A simplified momentum conservation analysis on transport reduction induced by zonal flow and turbulent dissipations

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Abstract: In the present paper, the linear relation between zonal flow and turbulent viscosity and that between zonal electric field and turbulent resistivity are for the first time derived from the momentum conservation of toroidal plasma system. Furthermore, the qualitative analysis shows that the electromagnetic micro-turbulence can produce the significant turbulent resistivity and viscosity at or near rational surface. Finally, we give the heat transport coefficients of ions and electrons, including the effects of the turbulent viscosity and resistivity, i.e., those of zonal flow and zonal electric field. It can be seen that the turbulent viscosity (equivalently, zonal flow) of plasma makes the heat transport coefficient of ions decrease significantly at or near rational surface while the turbulent resistivity (equivalently, zonal electric field) reduces largely that of electrons there. That is, the present results give a physical picture for the transport reduction induced by zonal flow and turbulent dissipations: Zonal flow is generated by turbulence nonlinearly and thus the turbulent energy is converted into the zonal flow energy. In the meantime, the zonal flow energy is converted into the thermal energies of ions and electrons via the turbulent resistivity and viscosity dissipations.

Zonal flow has been studied extensively in the past ten years and more [1, 2]. It has been showed both theoretically [3, 4] and experimentally [5-7] that zonal flow makes the decorrelation of turbulence and leads to the decrease of turbulence level and the formation of transport barrier. In the transport barrier, for example, the ion and electron heat diffusivities, χ_i and χ_e, reduce greatly. That is, the thermal energies of ions and electrons increase significantly. Up to now, however, it has not been clarified what channel the turbulent energy is converted into the thermal energies of ions and electrons via when zonal flow is generated. Therefore, it is a great challenge to magnetic confinement fusion researchers to find out the physical relations between zonal flow and χ_i and χ_e. In the present paper, the linear relation between the zonal flow ($\mathbf{V}_{ZF}$) and the turbulent viscosity (ν^{t}) and that between the zonal electric field ($\mathbf{E}_{ZF}$) and the turbulent resistivity (η^{t}) are for the first time derived on the basis of the momentum conservation of toroidal plasma system. Furthermore, the qualitative analysis shows that the electromagnetic micro-turbulence can produce the significant turbulent resistivity η^{t} and viscosity ν^{t} at or near rational surface. Thus, the self-generated momentum dissipation of ν^{t} and η^{t} can balance the self-generated momentum $\mathbf{V}_{ZF}$ and electric field $\mathbf{E}_{ZF}$ at or near rational surface. Finally, we give the χ_i and χ_e, including the effects of the turbulent viscosity ν^{t} and resistivity η^{t}. The present results indicate that ν^{t} makes the χ_i decrease significantly at or near rational surface while η^{t} reduces largely the χ_e there. In other words, $\mathbf{V}_{ZF}$ makes the χ_i decrease significantly at or near rational surface while $\mathbf{E}_{ZF}$ reduces largely the χ_e there.

At first, the force balance equation, i.e., momentum conservation one, is given when the effects of zonal flow are considered,

$$(\mathbf{E}_0 + \mathbf{E}_{ZF}) + (\mathbf{V}_0 + \mathbf{V}_{ZF}) \times \mathbf{B}_0 - \nabla P_{0i} / en = \eta \mathbf{J}_0 + (\nu / en) \Delta \mathbf{V}_{ZF} , \quad (1)$$

where $\mathbf{V}_0$ and $\mathbf{E}_0$ are the fluid velocity and the electric field produced by other factor, such as the neutral beam injection; P_{0i} is the equilibrium pressure of ions, $\mathbf{J}_0$ is the equilibrium current density, and $\mathbf{B}_0$ is the equilibrium magnetic field; η is the collisional resistivity, and ν is the collisional viscosity. Generally, the two terms on the right side of Eq. (1) are very small and even can be dropped out. Here the functionary time or lifetime of $\mathbf{V}_{ZF}$ and $\mathbf{E}_{ZF}$ is of confinement time scale (macro-time scale) even if the time change or formation time of $\mathbf{V}_{ZF}$ and $\mathbf{E}_{ZF}$ is of turbulent time scale. In addition, experiments show that the radial electric field observed (equilibrium quantity) is mainly contributed by the zonal flow during L-H transition [8]. Therefore, in Eq. (1) we approach both the $\mathbf{V}_{ZF}$ and $\mathbf{E}_{ZF}$ as equilibrium quantities. However, it is still difficult to understand why the momentum of plasma system is conservative from Eq. (1), in which there is only the self-generated momentum (i.e., zonal flow) but no momentum dissipation. In order to make the momentum of plasma system conservative, the self-generated momentum dissipation is required to balance the self-generated momentum. When the self-generated momentum dissipations, i.e., induced by the turbulent resistivity η^{t} and viscosity ν^{t}, is considered, the following momentum conservation equation is derived,

$$(\mathbf{E}_0 + \mathbf{E}_{ZF}) + (\mathbf{V}_0 + \mathbf{V}_{ZF}) \times \mathbf{B}_0 - \nabla P_{0i} / en = \eta^t \mathbf{J}_0 + \nabla \cdot \mathbf{R} / en , \quad (2)$$

with

$$\mathbf{R} = \overline{\delta\mathbf{b}\delta\mathbf{b} - \delta\mathbf{v}\delta\mathbf{v}} = \nu^t (\nabla \mathbf{V} + \nabla \mathbf{V}^+) , \quad (3)$$

where $\mathbf{R}$ is the turbulent stress, $\mathbf{V} = \mathbf{V}_0 + \mathbf{V}_{ZF}$, $\delta\mathbf{b}$ is the magnetic field fluctuation in plasma, and $\delta\mathbf{v}$ is the velocity fluctuation of plasma fluid. Here we have taken advantage of a bar denote the ensemble mean. Neglecting the time change of $\mathbf{V}_0$, $\mathbf{E}_0$, $\mathbf{B}_0$, $\mathbf{J}_0$ and P_{0i}, Eq. (2) yields

$$\mathbf{E}_0 + \mathbf{V}_0 \times \mathbf{B}_0 - \nabla P_{0i} / en = 0 . \quad (4)$$

Thus, we obtain from Eqs. (2) and (4)

$$\mathbf{E}_{ZF} + \mathbf{V}_{ZF} \times \mathbf{B}_0 = \eta^t \mathbf{J}_0 + \nabla \cdot \mathbf{R} / en. \quad (5)$$

Equation (2) involves the two time scales. One is the macro-time scale of equilibrium quantities $\mathbf{V}_0$, $\mathbf{E}_0$, $\mathbf{B}_0$, $\mathbf{J}_0$ and P_{0i}, and the other the turbulent time scale of zonal flow (or zonal electric field). The two time scales are parted by Eqs. (4) and (5). It should be noted that in Eqs.(2) and (5) both the zonal flow and zonal electric field are still regarded as macro-physical quantities on the basis of the experimental fact [8] that the observed radial electric field (macro-physical quantity) is mainly contributed by the zonal flow during L-H transition.

Physically, the resistivity mainly dissipates the electric field while the viscosity largely does the flow. Hence, Eq. (5) can be qualitatively written as the following two parts

$$\mathbf{E}_{ZF} = \eta^t \mathbf{J}_0, \quad (6)$$

$$\mathbf{V}_{ZF} \times \mathbf{B}_0 = \nabla \cdot \mathbf{R} / en. \quad (7)$$

Equations (6) and (7) for the first time give the linear relation between the zonal electric field $\mathbf{E}_{ZF}$ and turbulent resistivity η^t and that between the zonal flow $\mathbf{V}_{ZF}$ and turbulent viscosity ν^t, respectively. Thus, the larger the zonal electric field $\mathbf{E}_{ZF}$ the larger the turbulent resistivity η^t and just the same, the larger the zonal flow $\mathbf{V}_{ZF}$ the larger the turbulent viscosity ν^t. Physically, Eqs. (6) and (7) indicate that the generation of zonal flow and zonal electric field is always accompanied by that of the turbulent viscosity and the turbulent resistivity η^t.

The zonal flow and zonal electric field have been extensively studied through various means of theories, numerical simulations and experiments. As a result, their properties, effects and dynamics have been understood satisfactorily [2]. However, we only have less knowledge about the properties of turbulent resistivity η^t and turbulent viscosity ν^t in tokamak plasmas. In particular, it is important to know whether there exist the turbulent resistivity η^t and turbulent viscosity ν^t sufficient large to balance the self-generated momentum of plasma system. Therefore, it is necessary to further understand their properties in tokamak plasmas. The formulas for turbulent resistivity and viscosity have been derived [9], and then applied to the reversed field pinch plasmas [10] and tokamak ones [11], respectively, whose concrete expressions are

$$\eta^t = 0.2K^2 / \varepsilon, \quad (8)$$

$$\nu^t = 0.07K^2 / \varepsilon, \quad (9)$$

with

$$K = (1/2)[\overline{(\delta\mathbf{b})^2} + \overline{(\delta\mathbf{v})^2}], \tag{10}$$

$$\varepsilon = \eta\overline{\left(\frac{\partial\delta b^a}{\partial x^b}\right)\left(\frac{\partial\delta b^a}{\partial x^b}\right)} + \nu\overline{\left(\frac{\partial\delta v^a}{\partial x^b}\right)\left(\frac{\partial\delta v^a}{\partial x^b}\right)}, \tag{11}$$

where a, $b = r$, θ, and z, the summation convention is adopted for the repeated superscripts; Thus, electromagnetic micro-turbulences, such as electromagnetic ITG and ETG modes or their coupling instabilities, all can make contribution to the generation of η^{t} and ν^{t}.

In what follows, according to the basic properties of micro-turbulence we qualitatively analyze η^{t} and ν^{t} generated by the turbulence only at the rational surface and the position $\theta = 0$, no matter which kind of micro-turbulence induces them. It is well known that micro-turbulence has a maximum at $\theta = 0$, that is, $\partial\mathbf{f} / \partial\theta\big|_{\theta=0} = 0$, where $\mathbf{f} = \delta\mathbf{b}$ or $\delta\mathbf{v}$. In addition, at the rational surface $k_{//} = 0$. In other words, micro-turbulence changes very slowly in the direction of $\hat{z}$. It means $\partial\mathbf{f} / \partial z \approx 0$ at a rational surface. Thus, micro-turbulence changes fast only in the radial direction. However, micro-turbulence is usually excited at rational surface $r = r_s$. Thus it is reasonable to assume $\partial\mathbf{f} / \partial r\big|_{r=r_s} = 0$. As a result, Eq. (11) yields $\varepsilon \approx 0$. Hence, from Eqs. (8) and (9) we come to the conclusion that the electromagnetic micro-turbulence can induce the significant resistivity η^{t} and viscosity ν^{t} at rational surface. As a result, the self-generated momentum dissipation due to the turbulent resistivity η^{t} and viscosity ν^{t} can balance the self-generated momentum due to the zonal flow.

It is well known that the plasma resistivity heats electrons mainly while the plasmas viscosity heats ions mainly. Thus, the turbulent resistivity can be regarded as a heat source of electrons. The radial scale length of the heat source is the electron temperature gradient scale length L_{Te}. The power of the heat source is $< \eta^t \mathbf{J}_0{}^2 >$. Then, it is easy to obtain the thermal transport coefficient of electrons induced by the heat source

$$- < \eta^t \mathbf{J}_0{}^2 > L_{Te}^2 / P_{0e}, \tag{12}$$

where P_{0e} is the electron pressure. If χ_e^{cl} is the conventional electron heat transport coefficient of electrons, including the collisional and the anomalous and so on, the effective heat transport coefficient of electrons is qualitatively [11]

$$\chi_e^{eff} = \chi_e^{cl} - < \eta^t \mathbf{J}_0{}^2 > L_{Te}^2 / P_{0e}, \tag{13}$$

Similarly, we obtain the effective heat transport coefficient of ions after the plasma turbulent viscosity is considered [10],

$$\chi_i^{eff} = \chi_i^{cl} - < \nu^t \nabla \mathbf{V} : \nabla \mathbf{V} > L_{Ti}^2 / P_{0i} . \tag{14}$$

Equations (13) and (14) indicate that η^t makes χ_e^{eff} reduce and ν^t does χ_i^{eff} decrease, respectively. It is no strange that as a heat source of electrons, the turbulent resistivity η^t heats electrons locally and leads to the decrease of χ_e^{eff} while the turbulent viscosity ν^t heats ions locally and makes χ_i^{eff} decrease. Furthermore, due to the linear relation between η^t and $\mathbf{E}_{ZF}$ and that between ν^t and $\mathbf{V}_{ZF}$, as showed in Eqs. (6) and (7), it can be seen from Eqs. (13) and (14) that the lager the zonal electric field the smaller the effective heat transport coefficient of electrons and the lager the zonal flow the smaller the effective heat transport coefficient of ions. The previous arguments show that electromagnetic micro-turbulence can induce the significant turbulent resistivity η^t and turbulent viscosity ν^t at or near rational surface. Thus, they strongly heat the ions and electrons at or near rational surface so that the χ_i^{eff} and χ_e^{eff} decrease to form the thermal transport barriers of ions and electrons there. The conclusion could a cause underlying the experimental fact that the transport barriers are usually observed at or near rational surface. During the formation of transport barrier or the fast increase of η^t, ν^t, $\mathbf{E}_{ZF}$ and $\mathbf{V}_{ZF}$, the increase of electron temperature induced by η^t is faster than that of ion temperature, because the mass of electron is much smaller than that of ions and $\eta^t > \nu^t$ [see, Eqs. (8) and (9)]. Also the conclusion is coincide with the experimental result observed in H mode experiments [8, 12]. These agreements with experiments, in turn, show that the turbulent resistivity η^t and turbulent viscosity ν^t could indeed play important roles in forming the thermal transport barriers of ions and electrons. The present results indicate that the effective resistivity and viscosity should be much larger than collisional resistivity and viscosity especially at or near rational surface during or after the formation of transport barrier, which waits to be tested by experiments. In addition, it can be understood that the linear growth time (from the excitation of turbulence to its saturation) of the micro-turbulence, which generate the significant resistivity and viscosity at the rational surface, is the formation time of transport barrier, that is, the fast growth time of zonal flow [2] and zonal electric field, or turbulent resistivity and turbulent viscosity. For the simplicity, we assume that the turbulent resistivity and viscosity are mainly generated by electromagnetic ETG instability. The previous work [11] indicates that the linear growth time of the electromagnetic ETG instability is about $10^{-6} \sim 10^{-7}$ s, which is qualitatively in agreement with the formation time of transport barrier or H mode observed in the experiments [8].

In summary, based on the analyses of the momentum conservation and turbulent dissipation in toroidal plasma system, a physical picture for the transport reduction, induced by zonal flow and turbulent dissipation, is given: Zonal flow is generated by turbulence nonlinearly and thus the turbulent energy is converted into the zonal flow energy. In the meantime, the zonal flow energy is converted into the thermal energies of ions and electrons via the turbulent resistivity and viscosity dissipations. In this paper, we for the first time derived the linear relation between the zonal flow and turbulent viscosity and that between zonal electric field and turbulent resistivity. In addition, it is showed that electromagnetic micro-turbulence can induce the significant turbulent resistivity and turbulent viscosity at or near rational surface. If the large turbulent resistivity and viscosity, equivalently, the large zonal electric field and zonal flow, are generated at or near rational surface as stated above, both χ_i^{eff} and χ_e^{ff} will reduce significantly there.

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