Radio-Frequency Heating of Sloshing Ions in a Straight Field Line Mirror

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Abstract. Sloshing ions, the energetic ions with a velocity distribution concentrated to a certain pitch-angle, play an important role in plasma confinement in mirrors. They are normally produced in mirror traps with neutral beam injection. They also could be generated by ion-cyclotron heating. In the present report two radio-frequency heating scenarios to sustain a sloshing ion population in a newly proposed mirror device, the straight field line mirror, are examined. The first one consists in the ion cyclotron heating in two-ion species plasma using longitudinal wave conversion and fundamental harmonic heating of deuterium ions in tritium plasma. This scheme provides efficient ion heating for high deuterium "minority" concentration without substantial power deposition to the electrons. The second scenario is based on second harmonic heating of deuterium ions. The study uses numerical 3D calculations for the time-harmonic boundary problem for Maxwell's equations. For the radio-frequency heating in both schemes, a simple strap antenna is used. Calculations show that it has low antenna Q and operates in the regime of global resonance overlapping. For fundamental harmonic heating scenario only a small portion of the wave energy transits through the cyclotron layer and penetrates to the central part of the trap. The power deposition is peaked at the plasma core. The calculations show that this scenario is prospective for practical implementation in large mirror devices. First results of numerical calculations for second harmonic heating are reported.

A straight field line mirror [1,2] is a prospective device for fusion applications. Some beneficial properties with such a device is the MHD (magnetohydrodynamic) stability provided by a minimum B field, the minimum flux tube ellipticity and the local omnigenious property, with zero banana orbit widths, to the first order in the plasma β [3].

With a sloshing ion population, the straight field line mirror may serve as fusion reactor with relatively high Q factor (energy gain factor) [4]. Sloshing ions are the energetic ions with a velocity distribution concentrated to a certain pitch-angle, and they "slosh" between the magnetic mirrors where their concentration increases. The established technique to produce sloshing ions in mirror traps is neutral beam injection (see e.g.[5]). An alternative approach, although not yet tested experimentally with sufficient heating power [6], would be to sustain sloshing ions by radio frequency heating.

1. Fast wave propagation in mirrors and fundamental harmonic heating

The character of the fast wave propagation in open trap plasma is that along x and y, the directions perpendicular to the magnetic field, a fast wave normally form a standing wave structure with a low number of nodes, while in the longitudinal direction the number of oscillations is quite high (see e.g. Ref. [7]). We assume that the perpendicular node structure of the wavefield remains as the wave propagates along the plasma column [8]. Under this condition, the major features of the fast wave propagation could, to a certain extent, be described by the dispersion relation

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$$k_{\parallel}^2 = k_0^2 \varepsilon_{\perp} - k_{\perp}^2 / 2 \pm \sqrt{k_{\perp}^4 / 4 + k_0^4 g^2} \tag{1}$$

where $k_{\parallel}$ and $k_{\perp}$ are the parallel and perpendicular components of the wave vector, $k_0 = \omega / c$, $\varepsilon_{\perp} = \varepsilon_{11} = \varepsilon_{22}$, $g = -i\varepsilon_{12} = i\varepsilon_{21}$ and ε_{ik} is the cold plasma dielectric tensor. Following this formula, Fig.1 shows the dependence of the normalized parallel wavenumber $\bar{k}_{\parallel} = k_{\parallel} c / \omega_{pH}$ on the normalized magnetic field $\bar{B} = \omega_{cH} / \omega$ in a D-T (deuterium-tritium) plasma.

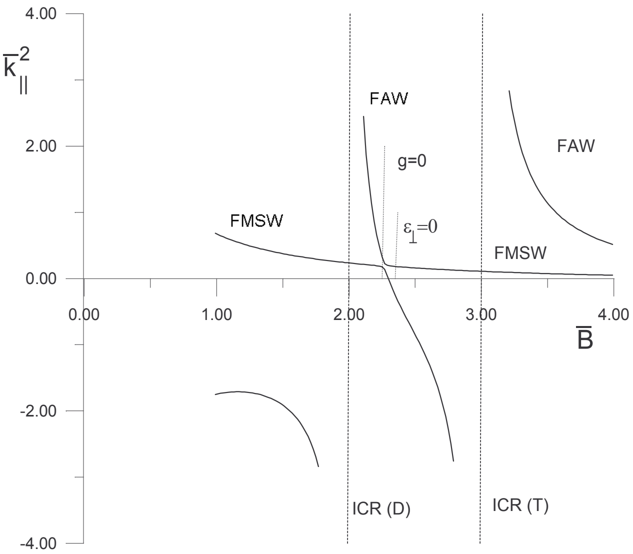

Figure 1 Normalized $\bar{k}_{\parallel}^2$ as a function of normalized magnetic field strength $\bar{B}$ for $\bar{k}_{\perp} = 0.3$.

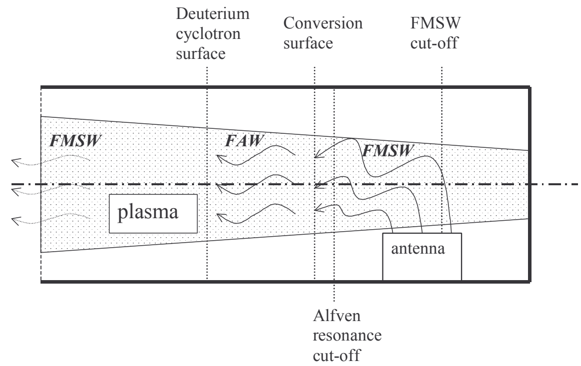

Figure 2 Sketch of rf heating scheme.

Based on (1), the following minority heating scenario is choosen: As sketched in Figure 2, a fast magnetosonic wave (FMSW) is excited by the antenna and launched towards lower magnetic field at a position where the magnetic field is lower than the resonant value for tritium ions and the real part of $\varepsilon_{\perp}$ is negative. This wave, traveling towards lower magnetic field, will convert to a fast Alfven wave (FAW). The FAW will reach the vicinity of the cyclotron zone for deuterium ions, where it will be damped by the cyclotron mechanism. As compared with the “magnetic beach” scheme, this approach has some advantages. The first is that it is easier to excite by the antenna the FMSW than the FAW, since there is less restriction for FMSW propagation in non-uniform plasma. Secondly, the antenna could be placed in the region where $\varepsilon_{\perp} < 0$, where it is easier to provide decoupling from the Alfvén resonances. It could be mentioned that the performance of this scheme could be decreased by reflections from the conversion zone and by over-barrier conversions. Numerical results for this fundamental frequency heating scheme are reported in Ref. [9].

The scenario for second harmonic heating is analogous. If $k_{\perp}$ is slowly varying along the mirror axis the effect of second harmonic cyclotron resonance could be modeled by the presence of minority with the half particle mass and the concntration $C_{\alpha 2} = 4 C_{\alpha} v_{T\alpha\perp}^2 / \omega_{H\alpha}^2$, where C_{α} is the concentration of the resonannt ion component. In such an approximation the second harmonic heating was studied in Ref. [7].

2. Numerical model

In the present study, electromagnetic calculations are used to analyze high-energy ion sustain in a reactor-scale straight field line mirror device. The magnetic field strength of that device is described by the following formula:

$$\vec{B} = B_0\left[\frac{\vec{e}_z}{1-z^2/c^2} - \vec{e}_x \frac{x}{c}\frac{1+z/c}{(1-z^2/c^2)^2} + \vec{e}_y \frac{y}{c}\frac{1-z/c}{(1-z^2/c^2)^2}\right], \quad (2)$$

where c is the half-distance between the left and right poles of the trap. At the poles at $z = \pm c$, the magnetic field strength goes to infinity. We consider a reactor-scale device with $c = 50$ m, $a = 150$ cm and $B_0 = 1.5$ T. Here, a is the plasma radius at the central plane where the magnetic surfaces have a circular cross-section. The trap confines deuterium and tritium sloshing ions with the mirror point at $B_m = 4B_0$ that corresponds to the coordinate $z/c = \sqrt{3}/2$. Following the scheme described above, the cyclotron resonance for deuterium should be located nearby this mirror point, and an estimate for the heating frequency is $\omega \approx 3\cdot 10^8$ s^{-1}. The computation domain is $z \in (0.82c, 0.9c)$, i.e. 41 m $< z <$ 45 m, which covers the deuterium cyclotron zone. Since the y component of magnetic field varies slowly, this variation may not seriously influence the rf heating and, we assume plasma and magnetic field uniformity along y.

To describe the transmission of rf power to sloshing ions, the boundary problem for Maxwell's equations

$$\nabla\times\nabla\times\left[\vec{E} - \vec{e}_{\parallel}(\vec{e}_{\parallel}\cdot\vec{E})\right] - k_0^2\hat{\varepsilon}\cdot\vec{E} = 4\pi i\omega\mu_0\vec{j}_{ext} \quad (3)$$

should be specified. Here $\vec{j}_{ext}$ is the external current, $\vec{e}_{\parallel} = \vec{B}/B$. The parallel electric field is neglected. The boundary condition at a metallic surface and at the left wall of the box at $z = 41$m are specified by $\vec{E}\times\vec{n} = 0$, $\frac{\partial}{\partial z}(\vec{E}\times\vec{e}_z) + ik_w(\vec{E}\times\vec{e}_z) = 0$, where $\vec{n}$ is the normal vector of the surface and k_w is a constant. The corrections caused by finite $k_{\parallel}$ are important only for the resonant part of the dielectric tensor $\hat{\varepsilon}$. This resonant component for ions is $\varepsilon_{++} = \vec{e}_+^{\,*}\cdot\hat{\varepsilon}\cdot\vec{e}_+$, where $\vec{e}_+ = (\vec{e}_\tau - i\vec{e}_y)/\sqrt{2}$ and $\vec{e}_\tau = \vec{e}_y \times \vec{e}_{\parallel}$, and we use the following expression:

$$\varepsilon_{++} = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega\left|k_{\parallel}v_{T\alpha\parallel}\right|}\left[F(\beta_\alpha) - \frac{i\sqrt{\pi}}{2}\exp(-\beta_\alpha^2)\right] \quad (4)$$

where $\beta_\alpha = (\omega - \omega_{c\alpha})/\left|k_{\parallel}v_{T\alpha\parallel}\right|$ and F is the Dawson's integral.

For second harmonic heating, the resonant correction to the dielectric tensor also contributes to ε_{++}, and it is not a function in space, but the differential operator (see e.g. [7])

$$\delta\varepsilon_{++}E_+ = \frac{\partial}{\partial r_+}\tilde{\varepsilon}_{2\alpha}\frac{\partial}{\partial r_-}E_+, \quad (5)$$

where $r_\pm = \tilde{x} \pm iy$, $\tilde{x}$ is a local coordinate perpendicular to the magnetic field $\nabla\tilde{x} \approx \vec{e}_\tau$ and

$$\tilde{\varepsilon}_{2\alpha} = \sum_\alpha \frac{4\omega_{p\alpha}^2 v_{T\alpha\perp}^2}{\omega |k_\parallel v_{T\alpha\parallel}| \omega_{H\alpha}^2} \left[F(\beta_{2\alpha}) - \frac{i\sqrt{\pi}}{2} \exp(-\beta_{2\alpha}^2) \right] \left[1 + (1 - 2\omega_{H\alpha}/\omega)(v_{T\alpha\perp}^2 / v_{T\parallel}^2 - 1) \right] \tag{6}$$

with $\beta_{2\alpha} = (\omega - 2\omega_{c\alpha}) / |k_\parallel v_{T\alpha\parallel}|$.

3. Results of calculations

We choose an antenna consisting of the strap elements oriented along the y axis. For good coupling to the plasma, the surface of the strap should be positioned as close as possible to the plasma surface. This determines the size of the strap in the x direction. As for the antenna positioning in the z direction, it would be better to place it near the wave cut-off where the wavefield comes out the plasma column to the maximum extent and where it is easier to couple it to the antenna. The antenna layout is shown in Figure 3.

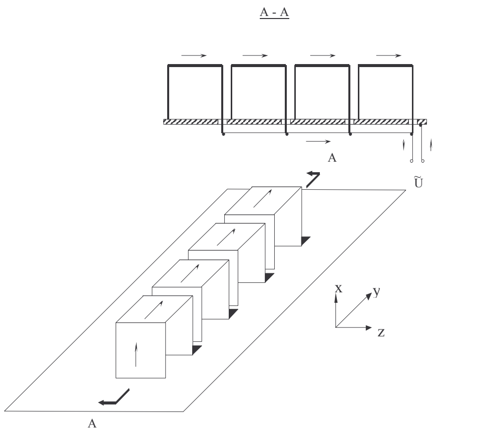

Figure 3 Strap antenna layout with the scheme of electric connection.

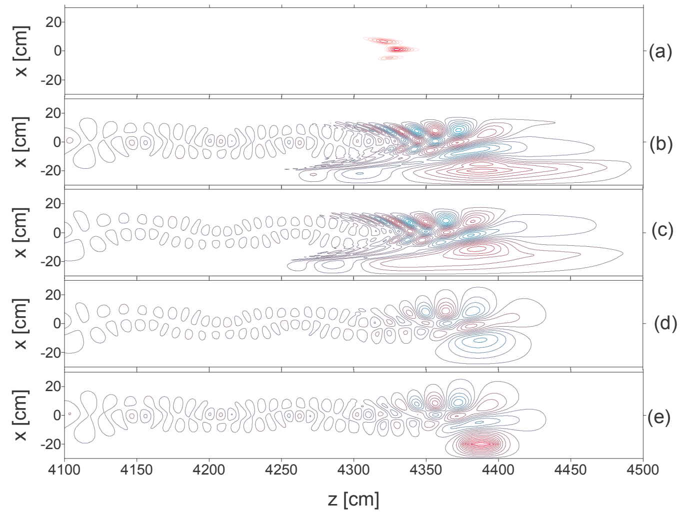

Figure 4 Contours of a) deposited power density b) $\mathrm{Re}\, E_\tau$, c) $\mathrm{Im}\, E_\tau$, d) $\mathrm{Re}\, E_y$ and e) $\mathrm{Im}\, E_y$ at the cross-section $y = 0$. Blue color corresponds to -0.5 V/cm and red is -0.5V/cm. Antenna current is 1A.

In the numerical calculations, the following set of parameters is chosen: Plasma density (in its maximum) is $n_{p0} = 5 \cdot 10^{14}\,\mathrm{cm}^{-3}$, heating frequency is $\omega = 2.5 \cdot 10^8\,\mathrm{s}^{-1}$, deuterium and tritium parallel thermal velocities at z-axis are $v_{TD\parallel} = v_{TT\parallel} = 5 \cdot 10^5$ m/s, the deuterium concentration is $C_D = 0.4$, $k_\parallel = 0.4\,\mathrm{cm}^{-1}$ and $k_w = 0.1$ cm^{-1}. For this set of parameters the electromagnetic field patterns are presented in Figure 4. In spite that the antenna currents are assumed real, both the real and the imaginary parts of the fields amplitude are of the same order. In accordance to the antenna current shape, the electromagnetic fields do not form an oscillational structure in the y direction. In the x direction, the wavefield has two nodes which indicates the dominance of the third radial mode. A FMSW wavefield near the antenna location ($z = 4350...4400$ cm) can be identified. As z decreases the amplitude of the E_y field component decreases having almost a FMSW structure while the E_τ component does not decrease up to the zone of strong cyclotron damping. Its structure near the tritium cyclotron resonance could be qualified as a FAW structure. The front of the FAW is convex. This could be explained by the lower deuterium cyclotron zone width at the plasma edge where the FAW approaches closer to the ion cyclotron resonance surface. A slight excitation of Alfvén resonances is visible at the plasma periphery.

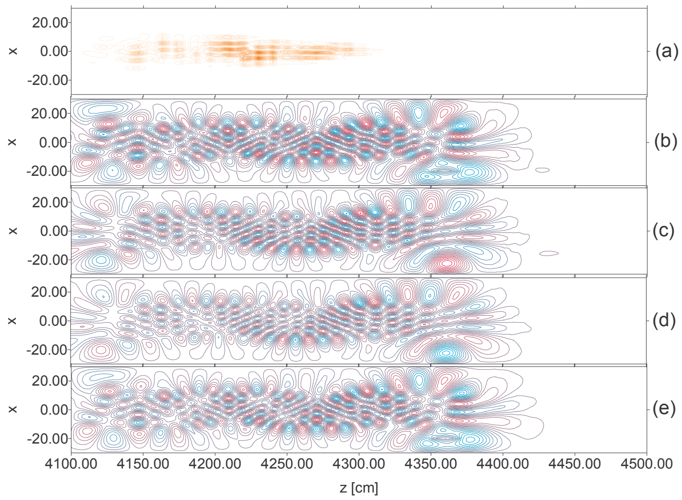

Figure 5 Contours of a) deposited power density, b) $\mathrm{Re}\,E_\tau$ *, c)* $\mathrm{Im}\,E_\tau$ *, d)* $\mathrm{Re}\,E_y$ *and e)* $\mathrm{Im}\,E_y$ *at the cross-section* $z = z_a$ *. Blue color corresponds to -0.5 V/cm and red is 0.5 V/cm. Antenna current is 1A.*

For second harmonic tritium heating calculations, the parameters are the following: Plasma density (in its maximum) is $n_{p0} = 5 \cdot 10^{14}$ cm^{-3}, heating frequency is $\omega = 3.1 \cdot 10^8$ s^{-1}, deuterium and tritium thermal velocities at z-axis are $v_{TD\|} = v_{TT\|} = 5 \cdot 10^5$ m/s and $v_{TD\perp} = v_{TD\perp} = 1.5 \cdot 10^6$ m/s, the deuterium concentration is $C_D = 0.6$, $k_\| = 0.4$ cm^{-1} and $k_w = 0.2$ cm^{-1}. The result of calculation is shown in Figure 5. In comparison with the minority heating, the power deposition zone is broader. For this specific calculation, cyclotron zone shine-through is about 18% in power, what is larger than for the minority heating, but within the range of acceptance.

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