Stability boundaries for fast particle driven TAE in stellarators

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Abstract

The influence of fast particles on TAE modes in stellarators is investigated in the framework of a kinetic MHD approach. The theoretical model couples ideal MHD with a drift kinetic equation allowing the perturbative calculation of growth and damping rates due the several particle species. Here, the code considers passing particles only. Comparison is made between the numerical results and a local theory where the latter turned out to be a useful first guess approximation. While the fast particle drive is, as in tokamaks, mainly due to the coupling with the toroidal components of the modulus of the magnetic field, the ion contributions to the damping stem from the coupling with the helical components.

1. Introduction

Meanwhile, several experimental investigations of fast particle effects in stellarators as eg., in W7-AS [1, 2], have been made. Recently, the parameter space for the exitation of toroidal Alfvén eigen modes (TAE's) and energetic particle modes (EPM's) has been explored at LHD [3]. Regarding this progress in the quantitative assessment, it is desirable to develop according theoretical tools.

We will show that the theory can predict stability limits for Alfvén eigenmodes in three-dimensional equilibria from both the analytical and the numerical point of view.

A drift-kinetic extension [5] of the ideal magneto-hydrodynamic (MHD) stability code CAS3D [7] is applied to shot No. 39042 of W7-AS [1] and a W7-X equilibrium. The equilibrium for W7-X is a high-β ($\beta = 4.2\%$) practically island free equilibrium which has been obtained from a PIES calculation [4].

2. Theory

We start from a three- dimensional MHD description of plasma stability and use the CAS3D stability code [7]. It has been shown that a kinetic energy principle can be derived which couples the drift kinetic particle species (electron, ions, and a fast ion component) to the MHD stability equations via an expression for the pressure in the force balance equation [5].

To avoid following 3D particle orbits explicitly, a technique invented by Rewoldt et al. [9] which is being used in numerous 2D codes [10, 11] has been adopted. The particles move

along field lines feeling a bounce or transit averaged drift only. The radial extension of the particle orbits is not taken into consideration.
The particle-wave energy exchange by resonant interaction is given by

$$\begin{aligned}
\delta W_{\mathrm{s}} \;=\;& \frac{\pi}{M^2}\left\{\sum_{\sigma}\right\}\int ds\int d\varphi\int d\mu\, d\epsilon\left(-\int\frac{d\vartheta}{|v_{||}|}\sqrt{g}B\right)\\
&\times\sum_{\substack{n,m\\ n',m'}}\sum_{p=-\infty}^{\infty} e^{-i\frac{2\pi}{N_p}(n'-n)\varphi}\\
&\times\left(\frac{\partial F}{\partial\epsilon}\right)_{\mu}\frac{\omega-2\pi(\frac{n}{N_p}J-mI)\omega^*}{m\left\langle\omega_d^{\vartheta}\right\rangle+\frac{n}{N_p}\left\langle\omega_d^{\varphi}\right\rangle+\left\{{\sigma(p+nq)\atop p}\right\}\omega_{\left\{{t\atop b}\right\}}-\omega}\\
&\times L^{(1)*}_{m'n'}\mathcal{M}^{m'n'*}_{pn}\,L^{(1)}_{mn}\mathcal{M}^{mn}_{pn}
\end{aligned} \tag{1}$$

with the following definitions of $\mathcal{M}^{m'n'}_{pn}$:
for passing particles

$$\mathcal{M}^{m'n'}_{pn} = \left\langle e^{i[2\pi(m'+n'q)\vartheta''-(p+nq)\omega_t t'']}\right\rangle_{\vartheta''} \tag{2}$$

and for reflected particles

$$\mathcal{M}^{m'n'}_{pn} = \left\langle e^{2\pi i(m'+n'q)\vartheta''}[\cos^2(\frac{\pi}{2}p)\cos(p\omega_b t'')-i\sin^2(\frac{\pi}{2}p)\sin(p\omega_b t'')]\right\rangle_{\vartheta''}. \tag{3}$$

The perturbed particle Lagrangian is given by

$$L^{(1)} = -(Mv_{\|}^2-\mu B)\vec{\xi}_{\perp}\cdot\vec{\kappa}+\mu B\vec{\nabla}\cdot\vec{\xi}_{\perp}\,.$$

Here, $\langle\ldots\rangle$ denotes the transit or bounce average ($t''=t''(\vartheta'')$)
The complete energy integral can be written as

$$\omega^2\delta W_{\mathrm{kin}} = \delta W_{\mathrm{mag}}+\sum_{\mathrm{s=i,e,hot}}\delta W_{\mathrm{s}}(\omega) \tag{4}$$

and constitutes a nonlinear eigenvalue problem for the mode frequency ω. In this paper, we will restrict to passing particles and a perturbative approach, i.e. we assume that $\delta W_{\mathrm{s}} \ll \delta W_{\mathrm{mag}}$, where δW_{kin} and δW_{mag} are the constituents of the ideal MHD energy principle with vanishing pressure contribution ($\gamma_a \approx 0$).
In this linear model, each species contributes separately to the growth or damping rate of the mode which can be calculated perturbatively according to:

$$\Delta\omega_s + i\gamma_s \approx \frac{1}{2}\frac{\delta W_{\mathrm{s}}(\omega_0)}{\delta W_{\mathrm{mag}}}\omega_0 \tag{5}$$

using the MHD eigenfunctions and the MHD frequency ω_0 of the CAS3D result on the right hand side.
Obviously, the mode is unstable if the growth rate γ is larger than zero:

$$\gamma = \gamma_{\mathrm{hot}}+\gamma_{\mathrm{i}}+\gamma_{\mathrm{e}} > 0\,. \tag{6}$$

This condition allows the derivation of stability boundaries (i.e. a critical β_{fast}) with respect to different parameters.

3. Local limit and resonance condition for fast particles

In the limit of localized modes, i.e. very large aspect ratio the expression for the passing particle fraction in Eq. (5) corresponds to the local theory developed in [8]. Additionally, we allow for a temperature gradient of thermal electrons and ions and arbitrary couplings between the mode and the Fourier components of the equilibrium magnetic field but neglect the suggested reflected particle correction. Furthermore we correct a numerical error in the application of the local theory in an earlier publication [6].

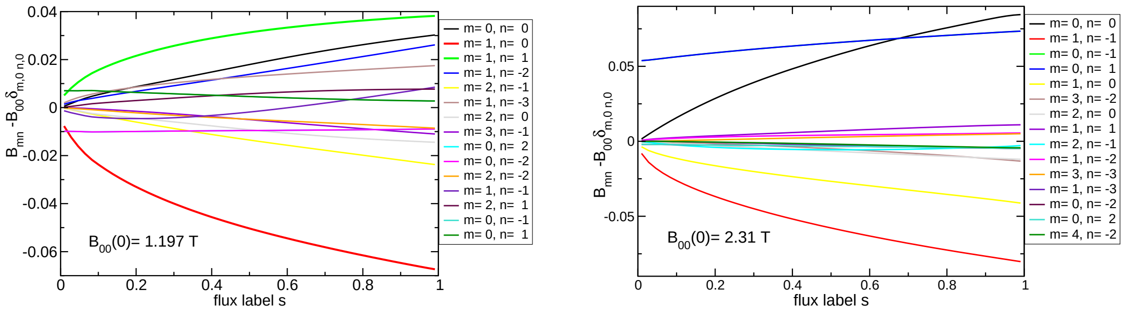

Figure 1: *Fourier components of the modulus of the magnetic field for W7-AS(#39042) (left) and W7-X (right)*

In the large aspect ratio limit the resonance condition is

$$v_{m'n'}^{\mathrm{res}} = v_A \left| 1 \pm \frac{m'\iota^* + n'N_p}{m\iota^* + n} \right|^{-1} \tag{7}$$

where N_p is the number of periods, ι^* labels the local value of the rotational transform where the Alfvén branches cross, m, n are the mode numbers of the Alfvén wave and m', n' label the component $B_{m'n'}$ to which the mode couples. For the local calculations, we consider the two main components of B_{mn} (see Fig. 1) only.

4. Results

We chose a W7-AS shot (No. 39042) which has already been discussed in the literature [1, 6]. Although, in this case, the fast ion drive is relatively small, the mode numbers and

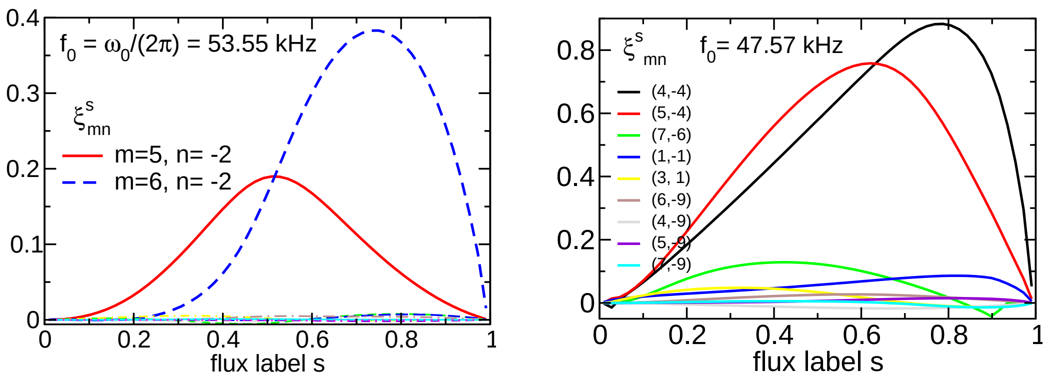

Figure 2: *mode structures of the W7-AS case (left) and the W7-X case (right)*

the mode structure were identified experimentally to be a toroidal Alfvén eigen mode (TAE) with its main Fourier components being $m = 5, n = -2$ and $m = 6, n = -2$.
The calculations have been performed resembling the experimental conditions as close as possible taking a slowing down distribution for the fast ions from the neutral beam injection. Additionally, the energy of the bulk and beam ions will be varied to distinguish different contributions to the growth rate and to calculate stability diagrams.

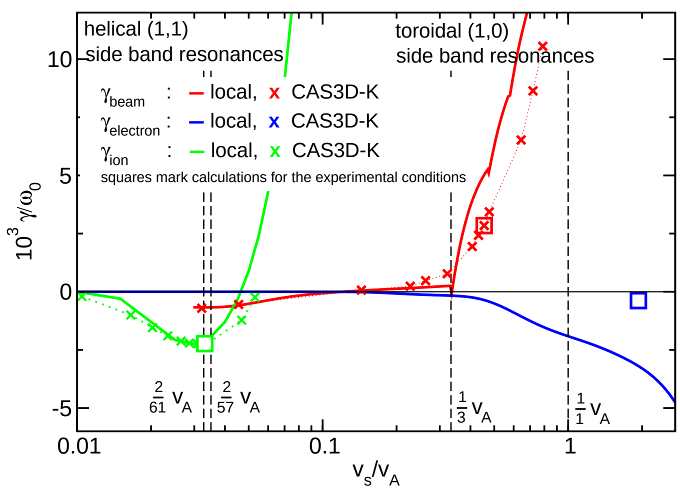

Figure 3: *Growth or damping rates, resp. of electrons, ions and beam ions vs. the ratio of velocity of the species to the Alfvén velocity. The lines of the major side band resonances have been indicated by dashed lines. Note, that β_{fast} was kept constant during the variation, while $\sqrt{\beta_i} = v_{th,i}/v_A$. The calculations for the experimental conditions have been marked with a square.* $v_A(s_0) = 4.65 \cdot 10^6\,\text{ms}^{-1}$

The growth or damping rates for bulk and fast ions, both being deuterons, are shown in Fig. 3. In general the agreement between both approaches is remarkably good considering the simplicity of the local approach.

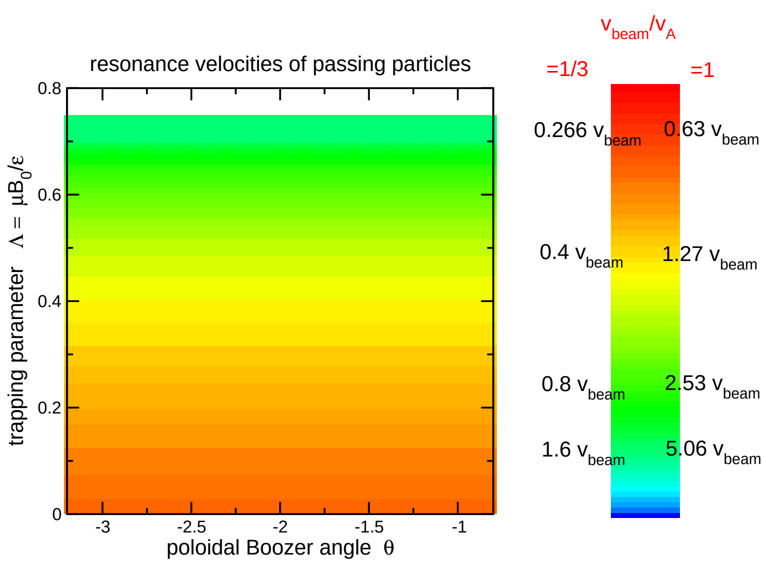

Figure 4: *Shown is the field line average of the transit frequency $< \omega_t >$ from Eq. (1), but normalized to the toroidal resonance velocities in units of v_{beam}/v_A. v_{beam} denotes the maximum beam velocity here. Therefore only those particles with $v_t|_r < 1.0 v_{beam}$ can contribute. Note, that the angle θ serves here as a field line label.*

We see that the main contributions to both, ion damping and fast ion growth rate, are connected with the resonance velocities due the coupling of the mode with the equilibrium magnetic field. For the hot particles, the growth rate increases rapidly when the maximum beam velocity exeeds the resonance velocity of $1/3 v_A$. The electron damping is weak ($\gamma_e = -3.82 \cdot 10^{-4}$) because the electron thermal velocity at the location of the mode ($\approx 1.933\, v_A(s_0)$) is beyond the important resonances.
To understand how the resonance condition looks like in the complex numerical model we look at the variation of the field line averaged transit frequency (Fig. 4): The

variation of this quantity with the field line label is so small that it is not resolved in the figure. It does mainly depend on the trapping parameter. In this sense, the picture shows a tokamak-like behaviour. On the other hand, we see that those particles which have a large $v_{||}$ (small Λ) can resonate with the side bands and contribute to the growth rate. This fits to the assumptions of the local theory and may partly explain its relative success. Another reason is presumably the relatively low shear of both equilibria. From eq. (7) we see that the smaller the change in ι the smaller is the radial variation of the resonance condition. The comparison of the stability diagrams (see Fig.5) with the

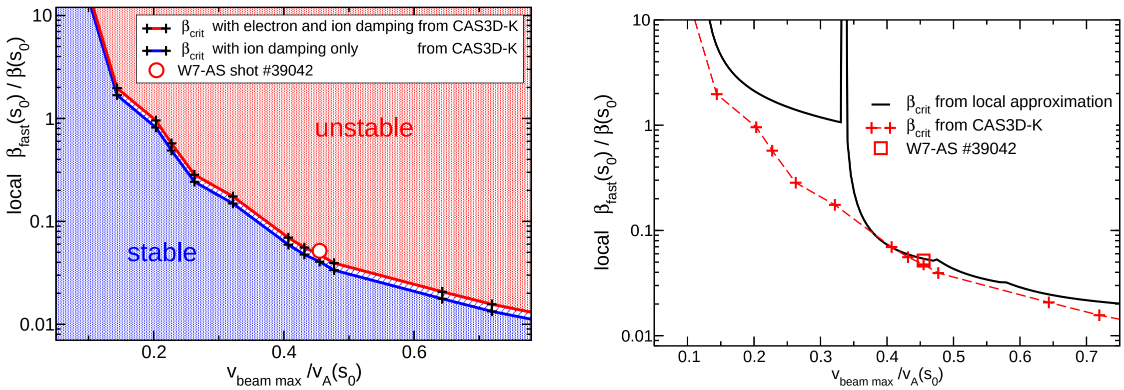

Figure 5: *The critical $\beta_{\text{fast}}/\beta$ vs. the ratio of maximum beam velocity to Alfvén velocity which constitutes a stability limit. The local approach following [8] is compared with the results from CAS3D-K.*

beam velocity shows differences for small beam velocities. The global code predicts a weak instability of $1.1\beta_{\text{fast crit}}$, being in good agreement with the local result. The same is true for the approximate threshold value in $v_{\text{beam max}}/v_A \approx 0.1$

For the W7-X mode, the agreement between local and global results is still better than a factor of two. Nevertheless, the numerical model again predicts the mode to be less stable. However, for the high β_i case considered here, the low energy part of the fast particle distribution function is expected to deviate from the simple slowing down model for envisaged neutral beam injection scenarios [12]. Therefore, to predict NBI stability, a refined model of the distribution function will be included in CAS3D-K.

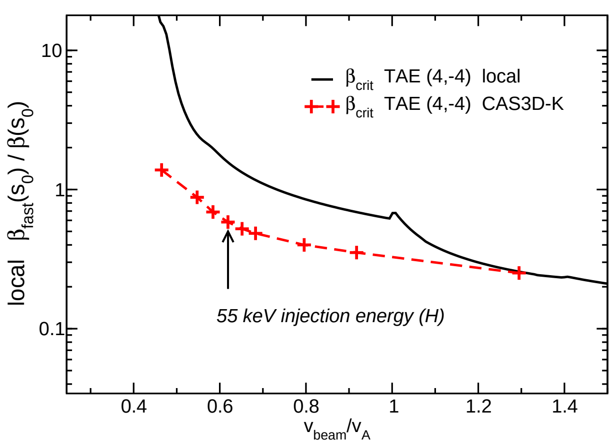

Figure 6: *The critical $\beta_{\text{fast}}/\beta$ vs. the ratio of maximum beam velocity to Alfvén velocity. As for the W7-AS case, the injection energy has been varied.*

5. Contribution of thermal ions

In this type of theory (kinetic MHD), it is possible that modes are destabilized by the temperature gradients of the thermal ions. This is illustrated in Fig. 7.

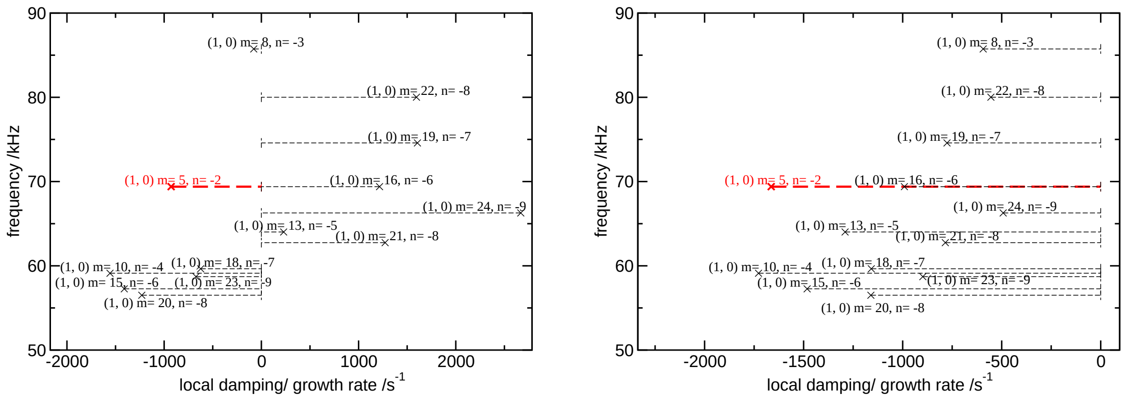

Figure 7: *local approach to ion damping rate with (left) and without (right) temperature gradient considered for several TAE modes for the W7-AS case*

It is an interesting question in how far this MHD model of kinetic growth and damping rates reflects the real behavior of the plasma. However, a decisive answer is not yet known, not even for tokamaks but can be expected from gyrokinetic codes as e.g. [11, 13, 15] see also [14]. In this paper we only took a closer look on modes which are stable also with an ion temperature gradient The extension of the model to include non-ideal

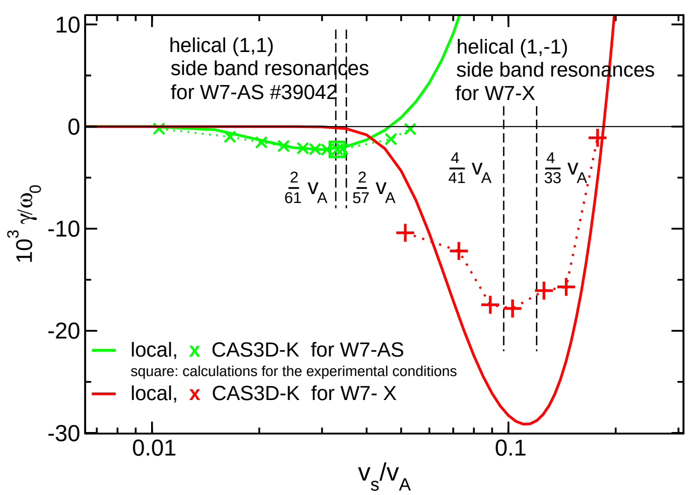

Figure 8: *Comparison of the ion damping for the W7-AS and the W7-X case. The main contributions come from the coupling to the helical components of B. Note, that the peak damping rate for W7-AS corresponds to $T_i = 400eV$, while the ion temperature for the W7X-case is $T_i = 3.8keV$. The higher ι of the W7-X equilibrium mainly accounts for the difference in the resonance conditions*

(i.e. finite gyro radius and finite $E_{\|}|$ effects) at least on the fluid side (for the equations see e.g. [16]) is underway and will allow access to cases where the gaps are closed.

6. Conclusion

The drift kinetic MHD growth and damping rates of a fast particle driven TAE mode have been calculated for realistic 3D conditions (W7-AS shot No. 39042).
Varying parameters as ion temperature and maximum beam velocity, stability diagrams have been calculated. The results indicate that this particular shot is close to marginality. These theoretical stability diagrams open up a possibility for more comparisons with the experimental exploration of the parameter space.

The results for the chosen W7-X equilibrium show that the local theory is a reasonable approximation but may deviate quantitatively, especially for small injection energies. The good agreement may be due to the small shear of the considered equilibria.
It is shown that resonances stemming from the coupling to the helical side bands contribute mainly to the ion damping for both equilibria. This constitutes a genuine effect of three dimensional equilibria.
The fast particle drive, on the other hand, comes mainly from the well known toroidal side band resonances at $1/3v_A$ and $1/1v_A$. The electron contribution to the damping is small because the electron velocity is larger than the Alfvén velocity.

Acknowledgment

The author wants to thank S. Zegenhagen for the good collaboration and many discussions, also J. Nührenberg, C. Nührenberg, R. Kleiber and M. Drevlak for valuable comments, A. Werner, H. Massberg and A. Weller for stimulating helpful discussions and informations on the experimental conditions.

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