Low-Frequency Global Alfvén Eigenmodes in Hybrids with Perpendicular Neutral Beam Injection

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Abstract. A novel global Alfvén eigenmode (GAE) has been predicted, with frequency well below the minimum of the Alfvén continuum. This GAE exists in the tokamak plasmas with broad low-shear central core and safety factor slightly exceeding unity in this region, and is capable of resonating with precession of the trapped energetic ions. This mode has the dominant numbers $m=n=1$, but the coupling with the upper toroidal sideband is crucial both for eigenmode formation and its excitation by energetic ions. The mode will be excited as the plasma pressure approaches ideal MHD stability limit, which minimizes the ion Landau damping. The properties of novel GAE are consistent with observations of the low-frequency $n=1$ mode driven by energetic ions in the “hybrid” discharges with record plasma pressures and perpendicular neutral beam injection on the JT-60U tokamak [N.Oyama, A.Isayama, G. Matsunaga *et al.*, Nucl. Fusion **49**, 065026 (2009)].

1. Introduction

The so called “hybrid” regime attracted much attention in tokamak research [1]. Such equilibria are characterized by the flat profile of the safety factor, q, in the wide central core, with $q_0 \approx 1$in this region. The main advantage of the hybrid shots is the absence of sawteeth, which are the main triggers of the harmful neoclassical tearing modes. For this reason the hybrid regime has been included as a third operational scenario for ITER [2].

Recently the $n = 1$ mode with extremely low frequency (few kilohertz in the plasma frame) has been observed in hybrid shots with record plasma pressures and perpendicular neutral beam injection (NBI) on the JT-60 upgrade tokamak [3]. At highest NBI powers, these modes not only deteriorate confinement of the energetic ions, but also destroy the *H*-mode pedestal [3]. Therefore, in order to avoid such detrimental effects, it is important to understand the nature of these modes and to estimate the fast ion pressure necessary for their excitation. This motivated the present work.

It is well known that, in the general case, the spectrum of the Alfvén waves in the torus is continuous [4]. There are, however, few important examples for modes of the discrete spectrum, which can be easily excited by energetic ions provided proper resonance conditions are matched. These include the global Alfvén eigenmodes (GAE) [5-9], the toroidal Alfvén eigenmodes [10,11] (including the low-shear family [12-15]), the beta-induced Alfven eigenmodes (BAE) [16,17], and the Alfvén cascades [18-21]. GAEs considered so far are basically cylindrical modes with frequency, ω_0, just below the minimum of the Alfvén continuum, $\omega_0 \leq (k_{||}V_A)_{min}$, and density inhomogeneity is the necessary condition for eigenmode existence [5]. In the present work it is shown that, when finite plasma pressure and toroidicity are taken into account, a novel GAE appears in the hybrid regimes. The frequency of the most unstable mode is quite low, $\omega_0 << (k_{||}V_A)_{min}$, and the mode can exist in plasmas with $V_A(r) = const$. Because of its low frequency, this mode can be excited by trapped energetic ions, such as α-particles in ITER, via precessional resonance. Due to the global character of this mode [$m = n = 1$ with $m(n)$ the poloidal (toroidal) mode number] such excitation can strongly deteriorate confinement of resonant ions and affect fusion burn in the hybrid scenario.

In the next section we outline derivation of the GAE dispersion relation, which closely follows MHD stability analysis presented in Ref.[22]. In Sec.III we calculate the minimal fast ion drive necessary to overcome the GAE continuum damping. Finally, in Sec.IV we discuss similarities between the novel GAE and the experimental observations presented in Ref.[3].

2. GAE Dispersion Relation

We assume that $(\omega_0 /\omega_A)^2 << \gamma_s\beta$ and $q_0 - 1\sim \varepsilon$, where $\omega_A=V_A/R$ with V_A as the Alfvén speed and R as the major radius of the torus, $\varepsilon=a/R$ with a the minor plasma radius, γ_s is the adiabatic index, and β is the plasma pressure normalized to the magnetic field pressure. Then the eigenmode equations in the central core take the form [22,23]

$$\frac{d}{dr}\left\{\varepsilon^{-2}\left[(\iota-1)^2-3\left(\frac{\omega_0}{\omega_A}\right)^2\right]r^3\frac{d\xi_1}{dr}\right\}-4\left(\frac{r}{4}\beta_p{}'+\beta_p\right)^2 r^3\xi_1=\left(\frac{r}{4}\beta_p{}'+\beta_p\right)\frac{d}{dr}\left(r^3\hat{\xi}_2\right), \tag{1}$$

$$\frac{d}{dr}\left(r^3\frac{d\hat{\xi}_2}{dr}\right)-3r\hat{\xi}_2=-4r^3\frac{d}{dr}\left[\left(\frac{r}{4}\beta_p{}'+\beta_p\right)\xi_1\right], \tag{2}$$

where radius r is normalized to the plasma minor radius a, $\iota=1/q$, $\beta_p=2\mu_0(\langle p\rangle - p)/B_p^2$, with $\langle\cdots\rangle=(2/r^2)\int_0^r(\cdots)\hat{r}d\hat{r}$, p is the plasma pressure, B_p is the poloidal magnetic field, prime denotes radial derivative, $\xi_2\equiv\varepsilon\hat{\xi}_2$, $\xi_{1(2)}$ is the amplitude of the $m = 1(2)$ poloidal harmonic of the plasma radial displacement, and it has been assumed that $V_A(r)=const$. The general solution of Eq.(2), which is regular on the magnetic axis, is given by

$$\hat{\xi}_2=r^{-3}\int_0^r\hat{r}^4\beta_p(\hat{r})\frac{d\xi_1}{d\hat{r}}d\hat{r}+\left[C-\beta_p(r)\xi_1(r)\right]r, \tag{3}$$

where C is an integration constant. Putting Eq.(3) into Eq.(1) and integrating, we find

$$\frac{d\xi_1}{dr}=\frac{\varepsilon^2 Cr\beta_p}{(\iota-1)^2-3(\omega_0/\omega_A)^2} \tag{4}$$

The dispersion relation can be obtained by matching the solution of the inner (shear-free) region to the solution in the outer (sheared) region. This procedure is accurate provided the transition between these regions is sufficiently abrupt [22]. Then in the outer region $|\iota-1|\sim 1$ and $\xi_1\sim\varepsilon^2$, as follows from Eq.(4). Therefore, in the outer region, we can neglect the toroidal coupling and write equation for the $m = 2$ harmonic in the form

$$\frac{d}{dr}\left[\left(\iota-\frac{1}{2}\right)^2 r^3\frac{d\hat{\xi}_2}{dr}\right]-3\left(\iota-\frac{1}{2}\right)^2 r\hat{\xi}_2=0\,. \tag{5}$$

Equation (5) has the following asymptotic solution in the shear-free region:

$$\hat{\xi}_2 \propto \frac{r}{r_2} + \sigma\left(\frac{r}{r_2}\right)^{-3}, \tag{6}$$

where $q(r_2)=2$ and the constant σ can be determined by integrating Eq.(5) through the outer region. Matching Eq.(6) with the asymptotic form of Eq,(3) in the outer region, we obtain the dispersion relation as follows:

$$\sigma = \left(\frac{r_2}{a}\right)^2 \int_0^a \frac{\left[\varepsilon\beta_p(r)\right]^2}{(\iota-1)^2 - 3(\omega_0/\omega_A)^2}\left(\frac{r}{r_2}\right)^5 d\left(\frac{r}{r_2}\right). \tag{7}$$

Note that, for $3(\omega_0/\omega_A)^2<(\iota_0-1)^2$ and $q_0>1$, the integral on the right-hand side (RHS) of Eq.(7) converges, which justifies interpretation of the novel mode as GAE.

The constant σ can be calculated analytically for the following model profile of the rotational transform:

$$\iota = \frac{1}{2} + \left(\iota_0 - \frac{1}{2}\right)\left[1-\left(\frac{r}{r_2}\right)^{2\lambda}\right], \tag{8}$$

where abruptness of the transition requires $\lambda >> 1$. Then the general solution of Eq.(5) is given by

$$\xi_2 = \frac{r}{r_2}\left\{C_1 F\left[1+\frac{3}{2\lambda},\frac{1}{2\lambda};1+\frac{2}{\lambda};\left(\frac{r}{r_2}\right)^{2\lambda}\right] + \left(\frac{r}{r_2}\right)^{-4} C_2 F\left[1-\frac{2}{\lambda},-\frac{3}{2\lambda};1-\frac{2}{\lambda};\left(\frac{r}{r_2}\right)^{2\lambda}\right]\right\}, \tag{9}$$

where $F(a,b;c;z)$ is the hypergeometric function. For $r \to r_2 - 0$, Eq.(9) yields [24]

$$\begin{aligned}\xi_2 = {} & C_1 \frac{\Gamma\left(1+\frac{2}{\lambda}\right)}{\Gamma\left(1+\frac{3}{2\lambda}\right)\Gamma\left(\frac{1}{2\lambda}\right)}\left\{2\psi(1) - \psi\left(1+\frac{3}{2\lambda}\right) - \psi\left(\frac{1}{2\lambda}\right) - \ln\left[1-\left(\frac{r}{r_2}\right)^{2\lambda}\right]\right\} \\ & + C_2 \frac{\Gamma\left(1-\frac{2}{\lambda}\right)}{\Gamma\left(1-\frac{1}{2\lambda}\right)\Gamma\left(-\frac{3}{2\lambda}\right)}\left\{2\psi(1) - \psi\left(1-\frac{2}{\lambda}\right) - \psi\left(-\frac{3}{2\lambda}\right) - \ln\left[1-\left(\frac{r}{r_2}\right)^{2\lambda}\right]\right\},\end{aligned} \tag{10}$$

where $\Gamma(z)$ is the gamma function and $\psi(z)\equiv\Gamma'(z)/\Gamma(z)$. Regularity of Eq.(10) at $r=r_2$ yields

$$\sigma = \frac{C_2}{C_1} = -\frac{\Gamma\left(1+\frac{2}{\lambda}\right)}{\Gamma\left((1-\frac{2}{\lambda}\right)}\frac{\Gamma\left(1-\frac{1}{2\lambda}\right)}{\Gamma\left(1+\frac{3}{2\lambda}\right)}\frac{\Gamma\left(-\frac{3}{2\lambda}\right)}{\Gamma\left(\frac{1}{2\lambda}\right)} = \frac{1}{3}\frac{\Gamma\left(1+\frac{2}{\lambda}\right)}{\Gamma\left(1-\frac{2}{\lambda}\right)}\frac{\Gamma\left(1-\frac{1}{2\lambda}\right)}{\Gamma\left(1+\frac{1}{2\lambda}\right)}\frac{\Gamma\left(1-\frac{3}{2\lambda}\right)}{\Gamma\left(1+\frac{3}{2\lambda}\right)}. \tag{11}$$

For the safety factor profile given by Eq.(8) and pressure profile given by $p(r)=p_0[1-(r/a)^{2\nu}]$, so that $\beta_p(r) \propto r^{2\nu-2}$, one can obtain from Eq.(7) in the limit $(\omega_0/\omega_A)^2 << (\iota_0-1)^2$

$$\omega_0 \cong \omega_A \sqrt{A(\iota_0 - 1)^2 - B(\varepsilon\beta_{p1})^2}\ , \tag{12}$$

where

$$A = [3(1-\zeta/3)(1-\zeta/2]^{-1}, \quad B = \frac{1}{2\sigma(2\nu+1)}\left(\frac{r_1}{a}\right)^2\left(\frac{r_1}{r_2}\right)^4 \frac{\pi\zeta(1-\zeta)}{\sin(\pi\zeta)},$$
$$\zeta = \frac{2\nu+1}{\lambda}, \quad r_1 = r_2\left(\frac{1-\iota_0}{\iota_0 - 0.5}\right)^{1/2\lambda}, \quad \beta_{p1} = \beta_p(r_1), \tag{13}$$

and σ is given by Eq.(11). Note that Eqs.(12,13) are valid only close to the ideal MHD stability limit.

3. GAE Excitation by Trapped Energetic Ions

The growth and damping rates of the GAE can be calculated perturbatively, using the lowest order eigenfunction and eigenvalue given by Eq.(4) and Eq.(12), respectively. The sum of the fluid and kinetic parts of the fast ion energy is given by [25]

$$\delta W_\alpha = \frac{m_\alpha \omega_{c\alpha}}{2}\omega_0 \int d^3 r r |\xi_r|^2 \int d\Gamma \frac{\omega_{d\alpha}}{\omega_0 - \overline{\omega}_{d\alpha}} \frac{\partial F_\alpha}{\partial r}\ , \tag{14}$$

where $\omega_{c\alpha}$ $(\omega_{d\alpha})$ is the fast ion gyrofrequency (magnetic drift frequency), overbar denotes bounce average, $d\Gamma$ is the velocity space volume element, F_α is the equilibrium distribution of the fast ions, and ξ_r is the radial component of the plasma displacement.

To calculate GAE continuum damping, we need expression for the toroidal coupling operator, C^+, near the *q=2* surface, where the mode frequency crosses the *m=2* cylindrical Alfvén continuum (see Fig.1). From Eq.(35) of Ref.[22] it follows that

$$C^+\{\xi_1\}\big|_{q\approx 2} \cong -\frac{1}{2}\left[r\iota'\Delta' + \frac{1}{4}\left(r\Delta'' + 3\Delta' - \frac{r}{R}\right)\right] r^2 \frac{d\xi_1}{dr} = \frac{R}{2}\frac{d\beta}{dr} r^2 \frac{d\xi_1}{dr}\ , \tag{15}$$

where Δ is the Shafranov shift and we used the relation [25]

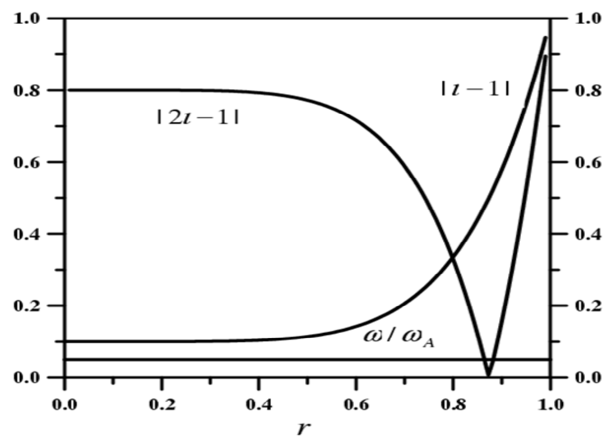

FIG. 1. Location of the GAE frequency with respect to the m=1 and m=2 cylindrical Alfvén continua

$$r\Delta''=\frac{r}{R}-(3-2s)+\alpha_p, \tag{16}$$

with $s=rq'/q$ and $\alpha_p= - q^2R\beta'$. Using Eq.(15), we can rewrite the equation for the $m=2$ harmonic near the $q=2$ surface in the form

$$\frac{d}{dr}\left\{\left[\left(\iota-\frac{1}{2}\right)^2-\left(\frac{3\omega_0}{2\omega_A}\right)^2\right]r^3\frac{d\xi_2}{dr}\right\}=\frac{R}{2}\frac{d}{dr}\left(\frac{d\beta}{dr}r^2\xi_1\right)-\frac{R}{2}\xi_1\frac{d}{dr}\left(\frac{d\beta}{dr}r^2\right), \tag{17}$$

where on the left-hand side we retained only the term with second derivative and we assumed $(\omega_0/\omega_A)^2 << \gamma_s\beta(r_2)$. Note that only the first term on the RHS of Eq.(17) contributes to the continuum damping.

Treating the fast ion drive and continuum damping perturbatively, we obtain the following expression for the corresponding shift of the eigenvalue, $\delta\omega$:

$$\frac{6\omega_0\delta\omega}{\omega_A^2}\int_0^a r^3\left(\frac{d\xi_1}{dr}\right)^2=-\frac{I(\kappa_0^2)}{3}\frac{R^{3/2}}{r_1}i\pi\frac{\omega_0}{\varpi_{dm}}\int_0^a r^{3/2}\frac{d\langle\beta_\alpha\rangle}{dr}\xi_1^2dr-\frac{R}{2}\left(\frac{d\beta}{dr}r^2\xi_1\right)_{r=r_2}\sum_{i=1,2}\int_{r_{Ai}-0}^{r_{Ai}+0}\frac{d\xi_2}{dr}dr. \tag{18}$$

In calculating the fast ion drive [the first term on the RHS of Eq.(18)], we have chosen F_α in the form of a slowing down energy, E, distribution with a δ-function in the pitch angle $\Lambda=\mu B_0/E$, and we retained only the imaginary part of the fast ion response, which is associated with precessional resonance $\omega_0=\varpi_{d\alpha}$. Furthermore, β_α is the fast ion beta, angular brackets denote the flux surface average, $I(\kappa_0)=2\mathrm{E}(\kappa_0)/\mathrm{K}(\kappa_0)-1$, E and K are the complete elliptic integrals, $\kappa_0=\kappa(\Lambda_0)$ with $\kappa^2(\Lambda)\equiv(R/2r)(1/\Lambda-1+r/R)$, $\varpi_{dm}=(E_\alpha/m_\alpha\omega_{c\alpha}r_1R)I(\kappa_0)$ with E_α the injection energy, r_{Ai} are the radii of the Alfvén resonance given by

$\iota(r_{Ai}) = 0.5 \pm 1.5\omega_0 / \omega_A$, and $\xi_1(\omega_0)$ is given by Eq.(4) [Eq.(12)]. Using Eq.(17), we obtain for the GAE excitation threshold

$$-\frac{I(\kappa_0^2)}{3}\frac{R^{3/2}}{r_1}\frac{\omega_0}{\overline{\omega}_{dm}}\int_0^a r^{3/2}\frac{d\langle\beta_\alpha\rangle}{dr}\xi_1^2 dr = \frac{R^2}{6}r_2\left(\frac{d\beta}{dr}\xi_1\right)^2_{r=r_2}\frac{\omega_A}{\omega_0|\iota'|_{r=r_2}} \quad . \tag{19}$$

Using Eq.(4) with boundary condition $\xi_1(a)=0$, one can obtain from Eq.(19) for the model fast ion pressure profile $<\beta_\alpha>(r)=\beta_{\alpha 0}[1-(r/a)^2]^2$,

$$\beta_{\alpha 0}^{th} \cong \frac{7\nu^2(4\nu+7)(8\nu+7)}{2^8\lambda(\nu-2\lambda)^2}\beta_0^2\frac{\varepsilon^{-1/2}}{I(\kappa_0^2)}\frac{\omega_A\overline{\omega}_{dm}}{\omega_0^2}\left(\iota_0-\frac{1}{2}\right)^{\frac{\nu}{\lambda}-5}(1-\iota_0)^{4-(2\nu+1)/\lambda}\left[1-\left(\iota_0-\frac{1}{2}\right)^{2-(\nu/\lambda)}\right]^2, \tag{20}$$

where we have taken into account that $\iota(a)=0$.

As a particular example, we consider the following set of parameters: $\varepsilon=1/3$, $\beta_0=0.1$, $\iota_0=0.9$, $\nu=1$ [$\beta_p(r)=const$], $\lambda=3$, $\overline{\omega}_{dm} = \omega_0$, and $\kappa_0=0$ (deeply trapped particles). From Eq.(12) it follows that $\omega_0 / \omega_A \approx 3.2\times 10^{-2}$ and Eq.(20) yields $\beta_{\alpha 0}^{th} \approx 2.8\times 10^{-3}$, a fairly low value.

4. Discussion and Summary

In the present work we ignored the ion diamagnetic drift effects. This is consistent with the ordering $(\omega_0 /\omega_A)^2 << \beta$ provided $\rho_i /L_p << \varepsilon$, where $\rho_i(L_p)$ is the thermal ion gyroradius (pressure gradient scale length). The frequency ordering implies also that $\omega_0 << \omega_{ti}$, where ω_{ti} is the thermal ion transit frequency. This means that the ion Landau damping of the present GAE should be weak due to the small gradient in velocity space of the resonant ions, and the electron Landau damping can be neglected.

The following properties of the described GAE are consistent with recent observations in JT-60U tokamak [3]: (i) the mode certainly driven by trapped energetic ions and its frequency correlates with precession frequency of the injected ions; (ii) the mode structure shown in Fig.14 of Ref.[3] is peaked on axis and decreases abruptly at the boarder between shear-free and finite shear regions [see Fig.6(c) of Ref.[3] for the *q*-profile], consistent with Eq.(4); (iii) the multi-harmonic excitation at the highest NBI power shown in Fig.11 of Ref.[3] is consistent with excitation of GAEs with $m=n>1$, which can be shown to have eigenfrequencies (for the parabolic pressure profile)

$$\omega_{n>1} = \omega_A\sqrt{\frac{1}{3}\left(\frac{1}{q_0}-1\right)^2 - K_m\left[\left(\frac{\varepsilon}{n}\right)\beta_p\right]^2},$$

where K_m decreases with m. The reason why higher harmonics are excited only at highest NBI power is clear: the pair of the Alfvén resonances around the $q = (n+1)/n$ surface is located much closer to the shear-free core, so that modes with $n > 1$ suffer from higher continuum damping. The abrupt disappearance of higher harmonics when one of the beams slanted in the co-current direction is replaced by the beam slanted counter to the plasma current, as shown in Fig.11 of Ref.[3], can be explained by smaller fast ion content inside the shear-free core (for the same NBI power) in the latter case due to orbital effects.

In summary, it is shown that, in the hybrid tokamak discharges, the low-frequency global Alfvén eigenmode with dominant $m=n=1$ can be easily excited by the trapped energetic ions. Except in the vicinity of the $q=2$ surface, the mode structure is identical to that of the quasi-interchange mode investigated in Ref.[22]. The analysis is restricted to plasmas slightly below the ideal MHD stability limit. The general case of plasmas well below this limit requires several complications, such as coupling to sound waves and geodesic compressibility effects [17, 21]. Although study of such BAE with $m=n=1$ would be interesting, one should bear in mind that for $q_0 \approx 1$ the condition $(\omega_0/\omega_A)^2 \sim (q_0-1)^2 \sim \beta$ is equivalent to $\omega_0 \sim \omega_{ti}$. Therefore, such modes should be suppressed by strong ion Landau damping. This is consistent with mentioned experiment [3], where the $m=n=1$ modes are observed only in hybrids with highest plasma pressure (note that, due to sub-critical injection, $T_i > T_e$ in the shots described in Ref.[3], which further enhances the ion Landau damping at lower plasma pressure).

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