Flow Generation Associate with RF Current Drive in a Tokamak Plasma

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Abstract: Radial electric field can be built by the charge accumulation due to resonant trapped electron pinch, when the rf wave is injected, and then drive the toroidal flow. The resonant pinch effect is evaluated for the parallel-drive scheme. It is found the radial electric field, is proportional to the rf power density. However, the efficiency of the radial electric field drive decays exponentially with respect to the square of the parallel phase velocity of the wave. For low hybrid waves, the parallel phase velocity is a few times of the electron thermal speed, therefore the momentum dissipated into trapped electron is rare otherwise the upshift of the $N_{//}$ spectrum is obvious. To generate typical radial electric field with order of 10kV/m, it is needed a few kW/m^3 power density deposited on thermal or sub-thermal electrons, which is a few percentage of 1MW/m^3 for current drive. However, to explain the strongly correlation between the rotation and the driven current observed in experiments, the power deposited to trapped electrons should be independent (or, at least weakly dependent) to resonant velocity instead of exponential dependence and the redistribution of current profile due to rf current drive should influence the particle transport significantly.

1. Introduction

Effects of flow on toroidal plasmas have been well addressed.[1] However, the neutral beam injection seems not enough to serve as the momentum source to drive the flow required in the ITER or other future reactors. Therefore, it is of great interest to investigate the method of flow drive without significant momentum input. Current drive by rf wave injection was successfully practiced in tokamaks.[2] At the mean time, in many rf experiments, the radial electric field and toroidal rotation can be generated.[3] Since in early experiment, the rotation is usually observed in the case of waves in the range of ion cyclotron frequency, the theories based on the orbit of energetic particles are developed. [4] However, similar rotations were found later in electron cyclotron radio frequency heating plasma or Ohmic plasmas. Then, more theories are developed from the view of neoclassical or turbulence transport. [5] A comprehensive and self-consistent description of many processes of toroidal flow generation and transport is provided in sequential time scales [6]. Also, an inter-machine comparison of intrinsic toroidal rotation is investigated. [7] One of the significant characters is the rotation is proportional to the stored energy but inverse proportional to the plasma current. At present, the theory [8] based on the turbulent momentum transport seems to be promising, since the rotation from a residual stress is confirmed by recent experiment at cylindrical laboratory plasma [9]. However, recent experiment on mode conversion flow drive [10] shows the different behavior, where the flow is significantly larger than the empirical scaling law of the intrinsic rotation and is rather sensitive to different rf-plasma interactions. Especially, another recent experiment [11] indicates that a toroidal flow can be generated by lower-hybrid-waves (LHW) injection, where the flow seems to be closely associated with the rf-driven current. Due to the dual effects on current and flow drive, and also, the technical flexibility of the wave injection, this new discovery of the toroidal rotation produced by the LHW is potentially important. The low hybrid current drive physics has been clear [12]. However, the physical mechanism of toroidal flow generation by the LHW is an open issue. A radial electric field is observed in the experiment, which implies an electron pinch possibly by the mechanism similar to the Ware pinch [13], but due to the rf-electron resonance. Therefore, it is of great interest to investigate whether this electron pinch effect induced by the LHW or other rf waves is relevant to producing a significant radial electric field and then driving a plasma rotation.

In this paper, we evaluate this effect of resonant trapped electron pinch and its relevance to the radial electric field needed for drive flow. It is found that two key factors to influence this effect strongly. One is the resonant velocity which is closely related to power absorbing spectrum, the other is the particle transport property. A rough comparison to the LHW flow drive experiment is presented. To generate typical radial electric field required with order of 10kV/m, a few kW/m^3 power density deposited on the thermal electrons or sub-thermal electrons. However, if the redistribution of current profile due to rf current drive influences the particle transport significantly, the close correlation between the flow drive and the current drive can be realized. Otherwise, these two processes are indeed competitive. A physical model of resonant trapped electron pinch is presented in Sec. 2. The magnitude of electric field driven by this mechanism is estimated in Sec. 3. Discussion based on the comparison to the LHCD experiment is given in Sec. 4 and the conclusion follows in Sec. 5.

2. Physical Model

We evaluated the pinch effect due to rf-resonant interaction for the parallel-drive scheme. Suppose that, during the time interval Δt, the electrons obtain the momentum Δp, parallel to the magnetic field, from the rf wave. This momentum gain is allocated to circulating electrons and trapped electrons separately, as $\Delta p = \Delta p_c + \Delta p_t$. For circulating electrons, a parallel current is determined by balancing the wave driving effects and the collision dragging effects.

$$\frac{\Delta p_c}{\Delta t} = \frac{\nu_e m_e J_{//}}{e} \tag{1}$$

However, the resonant trapped electrons, which absorb the parallel momentum as well, do not contribute to the current. Instead, the vector potential changes to keep the conservation of canonical annular momentum such that a radial current is produced as follows,

$$\frac{\Delta p_t}{\Delta t} = en_t v_r B_\theta \equiv eB_\theta \Gamma_{re} \tag{2}$$

This radial electron flux causes the negative charge accumulation. The charge density is decided by the balance between the rf-induced pinch flux and to the outward diffusion flux due to the slight increase in electron density gradient.

$$\Gamma_{re} = D_n \frac{\Delta q}{eL_n} \tag{3}$$

Here, $\Delta q = e\Delta n$, and it is assumed that the density gradient does not change too much so the change of density gradient is proportional to the change of electron density. Finally, the radial electric field E_r is decided by the Possion's law,

$$\nabla \cdot E_r = \frac{\Delta q}{\varepsilon_0} \tag{4}$$

Now let us connect the electric field to the rf power deposited into plasma. For parallel drive, the transferring energy is $\Delta W = v_{//}\Delta p$, so that the wave power absorbed is connected to the the momentum gain,

$$P_D = v_{//}\Delta p / \Delta t \tag{5}$$

Combine Eqs. (1) to (5), then we get

$$E_r = \frac{\kappa_t L_E L_n}{\varepsilon_0 D_n B_\theta v_{//}} P_D, \tag{6}$$

where $\kappa_t = \Delta p_t / \Delta p, L_E, L_n, \varepsilon_0, D_n, B_\theta, v_{//}, P_D$ are the fraction of momentum into trapped electrons, the radial scale length of electric field and density, the permittivity constant, the particle diffusion coefficient, the poloidal field, the parallel resonant velocity, the power density at given resonant velocity, respectively.

At the mean time, from Eq. (1), we can get the relation of the rf-driven current to the power density as

$$J_{//} = (1-\kappa_t)\frac{e}{m_e v_{//} \nu_e} P_D \tag{7}$$

Here, ν_e is the collisional rate at the given resonant velocity $v_{//}$. Then, we can also connect the radial electric field to $J_{//}$ by the relation

$$E_r = \frac{\kappa_t}{1-\kappa_t}\frac{L_E L_n \nu_e}{\varepsilon_0 D_n \Omega_\theta} J_{//}, \tag{8}$$

where Ω_θ is the electron cyclotron frequency at the poloidal field. Then if the poloidal rotation and the pressure profile is not changed too much due to the rf injection, the change of the toroidal rotation accompanying with the rf current drive may be written as

$$\Delta V_\phi = -E_r / B_\theta \ . \tag{9}$$

Then, a counter-current rotation is expected. In practice, the rf has its power spectrum, then Eqs. (6)-(9) can be consider the Green function to the differential element of rf power. However, it does not influence the order of the electric field and the rotation. In the next section, we will estimate the magnitude of the radial electric field by this mechanism.

3. Estimation of the magnitude of radial electric field

Let us examine Eq. (6). The scale length of density, L_n, is of the order of major radius. The scale length of electric field, L_E, is related to the self-consistent formation of the electric field, but we may estimate it with the order of a fraction of minor radius. The poloidal magnetic field is changed when the rf current is off-axis, for example in the LHCD, but the total current is usually kept constant, therefore, we may assume the separated influence of B_θ is not too much. At the mean time, the re-distribution of the poloidal magnetic field may influence the transport property, so it should be related to the particle diffusion coefficient D_n. However, to estimate D_n is complex, which is still an open issue in tokamak plasmas. We may left this problem in next section and here some typical values of D_n are chosen.

Now we turn to the fraction of momentum into trapped electrons, κ_t. For the resonant absorbing mechanism only, the κ_t is the same as the fraction of trapped particles for given resonant phase velocity $v_{//}$. Consider a large aspect ratio tokamak with the inverse aspect ratio $\varepsilon << 1$, the trapping condition of the particle is $v_\perp^2 \geq v_{//}^2 / 2\varepsilon$, the fraction of trapped particles can be roughly written as

$$\kappa_t(v_{//}) \approx \exp\left[-v_{//}^2(1-\varepsilon)/\left(2\varepsilon v_t^2\right)\right]. \tag{10}$$

Here we neglect the poloidal dependence of trapped condition and possible corresponding magnetic surface average, which decreases the magnitude of κ_t, but not by the difference of order. It is shown that the κ_t decrease exponentially with the increase of the square of resonant velocity. Therefore, the driven electric field decrease dramatically with the increase of resonant velocity, if it is assumed other parameters in Eq. (6) do not changed with the resonant phase. Figure 1 (a) shows this dependence for $\varepsilon = 1/6$. For comparison, the driven current $J_{//}$ are plotted in figure 1(a) with arbitrary unit as functions of resonant velocity. Here, we use the simple 1-D formula, $\nu_e = \nu_0 (v_{//}/v_{te})^{-3}$, for the collisional rate at the given resonant velocity.

Also, adopting the parameters such as $L_n = 0.3m$, $L_E = 0.03m$, $B_\theta = 0.5T$, $T_e = 3keV$, and $\varepsilon = 1/6$, the magnitude of electric field E_r is plotted as functions of power density P_D for in figure 1(b). Two diffusion coefficients are chosen: one is the typical anomalous value $D_n \sim 1m^2/s$, the other is the neoclassical value $D_n \sim 3\times10^{-3} m^2/s$ (assuming $n = 1\times10^{20} m^{-3}$, $B = 5T$, $q = 3$).

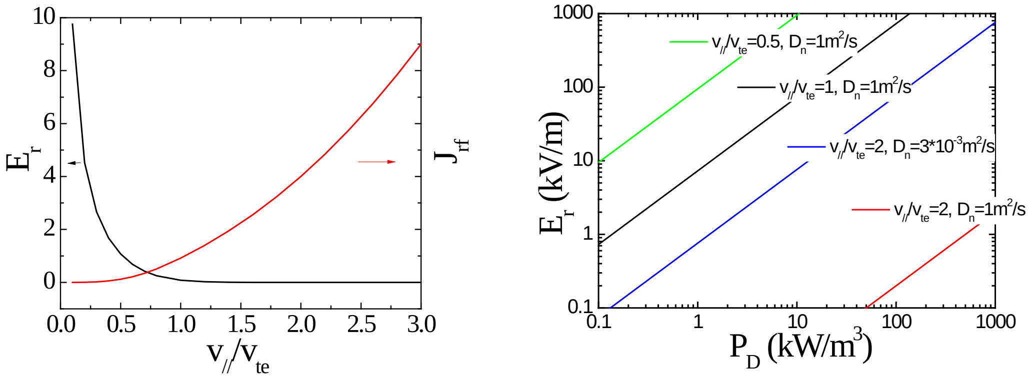

Figure1. (a) E_r and $J_{//}$ as functions of resonant velocity; (b) the magnitude of the driven electric field as functions of the power density for different resonant velocities and diffusion coefficient at typical parameters.

3. Discussion

In this section, we may compare above results from the model of resonant trapped electron pinch to the observation of toroidal rotation in C-Mod LHCD experiments.

Firstly, from Eq. (9), the resonant trapped electrons pinch can induce a counter-current rotation, which is consistent to the experiment observation.

Secondly, to driven the flow observed in experiment, a radial electric field with order of 10kV/m is required. For typical LHCD experiment, the parallel phase velocity is several times of the electron thermal speed. For example the spectrum $N_{//} = 2$ corresponds to $v_{//res} \sim 6v_{te}$ for $T_e = 3keV$. Therefore the momentum deposited into trapped electron is rare. Even the so-called $N_{//}$ spectrum upshift [14] occurs, for example, for $v_{//res} \sim 2v_{te}$ (the red line in Fig. 1(b)), to generate typical radial electric field with order of 10kV/m need a power density beyond 1MW/ m^3. However, if there are some mechanisms to deposit the power or the momentum directly on thermal or sub-thermal electrons, the typical electric field can be expected. For example, to generate a radial electric field with order of 10kV/m, it is needed a few kW/m^3 power density deposited on the thermal electrons [the black line in Fig 1(b)], which is a few percentage of 1MW/ m^3 for current drive. If the rf power is deposited on trapped electron directly, hundred Walt is enough to driven the expected electric field [the green line in Fig 1(b)]. This kind of power deposition can be realized by the so-called parametric process coupling to low frequency wave such as ion cyclotron waves or sound waves. [15] Recent study [16] shows the trapped electron mode can be destabilized by the parametric processes induced by the low hybrid injection. The anomalous Doppler broadening effect can scatter fast electrons along the transverse velocities [17] and then realize the desired momentum deposition. On the other hand, if the particle confinement is improved the requirement to the $N_{//}$ absorbing spectrum will be loosened. The blue line indicates the case of $v_{//res} = 2v_{te}$ and the neoclassical diffusion level. It is shown that tens kW/m^3 power density can drive tens kV/m electric field.

The third problem is concerning the strongly correlation between the rotation and the internal inductance observed in C-Mod LHCD experiments. Although Eq. (8) may claim that the driven rotation is proportional to the driven parallel current throughout the power density, these two roles of rf are indeed competitive if we consider their dependence on resonant velocity. Figure 1 (a) indicates this point that, when the rf driven current increases with the resonant velocity, the radial electric field decreases. This is quite opposite to the experiment observation, where both the current and the rotation increases with the decrease of $N_{//}$. Moreover, the rotation seems to lag behind the current distribution, which may indicate the causality. One possible way is to consider the effect of current profile on the particle transport. It is mentioned the article diffusion coefficient D_n we used is an effective parameter. It might include both diffusion and/or convection (pinch) through neoclassical and turbulence process. Usually it is thought the ion temperature gradient (ITG) mode and/or trapped electron mode (TEM) may be responsible to turbulence particle transport. We may neglect to distinguish the diffusion and convection, and directly write the unperturbed particle conservation equation as follows

$$S \equiv -D_n \nabla n = \Gamma_0 + \Gamma_t, \qquad (11)$$

where S is the source, Γ_0 is the flux due to neoclassical transport and other processes such as stochastic magnetic field, and Γ_t is the flux due to drift wave turbulence. Usually the Γ_0 is outward (positive D_n), a source is necessary to keep the density profile desired. Then, if the turbulent flux Γ_t is also outward, the effective diffusion coefficient D_n will be enhanced. On the contrary, if the Γ_t is inward, the D_n will decrease. The typical conclusion is that the curvature effect always induces an inward flux, but that the “thermo-diffusion” term can either be inward when the ITG is dominant and outward when the TEM is dominant. The injection of LHW can reduce the percentage of trapped electrons and then has a stabilizing effect on the TEM, but this effect is expected to be weak since only rare resonant electrons are near the trapped-passing boundary. However, if the ITG is dominant, the weakening of magnetic shear due to off-axis LHCD may influence the transport dramatically. Using the mixing length estimation, we can assume the flux due to the ITG turbulence can be described as

$$\Gamma_t \equiv -\alpha\left[R/L_{Ti} - \left(R/L_{Ti}\right)_c\right] \tag{12}$$

where $\alpha = 0$ for $R/L_{Ti} < \left(R/L_{Ti}\right)_c$ and $\alpha > 0$ for $R/L_{Ti} > \left(R/L_{Ti}\right)_c$ indicates the inward pinch flux due to unstable ITG modes. Neglecting the dependence of α, the critical threshold $\left(R/L_{Ti}\right)_c$ usually decreases with magnetic shear. A complex formula for the $\left(R/L_{Ti}\right)_c$ was obtained [18] based on numerical simulation result and a more simple critical gradient formula is given for toroidal electron temperature gradient (ETG) modes. [19] The latter may also serves as a good approximation if we considering the symmetry between ITG and ETG (roughly, only the role of ions and electrons are switched) . Here, we may give a rough estimation

$$\left(R/L_{Ti}\right)_c \approx C\left(1 + k_1\,\hat{s}/q\right), \tag{13}$$

where C is about 4~6 depending on other parameters, k_1 is of unity indicating the $\hat{s}/q$ dependence. When the rf is absent, the profiles will be kept quite close to $\left(R/L_{Ti}\right)_c$, so that We can assume $R/L_{Ti} = \left(R/L_{Ti}\right)_c + \delta$. The rf driven current decreases the value of $\hat{s}/q$. A rough estimation gives

$$\Delta\left(\hat{s}/q\right) = \frac{\mu_0 RS}{B_\phi \pi (r^2 - a^2)} J_{//} \equiv k_2 J_{//}, \tag{14}$$

where $J_{//}S$ is the total rf driven current. For typical parameters, $k_2 \sim 10^{-7} - 10^{-6}$. Then, considering $J_{//} \sim \eta \hat{v}_{//}^2$, the effective diffusion coefficient D_n with the rf can be determined from Eqs. (11) – (14).

$$D_n = \frac{\Gamma_0 - \alpha\delta - \alpha C k_1 k_2 J_{//}}{-\nabla n} \equiv D_0\left(1 - k\hat{v}_{//}^2\right), \tag{15}$$

where D_0 is the effective diffusion coefficient without rf and $k = \alpha C k_1 k_2 \eta / \left(\Gamma_0 - \alpha\delta\right)$. If there are some mechanisms to allocate the power to trapped electrons independent on the resonant velocity, the induced electric field will increases with $\hat{v}_{//}^2$ and the ratio of E_r and $J_{//}$ in Eq. (8) becomes $1/\left[\hat{v}_{//}^2\left(1 - k\hat{v}_{//}^2\right)\right]$. This ratio changes not much when $\hat{v}_{//}^2$ varies in wide regimes, which is consistent to experiment observations.

4. Conclusions

The mechanism of flow drive by resonant trapped electron pinch is evaluated. Radial electric field can be built by the charge accumulation due to resonant trapped electron pinch, when the rf wave is injected, and then drive the toroidal flow. Although a counter-current rotation can be induced, which is consistent to observations in C-Mod LHCD experiments, there is still two key factors prevent this mechanism explaining the experiment observations completely. Firstly, there should be enough momentum (or power) deposited to trapped electron. To generate typical radial electric field with order of 10kV/m, a few kW/m^3 power density deposited on the thermal electrons or sub-thermal electrons. Secondly, to explain the strongly correlation between the rotation and the driven current observed in experiments, the power deposited to trapped electrons should be independent (or, at least weakly dependent) to resonant velocity instead of exponential dependence and the redistribution of current profile due to rf current drive should influence the particle transport significantly. Otherwise, these two processes are indeed competitive when the resonant velocity changes.

For fast electron drive, the above critical conditions should be satisfied to build the radial electric field desired. However, for low frequency rf waves, the current drive efficiency is weakened due to the enhanced electron trapping at lower phase velocity, but the generation of electric field is of high efficiency. Therefore, the flow drive mechanism of resonant trapped electron pinch may be observed easily when injecting fast wave or Alfven wave into plasma.

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