Integrated Non-modal Linear and Renormalized Nonlinear Approach to the Theory of Drift Turbulence in Plasma Shear Flow

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1.Introduction

One of the most effective approaches to linear analysis of the evolution of plasma turbulence in shear flows is the Kelvins method of shearing modes or so-called non-modal approach. This method is equally reliable for the treatment of the plasma flows with arbitrary magnitude of the velocity shear[1], for spatially homogeneous or inhomogeneous velocity shear[2], for flows with time dependent velocity shear (for which it is unreasonable to apply the spectral expansion in time)[3]. The exceptional advantage of the non-modal approach is a possibility to perform the analysis of the plasma evolution on any finite time domain. This method appears to be very effective for the investigations of the sequences of the evolutionary processes in shear flows (see, for example Ref.[4]), which may incorporate different time scales and may appear at different stages of the temporal evolution. The essence of this approach, which originally was developed for fluid equations by Kelvin for flows with homogeneous velocity shear, consists in transforming the independent spatial variables from the laboratory frame to a frame convected with shear flow and studying the temporal evolution of the spatial Fourier modes of perturbation without any spectral expansions in time ([1-4] and references therein). The transformation to the coordinates convected with shear flow eliminates the explicit spatial dependence, related to shear flow, from the convective derivative in governing fluid equations. This transformation not only simplifies governing equations, but it is principally indispensable. The temporal evolution of the separate spatial Fourier harmonic with definite wave numbers may be considered only with convective coordinates; it is in contrast to the laboratory set of reference, in which spatial Fourier harmonics are coupled due to velocity shear.

Kinetic effects, such as finite Larmor radius effects, effects of Landau and cyclotron damping and resulted numerous kinetic instabilities, which, naturally, do not involved into fluid description of plasma shear flows, require the development of the kinetic description of the plasma shear flows, which has the Kelvins method of shearing modes or so-called non-modal approach as its foundation.

The investigation, results of which are presented in this report, have its objects 1)nonlinear renormalized non-modal fluid theory of drift turbulence in the case of the plasma flows with a moderate velocity shear (of the order of or greater than the instability growth rate, but less than the drift waves frequency), 2)the development of the non-modal approach to linear kinetic theory of plasma shear flows and its application to the investigation of the temporal evolution of the kinetic drift instability and 3) development of the non-modal renormalized kinetic theory of drift turbulence in plasma shear flows.

2. Renormalized Hydrodynamic Theory for Drift Turbulence in Plasma Shear Flows

We consider one of the simplest models describing drift turbulence at the edge of a magnetic confinement device, the Hasegawa-Wakatani equations for the dimensionless density $n = \tilde{n}/n_e$ and potential $\phi = e\varphi/T_e$ perturbations (n_e is the electron background density, T_e is the electron temperature),

$$\rho_s^2\left(\frac{\partial}{\partial t}+V_0'x\frac{\partial}{\partial y}-\frac{cT_e}{eB}\left(\frac{\partial\phi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\phi}{\partial x}\frac{\partial}{\partial y}\right)\right)\nabla^2\phi = a\frac{\partial^2}{\partial z^2}(n-\phi), \tag{1}$$

$$\left(\frac{\partial}{\partial t}+V_0'x\frac{\partial}{\partial y}-\frac{cT_e}{eB}\left(\frac{\partial\phi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\phi}{\partial x}\frac{\partial}{\partial y}\right)\right)n+v_{de}\frac{\partial\phi}{\partial y} = a\frac{\partial^2}{\partial z^2}(n-\phi), \tag{2}$$

$\mathbf{V}_0(x) = V_0'x\mathbf{e}_y$ is the velocity of the sheared flow, $a = T_e/n_0e^2\eta_\parallel$, $\eta_\parallel$ is the resistivity parallel to homogeneous magnetic field $\mathbf{B}_\parallel\mathbf{z}$, ρ_s is the ion Larmor radius at electron temperature T_e, $v_{de} = cT_e/eBL_n$ is the diamagnetic drift velocity, $L_n^{-1} = -d\ln n_{0e}(x)/dx$. With new spatial variables x_i, y_i, determined by the transform [1]

$$t = t, \qquad x_i = x, \qquad y_i = y - V_0'xt, \qquad z = z, \tag{3}$$

Eqs. (1) and (2) have a form

$$\rho_s^2\left(\frac{\partial}{\partial t}-\frac{cT_e}{eB}\left(\frac{\partial\phi}{\partial y_i}\frac{\partial}{\partial x_i}-\frac{\partial\phi}{\partial x_i}\frac{\partial}{\partial y_i}\right)\right)\nabla^2\phi = a\frac{\partial^2}{\partial z^2}(n-\phi), \tag{4}$$

$$\left(\frac{\partial}{\partial t}-\frac{cT_e}{eB}\left(\frac{\partial\phi}{\partial y_i}\frac{\partial}{\partial x_i}-\frac{\partial\phi}{\partial x_i}\frac{\partial}{\partial y_i}\right)\right)n+v_{de}\frac{\partial\phi}{\partial y_i} = a\frac{\partial^2}{\partial z^2}(n-\phi). \tag{5}$$

with time dependent Laplacian

$$\Delta = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} = \left(\frac{\partial}{\partial x_i}-V_0't\frac{\partial}{\partial y_i}\right)\left(\frac{\partial}{\partial x_i}-V_0't\frac{\partial}{\partial y_i}\right)+\frac{\partial^2}{\partial y_i^2}. \tag{6}$$

Transformed system (4)-(6) has two remarkable properties: it does not contain the linear spatially inhomogeneous convection terms and it contains the $E\times B$ non-linear convective derivatives in a form similar to the one in a plasma without any flows. The spatially homogeneous system (4), (5) may be Fourier transformed over variables x_i, y_i, z with electrostatic potential

$$\phi(x_i, y_i, z, t) = \int dk_x\int dk_y\int dk_z\phi(k_x, k_y, k_z, t)\,e^{ik_xx_i+ik_yy_i+ik_zz}. \tag{7}$$

For the perturbations with $k_y\rho_s < 1$, Eqs.(4) -(6) with transformation (3) introduce two characteristic times, $t_1 = (V_0')^{-1}$ and $t_2 = (V_0'k_y\rho_s)^{-1}$, conditioned by flow shear. With convective variables in times $t > t_1$, but less than t_2, the perturbations have the frequency and growth rate of the ordinary modal drift resistive instability without spatially inhomogeneous Doppler shift. The governing equations (4) – (6) on that time interval do not display any effects of the velocity shear. In the laboratory frame of reference spatial Fourier modes (7) are observed as sheared modes with time dependent component of the wave number $k_x = k_\perp - lv_0't$ directed along the velocity shear, and therefore are quite different from the normal mode assumption,

$$\phi(\mathbf{r},t)=\int dk_\perp \int dl \int dk_z \phi(k_\perp,l,0)\, g(k_\perp,l,t)\, e^{i\left((k_\perp - lV_0' t)x + ily + ik_z z\right)}. \qquad (8)$$

Solution (8) has a normal mode form only for a limited time, for which $|V_0' t| \ll 1$. When $V_0' \simeq \gamma$ non-modal effect becomes pronounced at times $t \geq \gamma^{-1}$ (γ is the modal instability growth rate). The comprehensive investigation in linear approximation the temporal evolution of the separate spatial Fourier mode of the electrostatic potential $\phi(k_x,k_z,k_z,t)$, was performed on the base of system (4), (5) in Ref.[1] in first. The renormalized nonlinear theory of the system (4), (5) was developed in first in Ref.[3]. The theory developed in Ref.[3] bases on the application the nonlinearly distorted coordinates, $\xi = x_i + \xi_1$, $\eta = y_i + \eta_1$ instead of coordinates x_i, y_i. The quantities ξ_1 and η_1 are determined by nonlinear relations

$$\xi_1 = -\frac{cT_e}{eB}\int_0^t \frac{\partial \phi}{\partial y_i} dt_1, \qquad \eta_1 = \frac{cT_e}{eB}\int_0^t \frac{\partial \phi}{\partial x_i} dt_1. \qquad (9)$$

With variables ξ,η the quadratic convective nonlinearity in Eqs.(4) and (5) becomes of the higher order. Omitting these nonlinear terms we come to linearized system (4), (5) with solution in variables x_i and y_i of a form [3]

$$\phi(x_i,y_i,t)=\int dk_x \int dk_y \phi(k_x,k_y,0)\, g(k_x,k_y,t_1)\, e^{ik_x x_i + ik_y y_i - ik_x \xi_1(t_1) - ik_y \eta_1(t_1)}, \qquad (10)$$

where wave numbers k_x, k_y are conjugate now to coordinates ξ, η, respectively, $\phi(k_x,k_y,0)$ is the initial data for perturbed potential, and $g(k_x,k_y,t_1)$ is unstable linear solution of Eqs.(4), (5) for electrostatic potential. Eq.(10) is in fact a nonlinear integral equation for potential ϕ, in which the effect of the total Fourier spectrum on any separate Fourier harmonic is accounted for. This form of solution, however, appears very useful for the analysis of the correlation properties of the nonlinear solutions to Hasegawa-Wakatani system and for the development of the approximate renormalized solutions to Hasegawa-Wakatani system, which accounted for the effect of the turbulent motions of plasma on the saturation of the drift-resistive instability. Assuming that the displacements $\xi_1(t)$, $\eta_1(t)$ obey the Gaussian statistics with mean zero, the renormalized form of the potential (10), in which the average effect of the random convection is accounted for, was obtained in Ref.[3],

$$\phi(x_i,y_i,t)=\int dk_x \int dk_y \phi(k_x,k_y,t_0)\exp\left(i\omega_d t + \gamma t - \int_0^t dt_1 C(k_x,k_y t_1) + ik_x x_i + ik_y y_i,\right) \qquad (11)$$

where $C(k_x,k_y,t)$ is determined by the integral equation

$$C(k_x,k_y,t)=\frac{c^2T_e^2}{e^2B^2}\int dk_{1x}\int dk_{1y} \left|\phi(k_{1x},k_{1y},t)\right|^2 \left|[\mathbf{k}_\perp \times \mathbf{k}_{1\perp}]\right|^2 \frac{C(k_{1x},k_{1y},t)}{\omega_d^2(k_{1x},k_{1y})}. \qquad (12)$$

and $\left|\phi(k_x,k_y,t)\right|^2 = \left|\phi(k_x,k_y,0)\right|^2 e^{2\gamma(k_x,k_y)t}$. In contrast to the case of plasma without shear flow, for which the steady state establishes on the level determined by the equation $\gamma(k_x,k_y)=C(k_x,k_y,t)$, that level is transient for plasma shear flows and holds only for a limited time $t \leq t_2$. The time evolution of the system (4), (5) at times $t \geq t_2$ characterizes by the non-modal effect of the enhanced dispersion, due to which the potential ϕ has a strongly non-modal form[1] with $g(k_x,k_y,k_z,t) \simeq (k_y \rho_s V_0' t)^{-2}$, for which Markovian approximation is not valid. Obtained results clearly show that the nonlinearity of the Hasegawa-Wakatani system of equations in variables x_i and y_i, with which frequency and growth rate are determined without spatially inhomogeneous Doppler shift and wave number is time independent, does not display effect of "the enhanced decorrelations provided by flow shear", which was considered in Refs.[5,6] as a main effect of the suppression of the drift turbulence in shear flows. As it follows from [5,6], the effect of the enhanced decorrelation is completely conditioned by convective derivative term $V_0' x(\partial/\partial y)$ in Eqs.(1) and (2) which is responsible for the Doppler shift. The effect of the enhanced dispersion of plasma displacements displays the variance of the plasma displacements resulted from the modes of the ordinary modal form, but observed in the laboratory frame.

3. Non-modal Approach to Kinetic Theory of Plasma Shear Flows

The results presented above display the effectiveness of the non-modal approach in the analysis of the linear and non-linear evolution of the plasma turbulence in shear flows. In this section we extend the non-modal approach onto kinetic theory of plasma shear flows. The transformation of spatial coordinates and velocity in Vlasov equation for the distribution function F_α to convective coordinates,

$$v_x = v_{\alpha x}, v_y = v_{\alpha y} + V_0' x_\alpha, v_z = v_{z\alpha}; x = x_\alpha, y = y_\alpha + V_0' x_\alpha t, z = z_\alpha, \tag{13}$$

(where $V_0' = dV_0/dx$, and it was assumed that shear flow appears at time $t=0$) excludes the spatial inhomogeneity introduced by shear flow from Vlasov equation,

$$\frac{\partial F_\alpha}{\partial t} + v_{\alpha x}\frac{\partial F_\alpha}{\partial x_\alpha} - \left(v_{\alpha y} - v_{\alpha x} V_\alpha' t\right)\frac{\partial F_\alpha}{\partial y_\alpha} + \omega_{c\alpha} v_{\alpha y}\frac{\partial F_\alpha}{\partial v_{\alpha x}} - \left(\omega_{c\alpha} + V_\alpha'\right) v_{\alpha x}\frac{\partial F_\alpha}{\partial v_{\alpha y}}$$
$$-\frac{e_\alpha}{m_\alpha}\left(\frac{\partial\varphi}{\partial x_\alpha} - V_\alpha' t\frac{\partial\varphi}{\partial y_\alpha}\right)\frac{\partial F_\alpha}{\partial v_{\alpha x}} + v_{\alpha z}\frac{\partial F_\alpha}{\partial z_\alpha} - \frac{e_\alpha}{m_\alpha}\frac{\partial\varphi}{\partial y_\alpha}\frac{\partial F_\alpha}{\partial v_{\alpha y}} - \frac{e_\alpha}{m_\alpha}\frac{\partial\varphi}{\partial z_\alpha}\frac{\partial F_\alpha}{\partial v_{\alpha z}} = 0. \tag{14}$$

With leading center and velocity space coordinates,

$$X_\alpha = x_\alpha + \frac{v_\perp}{\sqrt{\mu_\alpha}\,\omega_{c\alpha}}\sin\left(\phi_1 - \sqrt{\mu_\alpha}\,\omega_{c\alpha} t\right),$$

$$Y_\alpha = y_\alpha - \frac{v_\perp}{\mu_\alpha \omega_{c\alpha}}\cos\left(\phi_1 - \sqrt{\mu_\alpha}\,\omega_{c\alpha} t\right) - V_0' t\left(X_\alpha - x_\alpha\right), \qquad z_1 = z - v_z t, \tag{15}$$

$$v_{\alpha x} = v_\perp \cos\phi, \qquad v_{\alpha y} = \sqrt{\mu_\alpha}\, v_\perp \sin\phi, \qquad \phi = \phi_1 - \sqrt{\mu_\alpha}\,\omega_{c\alpha} t, \qquad v_z = v_{\alpha z}, \tag{16}$$

which correspond to more and more stretched with time along the shear flow Larmor orbit, the equation for the perturbation f_α of the equilibrium distribution function has a simple form,

$$\frac{\partial f_\alpha}{\partial t} = \frac{e_\alpha}{m_\alpha}\left[\frac{1}{\mu_\alpha \omega_{c\alpha}}\frac{\partial\varphi}{\partial Y_\alpha}\frac{\partial F_{0\alpha}}{\partial X_\alpha} - \frac{\sqrt{\mu_\alpha}\,\omega_{c\alpha}}{v_\perp}\frac{\partial\varphi}{\partial\phi_\alpha}\frac{\partial F_{0\alpha}}{\partial v_{\perp\alpha}} + \frac{\partial\varphi}{\partial z_\alpha}\frac{\partial F_{0\alpha}}{\partial v_{z\alpha}}\right]. \tag{17}$$

where $\mu_\alpha = 1 + V_0'/\omega_{c\alpha} > 0$. It is interesting to note, that the equilibrium distribution function $F_{0\alpha}$, which in laboratory frame contains the spatial inhomogeneity resulted from electric field $E_0(r)$, does not contain such inhomogeneity in coordinates (15),(16). In what follows we consider the equilibrium distribution function F_{i0} as a Maxwellian,

$$F_{0\alpha} = \frac{n_{0\alpha}\left(X_\alpha\right)}{\left(2\pi v_{T\alpha}^2\right)^{3/2}}\exp\left(-\frac{v_\perp^2 + v_z^2}{v_{T\alpha}^2}\right), \tag{18}$$

assuming the dependence on coordinate X_α of the density of plasma shear flow species. The spatially homogeneous, but time dependent, Eqs.(14) and (17) may be Fourier transformed over the variables $x_\alpha, y_\alpha, z_\alpha$ with conjugate wave numbers k_x, k_y and k_z and be traced upon the temporal evolution of the separate spatial Fourier mode of the perturbations of the distribution function, f_α and of the electrostatic potential, $\varphi(\mathbf{k},t)$. On this way we present the potential $\varphi(r,t)$ at the form

$$\varphi(x_\alpha, y_\alpha, z_\alpha, t) = \int \varphi(k_x, k_y, k_z, t) e^{ik_x x_\alpha + ik_y y_\alpha + ik_z z_\alpha} dk_x dk_y dk_z = \int \varphi(k_x, k_y, k_z, t)$$
$$\exp\left[ik_x X + ik_y Y + ik_z z_\alpha - i\frac{k_\perp(t) v_\perp}{\sqrt{\mu_\alpha}\,\omega_{c\alpha}} \sin\left(\phi - \sqrt{\mu_\alpha}\,\omega_{c\alpha} t - \theta(t)\right)\right] dk_x dk_y dk_z, \quad (19)$$

where $k_\perp^2(t) = \left(k_x - V_0' t k_y\right)^2 + k_y^2$ and $\tan\theta = k_y / (k_x - V_0' t k_y)$. The solution to Eq.(14) is calculated easily for any values of the velocity shear rate V_0'. Using that solution in the Poisson equation, $\Delta\varphi(\mathbf{r},t) = -4\pi \sum_{\alpha=i,e} e_\alpha \int f_\alpha(\mathbf{v}_\alpha, \mathbf{r}, t)\, d\mathbf{v}_\alpha$, we obtain the integral equation, which appears to be convenient and transparent for the analysis,

$$\left[\left(k_x - V_0' t k_y\right)^2 + k_y^2 + k_z^2\right]\varphi(\mathbf{k},t) = -\sum_{\alpha=i,e}\frac{1}{\lambda_{D\alpha}^2}\varphi(\mathbf{k},t) + \sum_{\alpha=i,e}\frac{1}{\lambda_{D\alpha}^2}\sum_{n=-\infty}^{\infty}\int_{t_0}^{t} dt_1 \frac{d}{dt_1}\Big\{\varphi(\mathbf{k},t_1)$$
$$\times I_n\left(k_\perp(t)k_\perp(t_1)\rho_\alpha^2\right) e^{-\frac{1}{2}\rho_\alpha^2\left(k_\perp^2(t)+k_\perp^2(t_1)\right) - in\left(\theta(t)-\theta(t_1)\right)}\Big\} e^{-\frac{1}{2}k_z^2 v_{T\alpha}^2 (t-t_1)^2 - in\sqrt{\mu}\,\omega_{c\alpha}(t-t_1)}$$
$$+\sum_{\alpha=i,e}\frac{i}{\lambda_{D\alpha}^2}\sum_{n=-\infty}^{\infty}\int_{t_0}^{t} dt_1 \varphi(\mathbf{k},t_1)\frac{k_y v_{d\alpha}}{\sqrt{\mu}}$$
$$\times I_n\left(k_\perp(t)k_\perp(t_1)\rho_\alpha^2\right) e^{-\frac{1}{2}\rho_\alpha^2\left(k_\perp^2(t)+k_\perp^2(t_1)\right) - in\left(\theta(t)-\theta(t_1)\right) - \frac{1}{2}k_z^2 v_{T\alpha}^2 (t-t_1)^2 - in\sqrt{\mu}\,\omega_{c\alpha}(t-t_1)}$$
$$-4\pi\sum_{\alpha=i,e} e_\alpha \delta n_\alpha(\mathbf{k},t,t_0) + \sum_{\alpha=i,e}\frac{1}{\lambda_{D\alpha}^2}\varphi(\mathbf{k},t_0) P_\alpha(t,t_0). \quad (20)$$

where

$$P_\alpha(t,t_0) = \sum_{n=-\infty}^{\infty} I_n\left(k_\perp(t)k_\perp(t_0)\rho_\alpha^2\right)$$
$$\times e^{-\frac{1}{2}\rho_\alpha^2\left(k_\perp^2(t)+k_\perp^2(t_0)\right) - in\left(\theta(t)-\theta(t_0)\right) - \frac{1}{2}k_z^2 v_{T\alpha}^2 (t-t_0)^2 - in\sqrt{\mu}\,\omega_{c\alpha}(t-t_0)}. \quad (21)$$

Note, that $P_\alpha(t_0,t_0) = 1$. The secular growth of the wavenumber $k_\perp(t)$ in Eq.(20) is the key element in the proper treatment of the long-time evolution of the perturbations in shear flow. Using the quasineutrality approximation with $\left(k_\perp^2(t) + k_z^2\right)\lambda_{D\alpha}^2 \ll 1$, and averaging this equation over the time $t \gg \omega_{ci}^{-1}$, we obtain from Eq.(20) the equation, which is relevant for the analysis of the evolution of low frequency drift perturbations in shear flow,

$$\int_{t_0}^{t} dt_1 \frac{d}{dt_1}\left\{\Phi(\mathbf{k},t_1)\left[(1+\tau) - I_0\left(k_\perp(t)k_\perp(t_1)\rho_i^2\right) e^{-\frac{1}{2}\rho_i^2\left(k_\perp^2(t)+k_\perp^2(t_1)\right)}\right]\right\}$$
$$-i\int_{t_0}^{t} dt_1 \Phi(\mathbf{k},t_1) k_y v_{di} I_0\left(k_\perp(t)k_\perp(t_1)\rho_i^2\right) e^{-\frac{1}{2}\rho_i^2\left(k_\perp^2(t)+k_\perp^2(t_1)\right)}$$

$$= -\int_{t_0}^{t} dt_1 \frac{d}{dt_1}\left(\Phi(\mathbf{k},t_1) I_0\left(k_\perp(t)k_\perp(t_1)\rho_i^2\right) e^{-\frac{1}{2}\rho_i^2\left(k_\perp^2(t)+k_\perp^2(t_1)\right)}\right)\left(1-e^{-\frac{1}{2}k_z^2 v_{Ti}^2 (t-t_1)^2}\right)$$
$$-i\int_{t_0}^{t} dt_1 \Phi(\mathbf{k},t_1) k_y v_{di} I_0\left(k_\perp(t)k_\perp(t_1)\rho_i^2\right) e^{-\frac{1}{2}\rho_i^2\left(k_\perp^2(t)+k_\perp^2(t_1)\right)}\left(1-e^{-\frac{1}{2}k_z^2 v_{Ti}^2 (t-t_1)^2}\right)$$
$$+\tau\int_{t_0}^{t} dt_1 \left(\frac{d\Phi(\mathbf{k},t_1)}{dt_1} + ik_y v_{de}\Phi(\mathbf{k},t_1)\right) e^{-\frac{1}{2}k_z^2 v_{Te}^2 (t-t_1)^2}, \tag{22}$$

where $\Phi(\mathbf{k},t) = \varphi(\mathbf{k},t)\Theta(t-t_0)$, $\Theta(t-t_0)$ is equal to zero for $t < t_0$ and equal to unity for $t \ge t_0$, and $\tau = T_i / T_e$. We have obtained by successive approximations approximate solutions to Eq.(22) for long wavelength, $k_\perp(t_0)\rho_i < 1$, perturbations in two limited cases without application the spectral transformation over time: in the case of the weak velocity shear, or a small time, for which condition $|V_0' t| \ll 1$ is met, and solution for the nonmodal stage, which is settled at $V_0' t \gg 1$. The weak ion Landau damping, for which $\left|1-e^{-\frac{1}{2}k_z^2 v_{Ti}^2 (t-t_0)^2}\right| \ll 1$, is assumed.

At the case $|V_0' t| \ll 1$, we can omit the time dependence of the wave number $\mathbf{k}_\perp(t)$. In that case Eq.(22) has an ordinary modal form.

$$\Phi(\mathbf{k},t) = C\exp\left(-i\omega(\mathbf{k})t + \gamma(\mathbf{k})t\right) \tag{23}$$

where $\omega(\mathbf{k})t$ and $\gamma(\mathbf{k})$ are the frequency and growth rate of the modal kinetic drift instability developed due to inverse electron Landau damping of drift waves. Now consider the time interval for which $(V_0')^{-1} \ll t \ll t_S = \left(V_0' k_\perp \rho_i\right)^{-1}$. If $k_\perp \rho_i < 1$ at time $t = 0$, at which the shear flow emerged, we will get $k_\perp(t)\rho_i < 1$ everywhere on that interval. By using the approximation

$$I_0\left(k_\perp(t)k_\perp(t_1)\rho_i^2\right) e^{-\frac{1}{2}\rho_i^2\left(k_\perp^2(t)+k_\perp^2(t_1)\right)}$$
$$\approx b_i + \left(k_x k_y V_0' \rho_i^2 (t+t_1) - \frac{1}{2}k_y^2 \rho_i^2 (V_0')^2\left(t^2+t_1^2\right)\right)\Theta(t), \tag{24}$$

in Eq.(22), where $a_i = \tau + k_\perp^2 \rho_i^2$ and $b_i = 1 - k_\perp^2 \rho_i^2$, we obtain the non-modal solution for the potential of drift-type perturbations, modified by the non-modal velocity shear driven effects, in the form

$$\Phi(\mathbf{k},t) = \Phi_0 \exp\left[-i\omega(\mathbf{k})\left(1 - \frac{1+\tau}{a_i b_i}\frac{t^2}{3t_s^2}\right)t + \gamma(\mathbf{k})t - \frac{t^2}{2a_i t_s^2}\right]. \tag{25}$$

As it follows from Eq.(25) these effects, which reveals in non-modal reduction of the frequency and growth rate, are negligible at $t \ll t_s$ and become dominant at $t \sim t_s$. Note, that for $\tau \gg k_\perp^2 \rho_i^2$ the time $a_i^{1/2} t_s$ is approximately equal to time $t_2 = \left(V_0' k_\perp \rho_s\right)^{-1}$ of the transition to strongly non-modal regime in the fluid theory of the drift turbulence of the plasma shear flow.

4. Renormalized Kinetic Theory for Drift Turbulence in Plasma Shear Flows

By application the methodology of the renormalization of the Vlasov equation, developed in Ref.[7], to plasma shear flow across the magnetic field, the renormalized version of the Eq.(17) is obtained, in which effect of nonlinear breakdown of phase of the potential (19) due to turbulent scattering of ions in electrostatic turbulence was accounted for. We find, that for the times $t < (V_0')^{-1}$ the main effect, which determines the nonlinear scattering of ions by long wavelength drift turbulence with $k_\perp \rho_i < 1$ is the scattering of the leading center coordinates, δX and δY. The non-modal effects are negligible at this time. At times $t > (V_0')^{-1}$ right the non-modal effects determine the nonlinear evolution of drift turbulence with dominant breakdown of phase of the potential due to scattering of the phase angle $\delta\phi$ in velocity space. For times $(V_0')^{-1} < t < t_s$ and for times $t > t_s$ we have, respectively

$$k_\perp \rho_i \delta\phi / k_x \delta X \sim k_y \rho_i (V_0' t)^3 \gg 1, \quad k_\perp \rho_i \delta\phi / k_x \delta X \sim (V_0' t)^2 \gg 1. \tag{26}$$

Applying the procedure of the solution of the integral equation (22) to the renormalized version of that equation, we obtain for $(V_0')^{-1} < t < t_s$ the renormalized solution (23) in the form

$$\Phi(\mathbf{k},t) = \Phi_0 \exp\left[-i\omega(\mathbf{k})\left(1 - \frac{1+\tau}{a_i b_i}\frac{t^2}{3t_s^2}\right)t + \left(\gamma(\mathbf{k}) - \frac{t}{2a_i t_s^2}\right)t - \int_0^t C(\mathbf{k},t_1)\,dt_1\right], \tag{27}$$

where $C(k,t)$ is determined by the equation

$$C(\mathbf{k},t) = \frac{c^2}{B^2} k_y^2 \rho_i^2 \frac{(V_0' t)^6}{8} \int d\mathbf{k}_1 \left|\varphi(\mathbf{k}_1,t)\right|^2 C(\mathbf{k}_1,t) \frac{k_{1y}^4}{\omega^2(\mathbf{k}_1)}. \tag{28}$$

If we omit linear non-modal terms in Eq.(27), the condition of the balance of the linear modal growth of the kinetic drift instability and non-linear non-modal dumping is determined by the equation $\gamma(k) = C(k,t)$. By using this equation in Eq.(28), we obtain the equation, which determines the time, at which that balance occurs,

$$\frac{\gamma(\mathbf{k})}{(V_0' t)^6} = \frac{c^2}{8B^2} k_y^2 \rho_i^2 \int d\mathbf{k}_1 \left|\varphi(\mathbf{k}_1,t)\right|^2 \gamma(\mathbf{k}_1) \frac{k_{1y}^4}{\omega^2(\mathbf{k}_1)}. \tag{29}$$

The effect of the shear flow reveals in the reducing with time as $(V_0' t)^{-6}$ the magnitude of the growth rate in the left part of the balance equation (29). That causes rapid asselerated suppression of the drift turbulence. This balance does not correspond to the steady state for drift turbulence in shear flow. The evolution of drift turbulence continues on times $t \geq t_s$. It follows by strongly non-modal way, where Markovian approximation, which is appropriate for the solution Eq.(23) when the growth rate and non-modal terms are small with respect to the frequency $\omega(k)$, ceases be valid.

5. Summary

The results presented in this report prove that any "universal rules" or "paradigms", that thoroughly determined the effect of the suppression of the turbulence by shear flow, are absent. The suppression of turbulence by shear flows is a mode dependent process, which, as a rule, includes the sequence of different non-modal linear and non-linear processes with different time scales for different parts of the spectrum of the unstable waves. The non-modal analysis of the resistive drift and kinetic (universal) drift instabilities reveals that linear non-modal effects lead to the decreasing the frequency and growth rate at time $t \leq t_2 = (V_0' k_y \rho_s)^{-1}$

and lead to rapid non-modal suppression of turbulence at time $t > t_2 = \left(V_0' k_y \rho_s\right)^{-1}$. Because of the secular growth of the component $k_x\left(t\right)$ of the wave number along the velocity shear, the results obtained above on the base of fluid equations have a limited validity in the investigations of long time evolution of the turbulence in plasma shear flows and the analysis on the base of the non-modal kinetic theory is necessary. The time dependence of the wavenumber $k_\perp\left(t\right)$ becomes the key element in the proper kinetic treatment of the long-time evolution of the perturbations in shear flow. In such kinetic analysis the nonlinear non-modal turbulent scattering of the phase angle of ion Larmor orbit may be the dominant effect, which determines rapid suppression of the drift turbulence by shear flow.

The "enhanced decorrelation by flow shear" considered in Refs.[5,6] have nothing in common with "enhanced suppression" of turbulence in shear flows (see also Ref.[3] for more extension discussion of that conclusion). The "universal rule $V_0' \simeq \gamma$" can't be considered as a condition for the suppression of turbulence by shear flow. Under that condition the perturbation, considered in convected set as a separate spatial Fourier mode with definite frequency and growth rate becomes observed in laboratory set in time $t \simeq \gamma^{-1}$ as a sheared mode with time dependent wave number. Only for the perturbations with $k_y \rho_i \simeq 1$ that time coincides with time t_2, at which strong nonmodal suppression of the drift turbulence occurs.

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[1] Mikhailenko, V.S., Mikhailenko, V.V., and Stepanov, K.N., Physics of Plasmas, **7**, 94 (2000)

[2] Mikhailenko, V.S., Mikhailenko, V.V., Stepanov, K.N., and Azarenkov, N.A., Physics of Plasmas, **15**, 072102 (2008)

[3] Mikhailenko, V.S., Mikhailenko, V.V., and Stepanov, K.N., Plasma Phys Controlled Fusion, **52**, 055007, (2010)

[4] Mikhailenko, V.S., Mikhailenko, V.V., Heyn, M.F., Mahajan, S.M., Physical Review E **66**, 066409 (2002)

[5] Biglari, H., Diamond, P.H., Terry, P.W., Phys.Fluids, **B2**, 1, (1990)

[6] Shaing, K.C., Crume, E.C., Houlberg, W., Phys.Fluids B **2**, 1492 (1990)

[7] Mikhailenko, V.S., Mikhailenko, V.V., Stepanov, K.N., and Azarenkov, N.A., Physics of Plasmas, **16**, 012305 (2009)