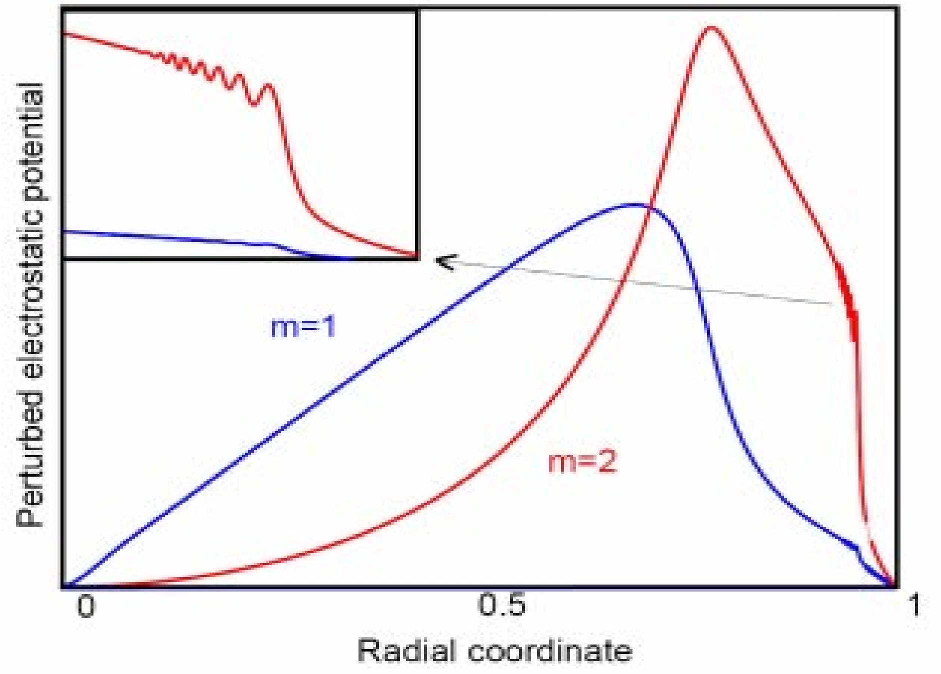
$$\nabla_\perp \cdot \frac{\omega^2}{v_A^2} \nabla_\perp \phi + \frac{\partial}{\partial s} \nabla_\perp^2 \frac{\partial \phi}{\partial s} = -r_{LT}^2 \frac{\omega^2}{\omega_A^2} \nabla_\perp^4 \phi$$

$$r_{LT}^2 = \frac{3}{4} \varrho_i^2 + \varrho_s^2 = \varrho_i^2 \left\{ \frac{3}{4} + \frac{T_e}{T_i} \frac{1 + i\hat{\nu} Z(\xi)}{1 + \xi Z(\xi)} \right\}$$

Coupling to the KAW

Singularity of the MHD operator is resolved by fourth order terms
Inwards propagating kinetic Alfvén wave is excited





‘Antenna-like’ version of LIGKA

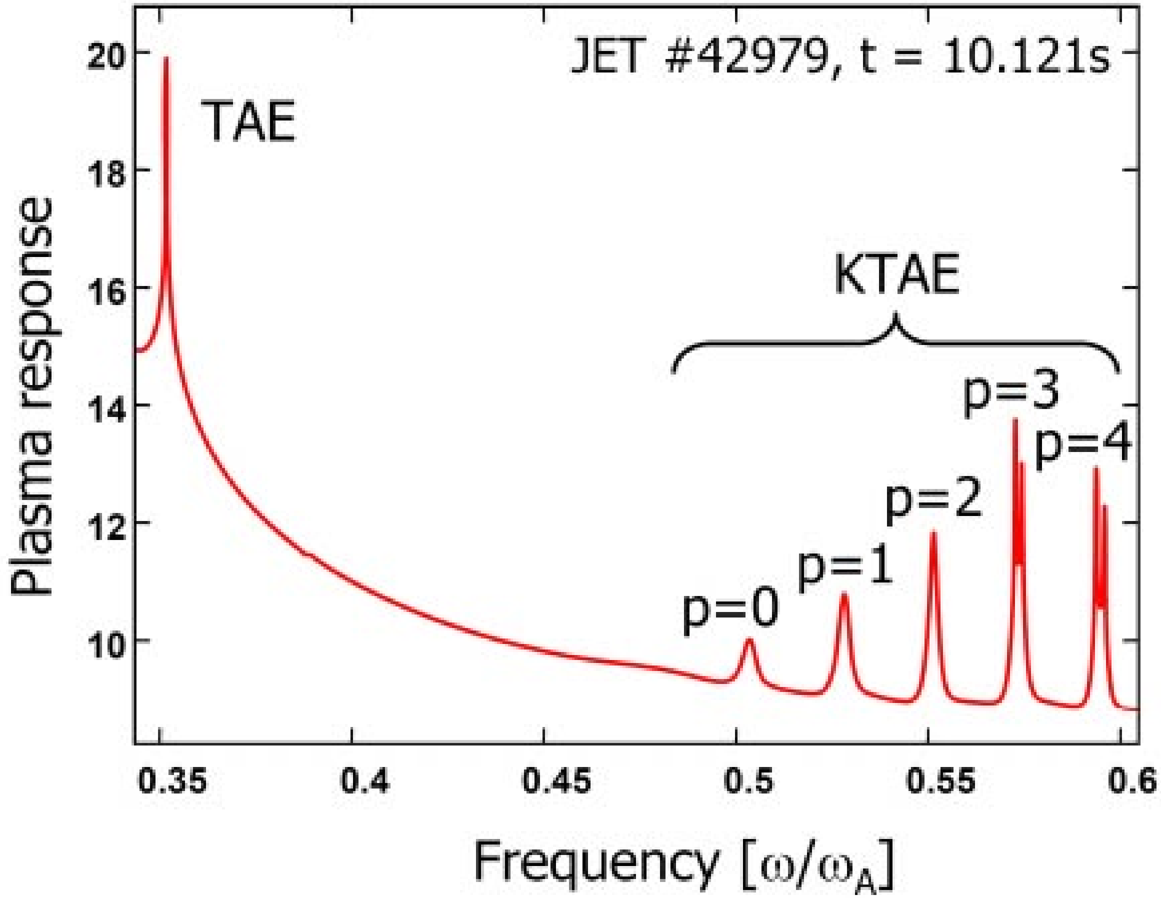
In order to find all modes around and in the gap:
drive vector d added artificially with d nonzero only at the plasma edge

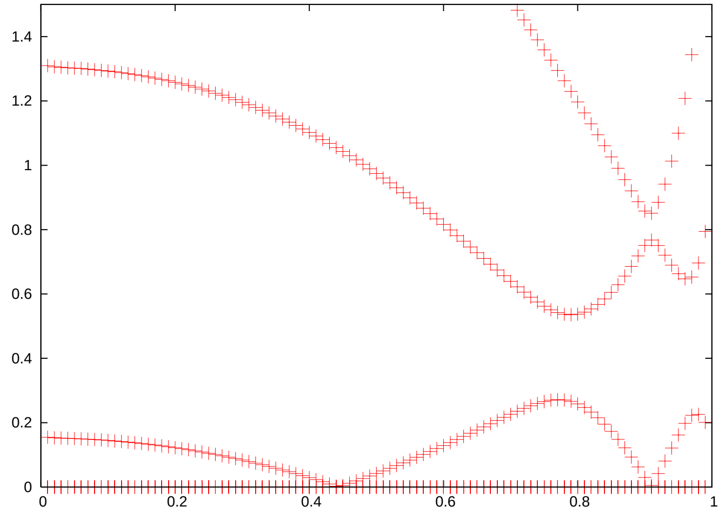
$$M(\omega)\begin{pmatrix}\phi\\ \psi\end{pmatrix}=\mathbf{d} \qquad \rightarrow \qquad \mathbf{I}\begin{pmatrix}\phi\\ \psi\end{pmatrix}=M(\omega)^{-1}\mathbf{d}$$

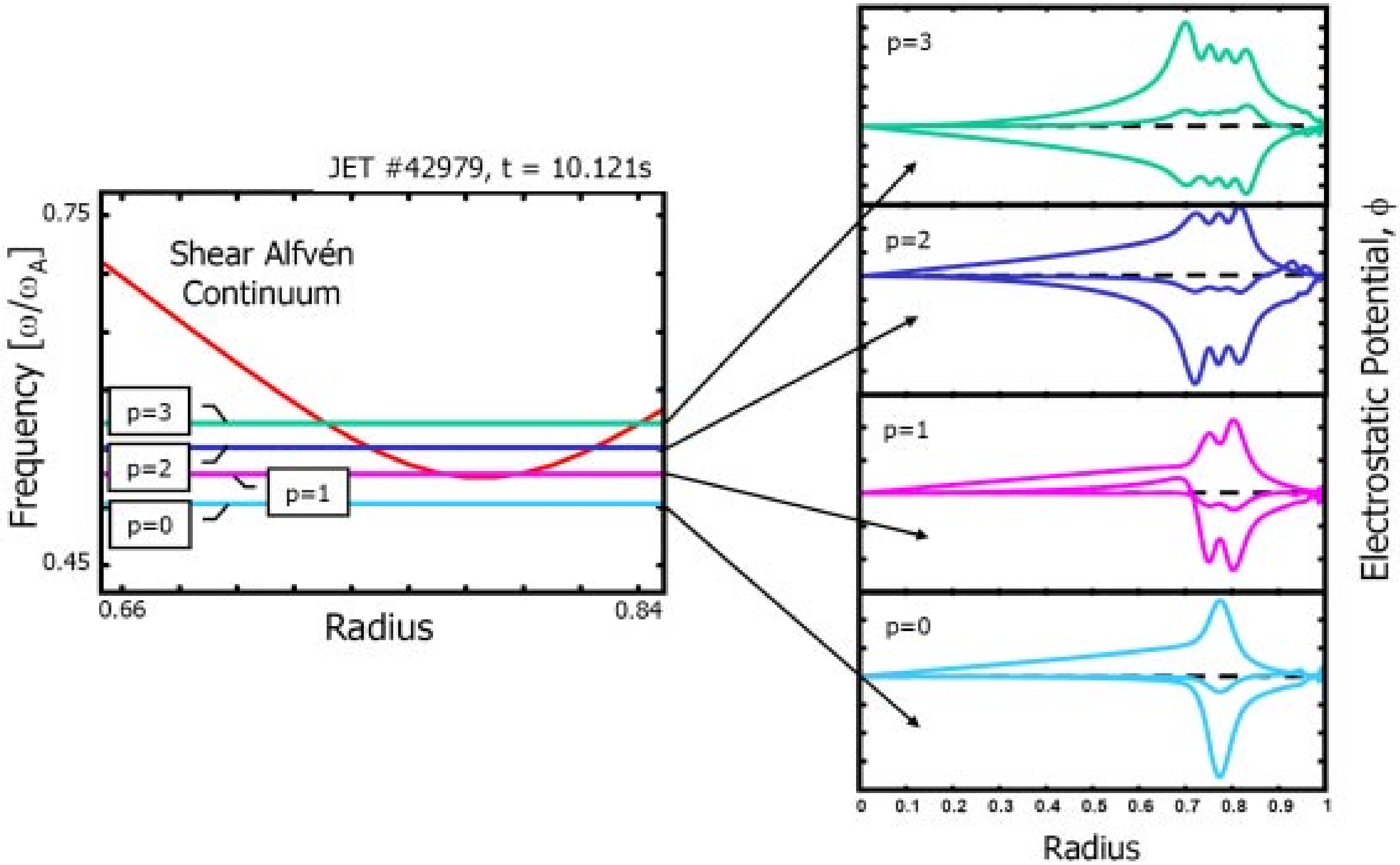
No vacuum region and also no proper boundary conditions (yet)
Plasma Response is measured by integral over Eigenfunction:

$$\mathcal{R}=\sum_m\int_0^a \phi_m\phi_m^* dr$$

Scan throughout the gap region







Two KAWs excited at the continuum intersections creating a standing wave: KTAE

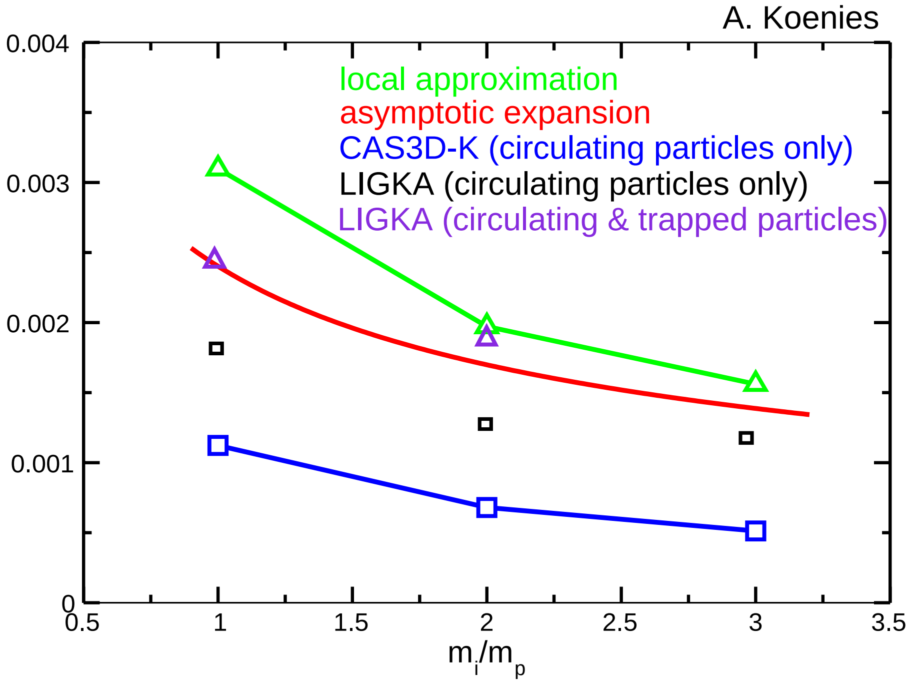
Damping Rates of Global TAE Modes

It is still an unresolved question what kind of damping mechanism is responsible for low (m,n) TAE modes:

- PENN: mode conversion of TAE into kinetic Alfvén wave (KAW) mainly in the plasma core
  measurements at JET agree with numerical prediction of PENN [Jaun,1999]
- fluid models predict a damping rate 25 times too small
  although the mass-scaling is predicted correctly $m^{-1/2}$
- CASTOR-K with FLR-effects (complex resistivity approximation [Connor, 1994]))
  mechanism: coupling of TAE to kinetic Alfvén wave (KAW) at the edge, also when gap is open
- Berk, IAEA 2004: model for TAEs in large aspect ratio plus FLR effects
  if gap is open (finite edge density), TAE is only weekly damped, no evidence of KAW conversion in centre
  otherwise strong damping due KAW coupling at the edge
- G. Fu, H.Berk, 2005: reduced kinetic model, no significant core damping found, damping rates up to 0.5%

TAE Damping Rates

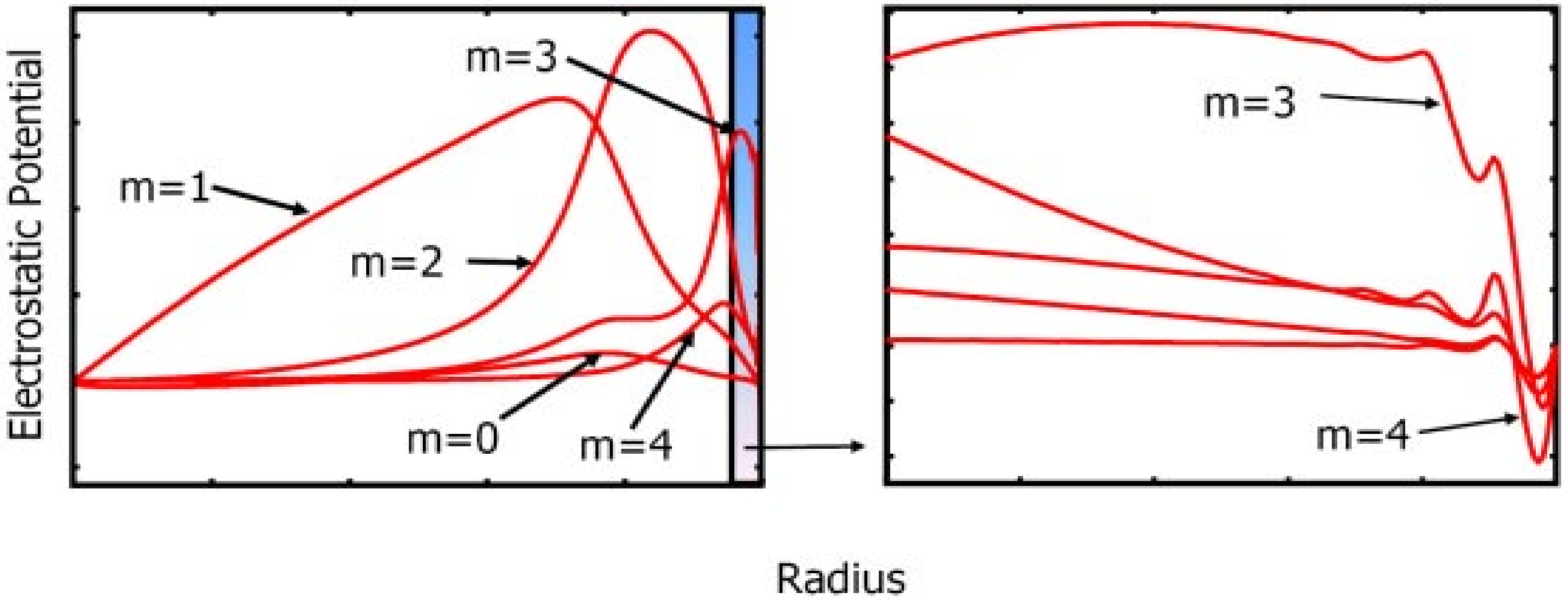
Benchmark with CAS3D-K for JET shot #42979@10.121s (A. Koenies, IPP Greifswald)
gyrokinetic terms turned off, electron Landau damping is main damping mechanism



remaining discrepancy due to $E_{\|}$-Effects

Damping Rates JET shot #42979@10.121s

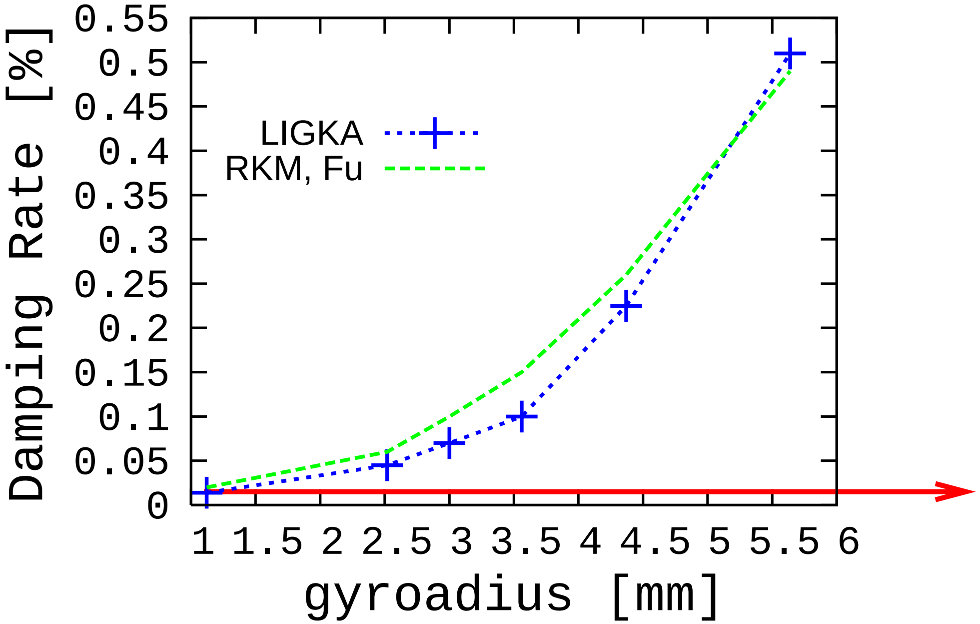
- with an open gap the calculated damping rates are about a factor of 8 too small [Fasoli et al.,(1999)]
- since the edge density profile measurements at JET are relatively poor, the gap could be closed: calculated damping rate: 0.70%

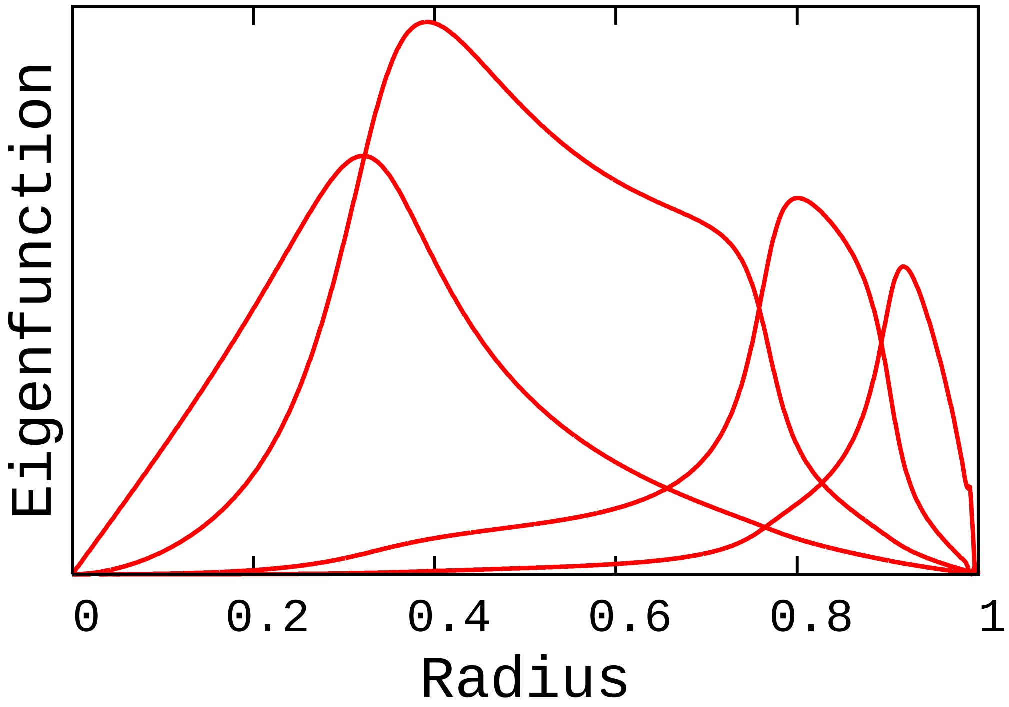


- no mode in the plasma centre is excited

Benchmark with Reduced Kinetic Model (G. Fu, PoP 12,2005):

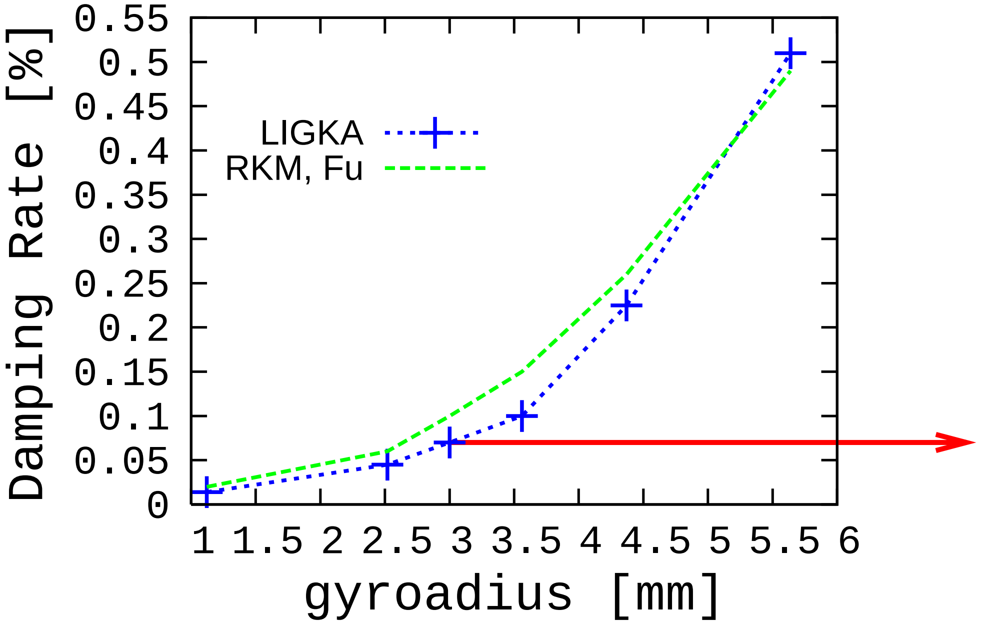
Based on JET Shot $\#38573@5.0s$
Very good agreement found! KAW 'tunnels into' the TAE

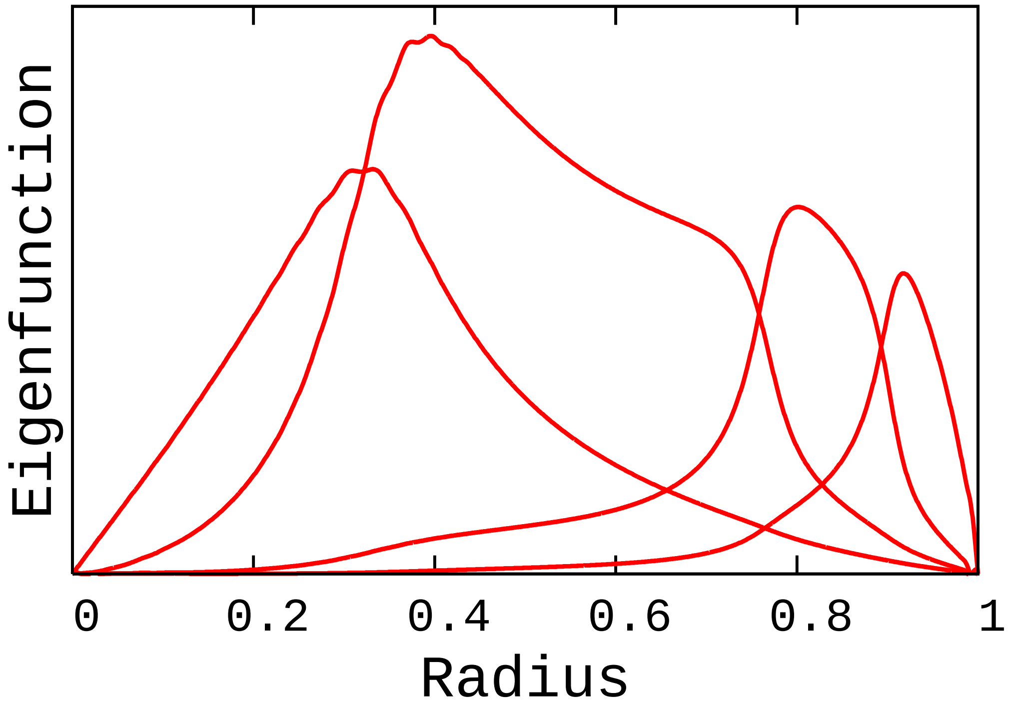




Benchmark with Reduced Kinetic Model (G. Fu):

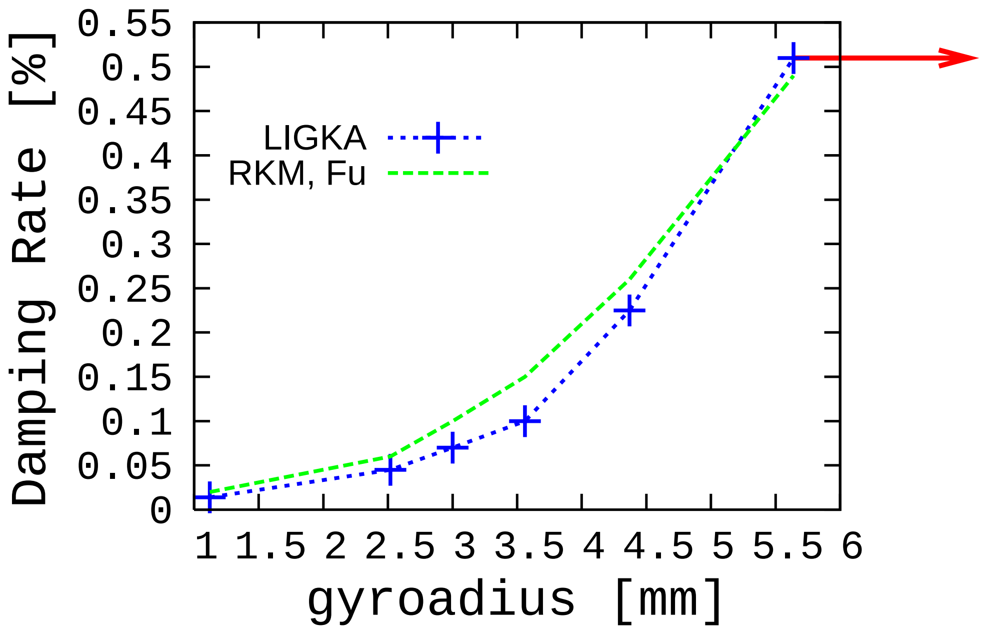
experimental values ($\varrho_i = 3\text{mm}$):
eigenmode structure starts to change compared to ideal case (perturbative treatment!)

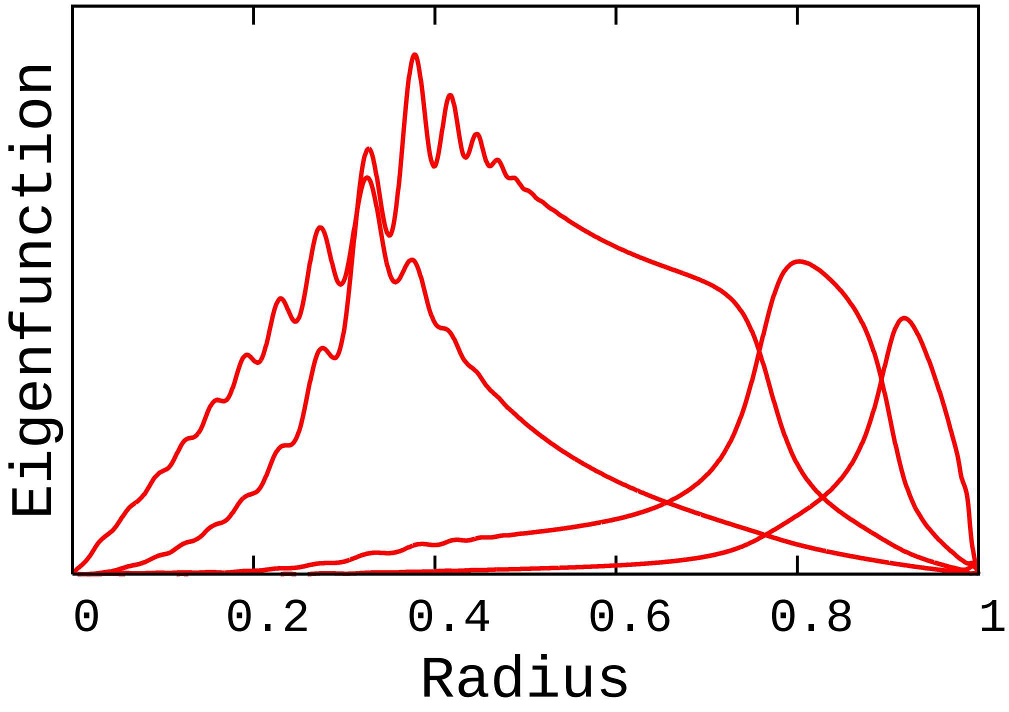




Benchmark with Reduced Kinetic Model (G. Fu):

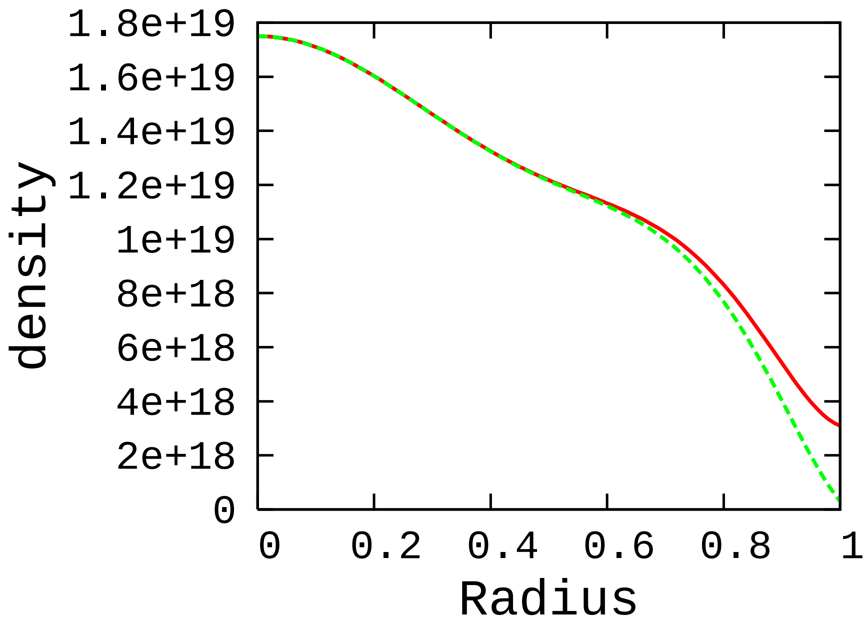
Significant changes in the eigenfunction:

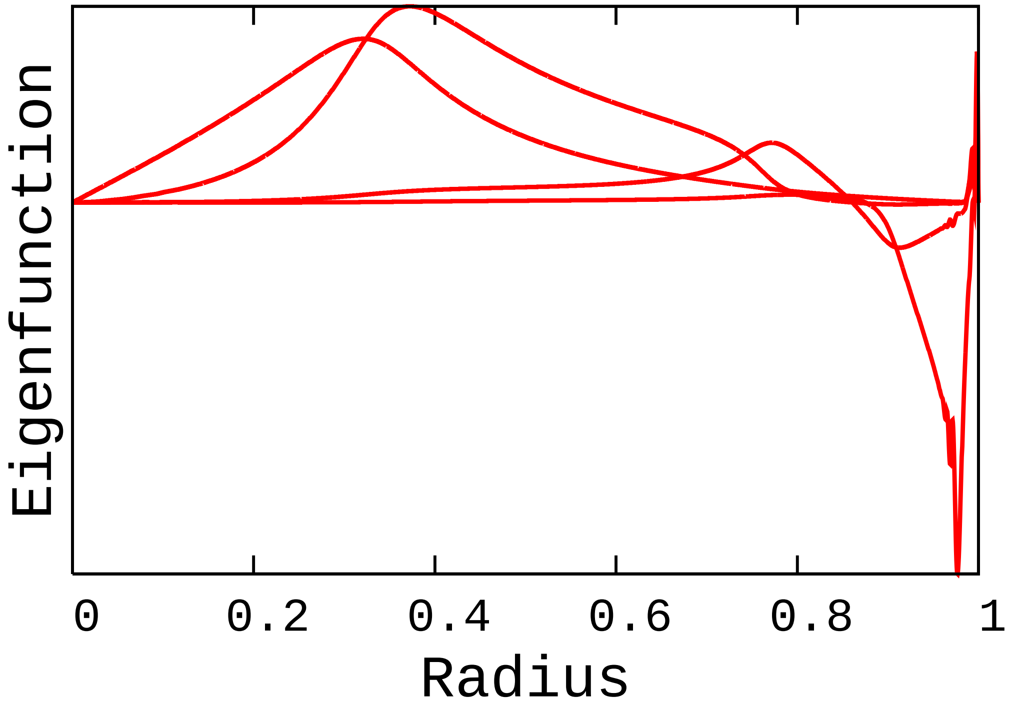




Benchmark with Reduced Kinetic Model (G. Fu):

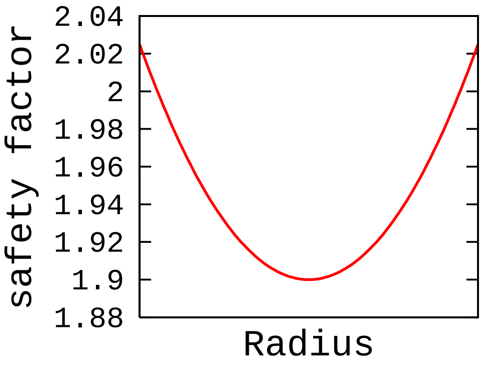
For a closed gap case (edge density very small) main damping comes from continuum damping at the edge.
Both codes agree relatively well: 0.5% (G.Fu) - 0.7% (LIGKA)

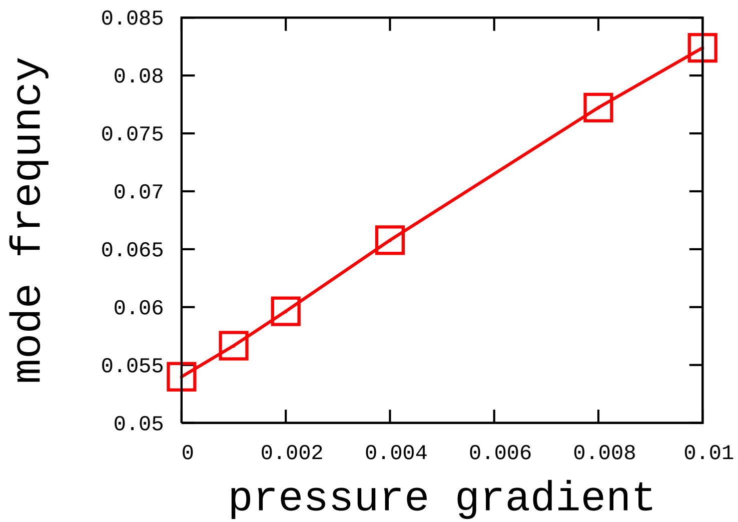


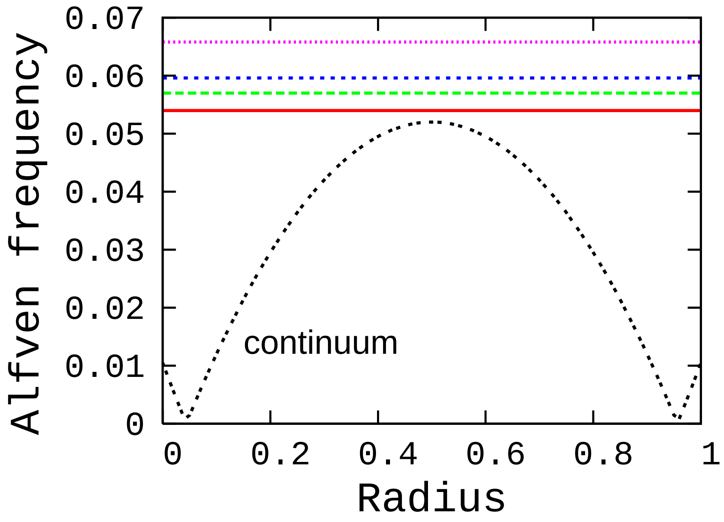


Finite β-Effect on Cascade Modes

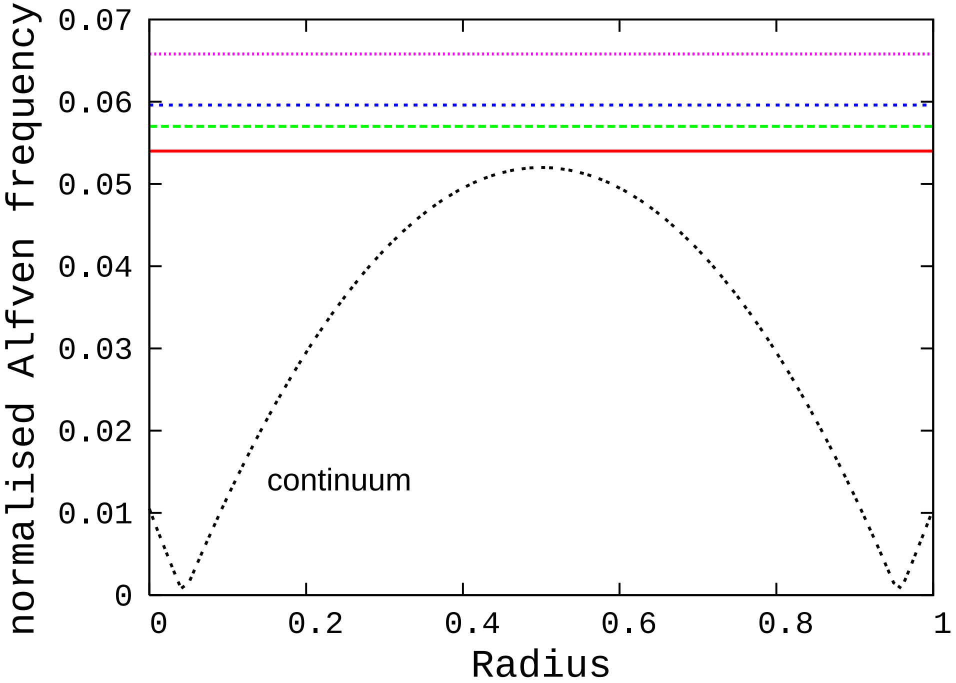
- first benchmark: simple model equilibrium: $q(s) = q_0 + 0.5q''(s - 0.5)^2$
- For an analytical shifted circle equilibrium, LIGKA finds that a larger pressure gradient helps the Cascade mode to exist
- agreement of LIGKA for a similar numerical equilibrium case with other codes (CASTOR)

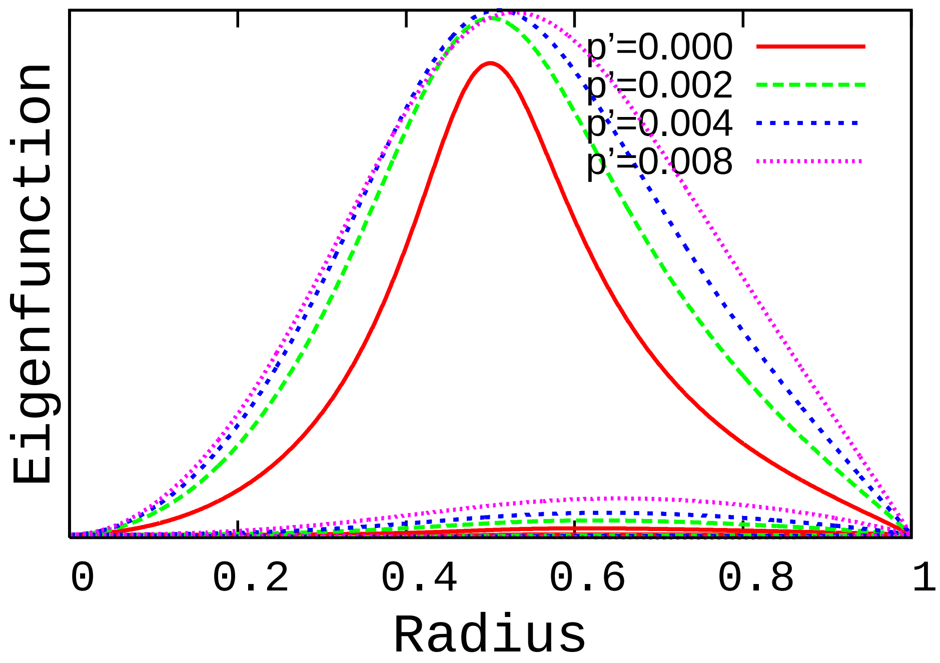






Cascade Modes: Eigenmode Structure





Here: radiative damping very small, because of small shear; no continuum damping

Conclusions:

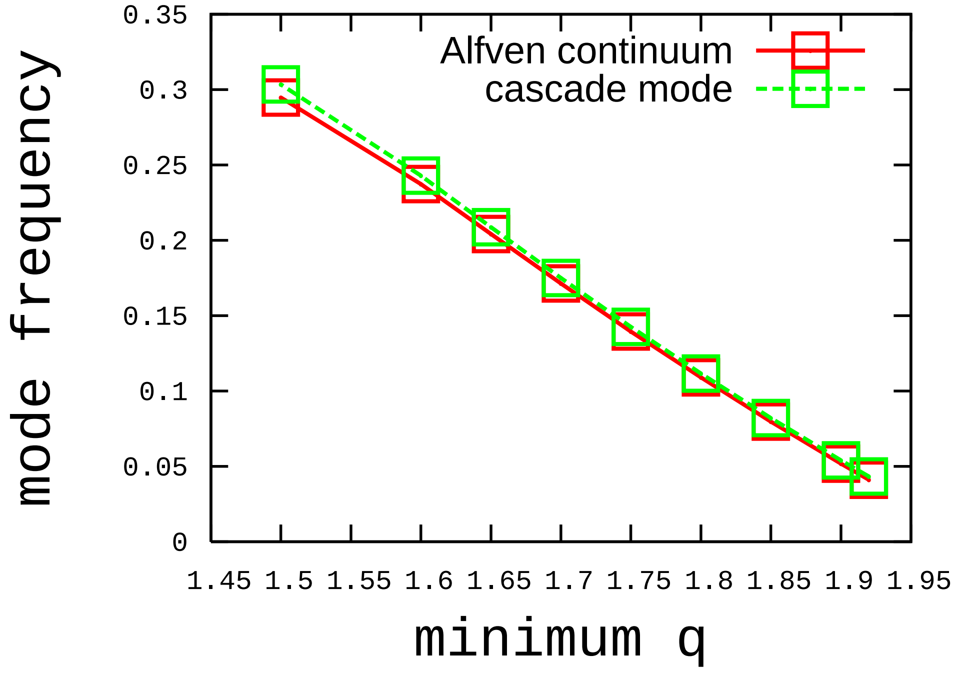
LIGKA is a comprehensive numerical model:

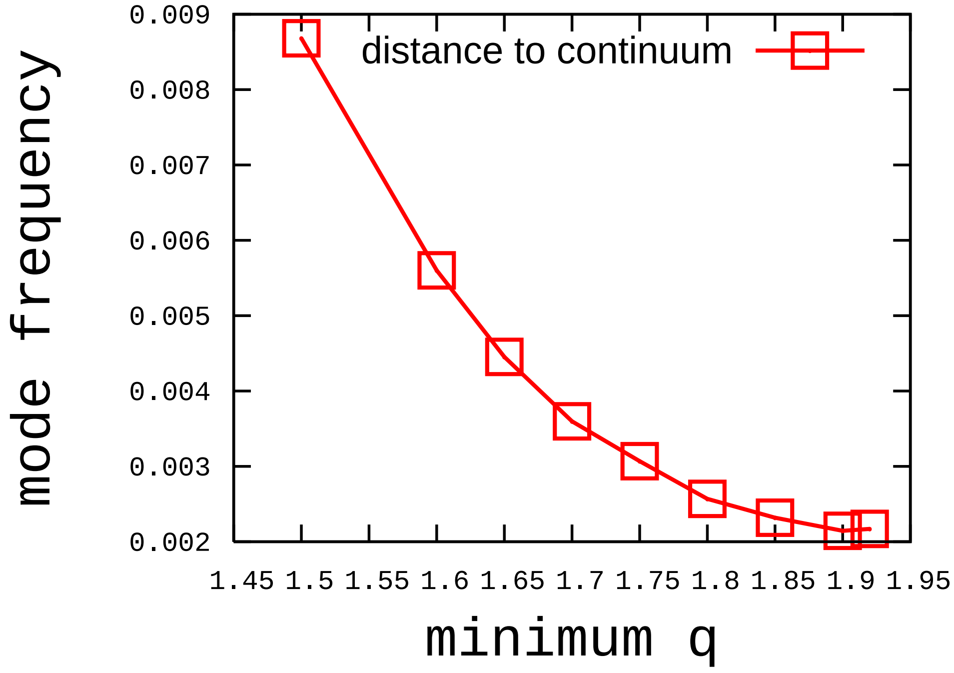
- non-perturbative, includes relevant damping mechanisms (except collisions), realistic tokamak geometry
- benchmarks for all important damping mechanisms successfully completed
- Analysis of JET results, no KAW in the centre found, higher (but still not high enough) damping rates (than perturbative code results) are found mainly due to continuum damping at the edge

Outlook:

- further model and code development:
  - collisions: simple model, collision operator
  - non-Maxwellian fast particle distribution function
- further benchmarking
- detailed investigation of edge damping effects (boundary conditions)
- internal kink mode physics
- extend to higher n modes (matrix reorganisation)
- non-ideal effects on cascade modes

Cascade Modes: Dependence on q_0





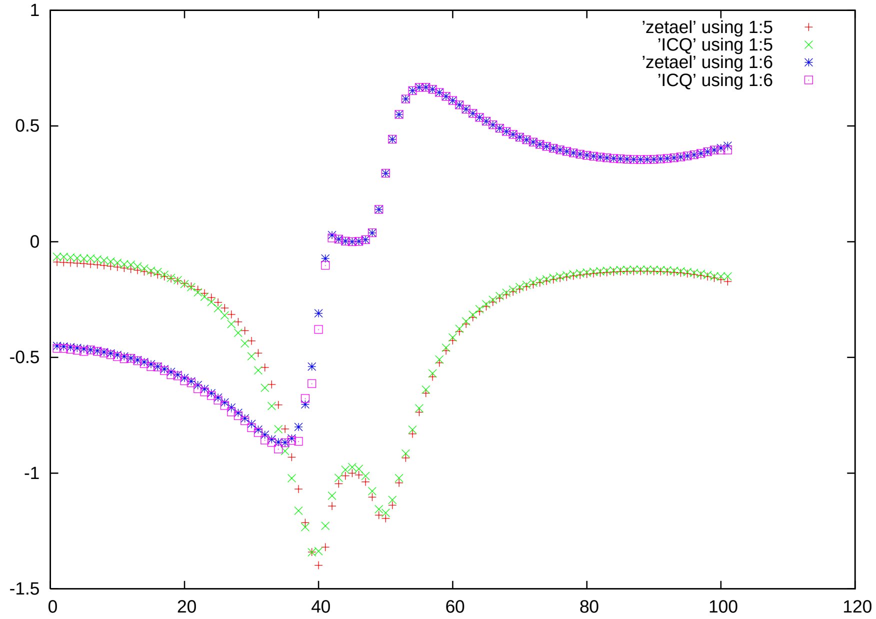
Cost and Computing Time

- time
  - Kinetic Integrals: 200 radial grid points x 2 variables x 2 equations x 2 Real,Complex x 2 trapped,passing x 2 particle species x m(5) x p(5) = 160000 Integrals
    Computing time (blade): 10 integrals per second per cpu
    $\longrightarrow$ 10 minutes on one 32 processor machine
    ca. 32-64 iterations necessary (Nyquist contour)
  - Eigenvalue problem: inverting a 4000 x 4000 sparse matrix: few minutes
- memory
  - Kinetic Integrals:
    Orbit integrals or trapped are calculated beforehand with HAGIS : $\approx$ 1 Gbyte per species
  - Eigenvalue problem:
    200 radial bins x 2 (Hermite polynomials) x 5 x 2 = 2000 x 2000 matrix $\leq$ 256 Mbytes

Numerical Resonance Denominator

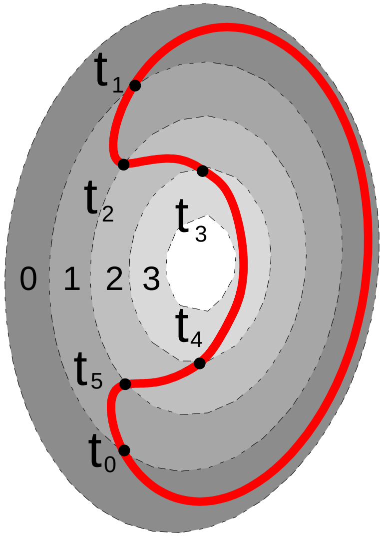
Thacher-Tukey algorithm for rational interpolation implemented in LIGKA
win of accuracy in several respects:

- taking into account exact bounce and drift frequencies (HAGIS), but still using constants of motion as integration variables
- taking into account exact position of pole in complex Λ-plane, no derivatives needed for calculating the residuum
- relatively simple form allows for frequent evaluations during Cauchy principal value calculation
- successful recovery of the analytical result $\xi Z(\xi)$ for the Landau problem
- grid refinement in both energy and pitch angle variable



Orbit Integrals: Trapped Ions

- linear problem: integration over unperturbed orbits $\Rightarrow$expanding perturbed potentials in finite elements before integration
- calculate time points of transit into neighbouring element with HAGIS: $t_j, \Delta t_j$
- $\sum$: partitioning of the periodic motion
- $\sum$: partitioning of one orbit in FE



$$\int_{-\infty}^{t} dt' e^{i\omega(t'-t)}\psi(r') = \sum_{\kappa=0}^{\infty} \int_{t-(\kappa+1)\tau_b}^{t-\kappa\tau_b} dt' e^{i\omega(t'-t)}\psi(r') =$$

$$\sum_{\kappa=0}^{\infty} \sum_{j=0}^{N-1} \int_{t-(\kappa+1)\tau_b+t_j}^{t-\kappa\tau_b+t_{j+1}-\tau_b} dt' e^{i\omega(t'-t)}\psi(r_j) = \frac{1}{i\omega(e^{i\omega\tau_b}-1)} \sum_{j=0}^{N-1} \left[e^{i\omega t_j}(e^{i\omega\Delta t_j}-1)\psi(r_j)\right]$$