Non-Conventional Fishbone Instabilities Driven by Circulating Energetic Ions

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A new kind of fishbone instability associated with circulating energetic ions is predicted. The considered instability is essentially the energetic particle mode and arises in plasmas with on-axis safety factor q_0 close to unity and extended shear-free central core, separated from the wall by a region with finite shear. The frequency of this "quasi-interchange" fishbone mode is $\omega \sim k_{0||}v_\alpha$ with $k_{0||}$ the parallel wave number in the shear-free core and v_α the injection velocity of energetic ions. "Infernal" fishbone modes with the same properties, but $m/n > 1$ with $m(n)$ the poloidal (toroidal) mode number, are investigated. A possibility to explain recent experimental observations of the $m=2$ fishbone oscillations accompanied by strong changes of the neutron emission during tangential neutral beam injection in the National Spherical Torus Experiment is shown.

1. Introduction

Some tokamak discharges are characterized by safety factor q close to unity in a wide region in the plasma core, which is separated from the wall by the edge region with large magnetic shear. This is the case for the "hybrid" regime, which is considered as a third operational scenario for ITER [1]. Furthermore, q-profiles with extended flat region and q_0 close to low-order rational are typical for high beta discharges in spherical tokamaks [2]. Since all mentioned discharges are accompanied by strong neutral beam injection (NBI), the problem of kinetic stability of such low-shear configurations in the presence of energetic ions represents considerable interest, which motivated present work.

In the framework of ideal MHD stability theory equilibrium with this type of q-profile is susceptible to a pressure-driven "infernal" modes [3,4]. A particular case of these instabilities is the quasi-interchange (QI) mode with poloidal and toroidal mode numbers $m=n=1$ [5-7]. The eigenfunction of this instability is of convective, or "cellular" character, in contrast with rigid kink displacement in the finite shear case. In the present work we show that this property, combined with finite orbit width of energetic ions, leads to the new kind of the fishbone mode with characteristic frequency of the Cherenkov resonance in the shear-free core, $\omega \sim k_{0||}v_\alpha$ with $k_{0||}=(m-nq_0)/q_0R$ and v_α injection velocity.

The purpose of the present work is to extend the ideal MHD theory of the QI mode in toroidal plasmas, developed in [6,7], to equilibrium with minor population of energetic ions. In the next section dispersion relation for the QI fishbone is derived and analyzed. The case of arbitrary (m, n) is considered in Sec.3. Section 4 is a short summary.

2. Dispersion Relation for the QI Fishbone Mode

The eigenmode equations for the QI fishbone can be obtained from the minimization of the total energy of the perturbation

$$E=\frac{R_0}{\pi^2 B_0^2}(\delta W_{MHD}+\delta W_k)-\frac{\omega^2}{\omega_A^2}N, \tag{1}$$

where δW_{MHD} is the ideal MHD potential energy [6,7], $\omega_A=V_A/R_0$ with $V_A(r)\approx$ const the Alfven speed and R_0 the major radius of the torus,

$$N=\frac{1}{2\pi^2 R_0}\int d^3r\,|\vec{\xi}_\perp|^2 \tag{2}$$

with $\vec{\xi}_\perp$ the transverse displacement, and δW_k is the kinetic part of the potential energy, which encapsulates information regarding resonant energy exchange between energetic ions and fishbone mode [8]

$$\delta W_k=\frac{1}{2}\int d^3r\vec{\xi}_\perp^*\bullet\nabla\Pi_\alpha^k=-\frac{\pi^2 m_\alpha}{\omega_{c\alpha}}\sum_{\sigma}\int v^3dv\int dP_\varphi\int d\Lambda\tau_b\frac{\partial F_\alpha}{\partial \mathrm{E}}\frac{\omega-\omega_{*\alpha}}{\omega-k_\| v_\|}\left|\left\langle\begin{matrix}\left(\frac{v_\perp^2}{2}+v_\|^2\right)\vec{\xi}_\perp\bullet\vec{\kappa}\times\\ \exp[i(\omega-k_\| v_\|)t]\end{matrix}\right\rangle\right|^2,$$

where $\delta\Pi_\alpha^k=\delta p_{\perp\alpha}^k\hat{I}+(\delta p_{\|\alpha}^k-\delta p_{\perp\alpha}^k)\vec{b}\vec{b}$ is the pressure tensor with $\delta p_{\|,\perp\alpha}^k$ the parallel and perpendicular pressure perturbations associated with non-adiabatic response of the energetic ions, $\sigma=v_\|/|v_\||$, $\Lambda=\mu_g B_0/\mathrm{E}$ with $\mathrm{E}\,(\mu_g)$ the particle energy (magnetic moment), τ_b is the particle transit time, $\omega_{*\alpha}=(\partial F_\alpha/\partial \mathrm{P}_\varphi)(\partial F_\alpha/\partial \mathrm{E})^{-1}$, $\vec{\kappa}$ is magnetic field curvature, and $\langle\ldots\rangle$ denotes orbit averaging.

Below we assume that the energetic-ion population consists of well-circulating particles with the equilibrium distribution function given by

$$F_\alpha=\frac{\sqrt{2}m_\alpha^{3/2}}{\pi\mathrm{E}_\alpha}p_\alpha(\bar{r})H(\mathrm{E}_\alpha-\mathrm{E})\mathrm{E}^{-3/2}\delta(\Lambda)\quad, \tag{3}$$

where $p_\alpha(\bar{r})$ is the beam particle pressure and *H(x)* is the unit step function. Furthermore, it is assumed that energetic ions are deposited in the shear-free core, that is $p_\alpha(\bar{r}>r_0)\approx 0$, where r_0 is the radius at the interface between shear-free and finite shear regions.

Omitting term odd in θ in $\vec{\xi}_\perp\bullet\vec{\kappa}$, which does not contribute to δW_k, we obtain

$$\vec{\xi}_\perp\bullet\vec{\kappa}=-\frac{1}{R_0}\xi_1\{r[\theta(t)]\}\cos[\theta(t)]\exp\{i[\theta(t)-\varphi(t)-\omega t]\}\quad, \tag{4}$$

where ξ_1 is the amplitude of the *m=1* radial displacement,

$$r[\theta(t)]=\bar{r}-\Delta_\alpha\cos[\theta(t)]\,,\ \Delta_\alpha=\frac{q(\bar{r})}{v_\|\omega_{c\alpha}}\left(\frac{v_\perp^2}{2}+v_\|^2\right),\ \theta(t)=\frac{v_\|}{q(\bar{r})R_0}t\,,\ \varphi(t)=\frac{v_\|}{R_0}t\quad.$$

We now assume $\Delta_\alpha<<\bar{r}$ and Taylor expand $\xi_1[r(\theta)]$ in Eq.(4). Substituting result to expression for δW_k together with Eq.(3), taking orbit average and velocity space integration, one can obtain

$$\frac{R_0}{\pi^2 B_0^2}\delta W_k=-\frac{2}{\pi^2}\rho_\alpha^3 R_0 F\left(\frac{\omega}{k_{0\|}v_\alpha}\right)\left|\frac{d\xi_1}{dr}\right|^2\frac{d\beta_\alpha}{dr}\quad, \tag{5}$$

where $\rho_\alpha=v_\alpha/\omega_{c\alpha}$,

$$F(\Omega) \equiv \frac{1}{5} + \frac{\Omega}{4} + \frac{\Omega^2}{3} + \frac{\Omega^3}{2} + \Omega^4 + \Omega^5 \ln\left(1 - \frac{1}{\Omega}\right) \quad , \tag{6}$$

and $\omega << \omega_{*\alpha}$ has been assumed. Note that it is the quasi-interchange character of the mode ($d\xi_1/dr \neq 0$ in the shear-free core), which allows for efficient power transfer at the Cherenkov resonance in this case. This is in contrast with rigid internal kink, when only particles crossing *q=1* surface in the course of their drift motion contribute to the non-adiabatic response [9].

For simplicity below we take radial fast ion distribution in the form $\beta_\alpha = \beta_{\alpha 0}[1-(r/r_0)^4]$. Rescaling the variable *r/a→r* with *a* the plasma radius, one can obtain from minimization of energy in Eq.(1) the following Euler equations in the shear-free core:

$$\frac{d}{dr}\left\{\varepsilon^{-2}\left[(\mu-1)^2 + l_\alpha(\omega,r) - \frac{\omega^2}{\omega_A^2}\right] r^3 \frac{d\xi_1}{dr}\right\} - 4\left(\frac{r}{4}\beta_p'' + \beta_p\right)^2 r^3 \xi_1 = \left(\frac{r}{4}\beta_p' + \beta_p\right)\frac{d}{dr}(r^3 \hat{\xi}_2) \quad , \tag{7}$$

$$\frac{d}{dr}\left(r^3 \frac{d\hat{\xi}_2}{dr}\right) - 3r\hat{\xi}_2 = -4r^3 \frac{d}{dr}\left[\left(\frac{r}{4}\beta_p' + \beta_p\right)\xi_1\right] \quad , \tag{8}$$

where $\mu=1/q$ with $|\mu-1| \sim \varepsilon$ in the shear-free core, $\xi_2 \equiv \varepsilon \hat{\xi}_2$ with ξ_2 the amplitude of the *m=2* radial displacement and $\varepsilon \equiv a/R_0$, $\beta_p(r) = -(8\pi R_0^2 / r^4 B_0^2)\int_0^r \hat{r}^2 (dp_c/d\hat{r}) d\hat{r}$ with p_c the background plasma pressure, and

$$l_\alpha(\omega,r) \equiv \frac{8}{\pi^2}\frac{\rho_\alpha^3 R_0}{r_0^4}\beta_{\alpha 0} F\left(\frac{\omega}{k_{0\parallel} v_\alpha}\right) H(r_0 - r) \quad . \tag{9}$$

The general solution of Eq.(8), which is regular on axis, has the form

$$\hat{\xi}_2 = r^{-3}\int_0^r \hat{r}^4 \beta_p(\hat{r}) \frac{d\xi_1}{d\hat{r}} d\hat{r} + [e - \beta_p(r)\xi_1(r)] r \quad , \tag{10}$$

where *e* is an integration constant. Substituting Eq.(10) into Eq.(7) and integrating, one can obtain

$$\frac{d\xi_1}{dr} = \frac{\varepsilon^2 e r \beta_p}{(\mu-1)^2 + l_\alpha(\omega,r) - \omega^2/\omega_A^2} \quad . \tag{11}$$

Dispersion relation can be obtained by matching the solution in the inner (shear-free) to the solution in the outer (sheared) region. In the latter region toroidal coupling in equation for ξ_2 can be neglected, since $|\mu-1| \sim 1$ and therefore $\xi_1 \sim \varepsilon^2$ [see Eq.(11)]. Equation for ξ_2 in the sheared region then takes the form

$$\frac{d}{dr}\left[\left(\mu - \frac{1}{2}\right)^2 r^3 \frac{d\hat{\xi}_2}{dr}\right] - 3\left(\mu - \frac{1}{2}\right)^2 r\hat{\xi}_2 = 0 \quad , \tag{12}$$

which has the asymptotic solution in the shear-free region

$$\hat{\xi}_2 \propto \frac{r}{r_2} + \sigma\left(\frac{r}{r_2}\right)^{-3} \quad , \tag{13}$$

where $\mu(r_2)=1/2$ and constant σ should be determined numerically by integrating Eq.(12) through the outer region. Matching Eq.(13) with asymptotic form of Eq.(10) in the outer region, we obtain dispersion relation

$$\sigma = \left(\frac{r_2}{a}\right)^2 \int_0^a \frac{[\varepsilon \beta_p(r)]^2}{(\mu-1)^2 + l_\alpha(\omega,r) - \omega^2/\omega_A^2} \left(\frac{r}{r_2}\right)^5 \frac{dr}{r_2} \quad , \tag{14}$$

where dimensions have been restored. Note that the integrand in Eq.(14) has the pole at the Alfven resonance in the outer region. The residue at this pole represents continuum damping of the fishbone mode. Away from the resonance the integrand becomes negligible in the sheared region. Thus, we can rewrite Eq.(14) in the form

$$\sigma = \left(\frac{r_2}{a}\right)^2 \frac{\varepsilon^2}{(\mu_0-1)^2 + l_\alpha(\omega) - \omega^2/\omega_A^2} \int_0^{r_0} \beta_p^2(r) \left(\frac{r}{r_2}\right)^5 \frac{dr}{r_2} + \sigma_{res}(\omega) \quad , \tag{15}$$

where

$$\sigma_{es}(\omega) = \left(\frac{r}{r_2}\right)^2 \varepsilon^2 \beta_p^2(r_A) \left(\frac{r_A}{r_2}\right)^5 \lim_{\eta\to 0} \int_{r_A-0}^{r_A+0} \frac{1}{(\mu-1)^2 - (\omega+i\eta)^2/\omega_A^2} \frac{dr}{r_2} =$$

$$i\pi \left(\frac{r_2}{a}\right)^2 [\varepsilon \beta_p(r_A)]^2 \frac{(r_A/r_2)^5}{r_2 \, |\partial/\partial r(\mu-1)^2 |_{r=r_A}|} \equiv i\sigma_1(\omega) \quad , \tag{16}$$

and $|\mu(r_A)-1| = \omega/\omega_A$. Equations (15,16) can be used to determine critical fast ion pressure and mode frequency at the onset of the fishbone mode for any particular profiles of the rotational transform and background plasma pressure.

3. Infernal Fishbone Modes with Arbitrary Mode Numbers

In the present section we generalize results of the previous section to arbitrary mode numbers. In the shear-free core with $m - nq_0 \sim \varepsilon$ the mode equations for general *m, n* are

$$\frac{d}{dr}\left\{\left[\left(\frac{\mu}{n}-\frac{1}{m}\right)^2 + \frac{l_\alpha}{(mn)^2} - \left(\frac{\omega}{\omega_A mn}\right)^2\right] r^3 \frac{d\xi_m}{dr}\right\} - (m^2-1)\left[\left(\frac{\mu}{n}-\frac{1}{m}\right)^2 - \left(\frac{\omega}{\omega_A mn}\right)^2\right] r\xi_m -$$

$$\frac{\varepsilon^2}{m^2}\left[\frac{1}{2}(r\beta_p' + 4\beta_p)^2 + \left(1-\frac{n^2}{m^2}\right)(r\beta_p' + 4\beta_p)\right] r^3 \xi_m = \tag{17}$$

$$\frac{\varepsilon^2 n}{2m^2(m+1)} r^{1+m} (r\beta_p' + 4\beta_p) \frac{d}{dr}(r^{2+m}\hat{\xi}_{m+1}) \quad ,$$

$$\frac{d}{dr}\left(r^3 \frac{d\hat{\xi}_{m+1}}{dr}\right) - [(m+1)^2 - 1] r\hat{\xi}_{m+1} = -\frac{m+1}{2n} r^{2+m} \frac{d}{dr}[(r\beta_p' + 4\beta_p) r^{1+m}\xi_m] \quad , \tag{18}$$

where again $\xi_{m+1} \equiv \varepsilon \hat{\xi}_{m+1}$. The general solution of Eq.(18), which is regular on magnetic axis, takes the form

$$n\hat{\xi}_{m+1} = -\frac{1}{2}(1+m) r^{-(2+m)} \int_0^r d\hat{r} (\hat{r}\beta_p' + 4\beta_p) \hat{r}^{2+m} \xi_m + e r^m \quad . \tag{19}$$

Substituting Eq.(19) into Eq.(17), one can obtain

$$\frac{d}{dr}\left\{\left[\left(\frac{\mu}{n}-\frac{1}{m}\right)^2+\frac{l_\alpha}{(mn)^2}-\left(\frac{\omega}{\omega_A mn}\right)^2\right]r^3\frac{d\xi_m}{dr}\right\}-(m^2-1)\left[\left(\frac{\mu}{n}-\frac{1}{m}\right)^2-\left(\frac{\omega}{\omega_A mn}\right)^2\right]r\xi_m - \frac{\varepsilon^2}{m^2}\left(1-\frac{n^2}{m^2}\right)\frac{d}{dr}(r^4\beta_p)\xi_m=\frac{\varepsilon^2}{m^2}e\frac{d}{dr}(r^4\beta_p)r^{m-1} \quad . \tag{20}$$

With pressure profile given by $p_c = p_0[1-(r/a)^{2\nu}]$ we have $\beta_p = \hat{\beta}_p r^{2\nu-2}$. Thus one can see that, for $|m-nq_0|\sim\varepsilon$, the ratio of the last term on the left-hand side of Eq.(20), which represents stabilizing effect of the average magnetic well, to the second term on the LHS of this equation is of the order of $(r_0/a)^{2\nu} \ll 1$, where r_0 is again the radius at the transition between shear-free and sheared regions, which is assumed to be sufficiently abrupt to allow for asymptotic matching of the in these regions. Neglecting the last term on the LHS, Eq.(20) can be easily integrated. Imposing boundary condition $\xi_m(1)=0$, one finds

$$\xi_m = \frac{2\varepsilon^2 e\hat{\beta}_p(\nu+1)(r^{2\nu}-1)r^{m-1}}{[(2\nu+m)^2-1]l_\alpha(\omega,r)/n^2+4\nu(\nu+m)[(m/nq-1)^2-(\omega/\omega_A n)^2]} \quad . \tag{21}$$

Note that ξ_m is again negligible in the sheared region, except in the vicinity of the Alfven resonance. The dispersion relation again can be obtained by matching the asymptotic form of the solution Eq.(19) with ξ_m given by Eq.(21) in the sheared region, to the shear-free limit of the outer Eq.(18) (with the right-hand side neglected):

$$\hat{\xi}_{m+1} \propto \left(\frac{r}{r_{m+1}}\right)^m + \sigma_m\left(\frac{r}{r_{m+1}}\right)^{-(2+m)} \quad , \tag{22}$$

where $\mu(r_{m+1})=n/(m+1)$. We find

$$\sigma_m = \frac{1+m}{n(\nu+m)}\left(\frac{r_{m+1}}{a}\right)^{-2(m+1)}\left(\frac{r_0}{a}\right)^{2(\nu+m)} \times \frac{\varepsilon^2\hat{\beta}_p^2(\nu+1)^2}{[(2\nu+m)^2-1]l_\alpha(\omega)/n^2+4\nu(\nu+m)[(m/nq_0-1)^2-(\omega/\omega_A n)^2]}+\sigma_{res} \quad , \tag{23}$$

where we have restored radius dimension. For the μ-profile given by

$$\mu = \frac{n}{m+1}+\left(\mu_0-\frac{n}{m+1}\right)\left[1-\left(\frac{r}{r_{m+1}}\right)^{2\lambda}\right]$$

with $\lambda \ge m+2$, we have [6]

$$\sigma_m \approx \frac{m}{m+2}\left(1-\frac{m+1}{\lambda}\right) \quad , \tag{24}$$

and expression for σ_{res} takes the form

$$\sigma_{res} = i\pi\frac{(m+1)^3}{8\lambda n}\frac{\varepsilon^2\hat{\beta}_p^2(\nu+1)^2}{\nu(\nu+m)}\left(\frac{r_{m+1}}{a}\right)^{2(\nu-1)}\left[\frac{(m+1)\omega}{n\omega_A}\right]^{\frac{\nu+m}{\lambda}-2} \equiv i\sigma_{1m} \quad . \tag{25}$$

To make contact with NSTX experiment [10], we take $v_\alpha/V_A=2$, $m=2$, $n=1$, $q_0=1.7$, $R_0/a=1.5$, and choose $\nu=6$, $\lambda=4$, so that $\sigma_2=1/8$, $\sigma_{12}(\omega)=const$, $r_3/a \approx 0.8$, $r_0/a \approx r_2/a \approx 0.6$. Considering plasma at the margin of the MHD stability in the absence of fast ions [taking $l_\alpha = \omega = 0$ in Eq.(23)], and taking into account that $\hat{\beta}_p = (\beta_o/\varepsilon^2)(m/n)^2[\nu/(\nu+1)]$, we obtain $\beta_0^{marg}=0.35$, and Eq.(25) yields $\sigma_{res} \approx 0.55$. Equation (23) then yields at the margin of the fishbone stability ($Im\omega=0$)

$$\hat{\beta}_\alpha^{crit} = \frac{192}{195}\frac{4\Omega^2(2\mu_0-1)^2}{\mathrm{Re}\,F(\Omega)+(\sigma_{12}/\sigma_2)\,\mathrm{Im}\,F(\Omega)} \quad , \tag{26}$$

$$\mathrm{Im}\,F(\Omega)\left[4\Omega^2\left(\frac{\sigma_{12}}{\sigma_2}+\frac{\sigma_2}{\sigma_{12}}\right)-\frac{\sigma_{12}}{\sigma_2}\right] = \mathrm{Re}\,F(\Omega) \quad , \tag{27}$$

where $\hat{\beta}_\alpha \equiv (8/\pi^2)(\rho_\alpha^3 R_0/r_0^4)\beta_{\alpha 0}$. Solution of Eqs.(26),(27) yields $\Omega \approx 0.6$, $\hat{\beta}_\alpha^{crit} \approx 2.5\times10^{-2}$. Taking into account that in NSTX $r_0 \approx 0.6a \approx 40cm$, $R_0 \approx 100\ cm$, $\rho_\alpha \approx 20\ cm$, we obtain $\beta_{\alpha 0}^{crit} \approx 0.1$. For the chosen fast ion pressure profile this corresponds to the volume averaged fast ion beta $\langle\beta_\alpha\rangle = (2/3)(r_0/a)^2\beta_{\alpha 0} \approx 2.4\times10^{-2}$. Taking into account that energetic ions are deutrons with injection energy *80 keV*, we obtain $f = 0.6(2\mu_0-1)v_\alpha/2\pi R_0 \approx 46kHz$. Both these values are in reasonable agreement with observed initial fishbone frequency in the plasma frame $f \approx 45\ kHz$ and volume averaged beam ion beta $\langle\beta_\alpha\rangle \approx 2\,\%$.

4. Summary

We have shown that in low-shear tokamaks fishbone modes with arbitrary mode numbers can be destabilized by the interaction with energetic circulating ions at the Cherenkov resonance. In contrast to the conventional $m=n=1$ circulating-ion-driven fishbone instability, the considered instability is caused mainly by particles with orbits inside the flux surface with the radius r_0 separating regions with low and finite magnetic shear. The efficient energy exchange between these particles and the perturbation takes place due to finite orbit width of the energetic ions and a radial gradient of the mode poloidal electric field. Both the mode frequency and critical fast ion pressure are in reasonable agreement with experimental observations of bursting $m=2$ fishbone oscillations accompanied by strong changes of the neutron yield in the NSTX spherical torus.

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