Tearing modes in electron magnetohydrodynamics

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Abstract. The dissipation mechanism of collisionless reconnection is analyzed, and the effects of anisotropy pressure gradient and guide field gradient on tearing mode are also analyzed in electron magnetohydrodynamics. It is found that either pressure-based dissipation or inertia-based dissipation dominates, has a great relation with the relative scaling orders between the electron thermal Larmor radius and electron inertia skin depth. The effects of pressure gradient also depend on the relative magnitude between parallel and perpendicular equilibrium pressure gradients. When the pressure-based dissipation is dominant, the condition that pressure drives or suppresses tearing mode instability also depends on the relative magnitude between parallel and perpendicular equilibrium pressure gradients. It is also shown that the guide field gradient has a significant influence on tearing mode. When the guide field gradient is smaller than the magnetic field shear at the magnetic null plane, the growth rate of tearing mode instability is enhanced by the guide field gradient and has no oscillatory component. When the guide field gradient is larger than the magnetic field shear, the guide field gradient can destabilize tearing mode instability dramatically. In this case, the growth rate is proportional to the guide field gradient.

Magnetic reconnection is one of the universal plasma phenomena in space and laboratory plasmas. It is a fundamental transport mechanism, and can support large scale transport by localized diffusive effects. Since the Sweet-Parker model [1, 2] and resistive tearing mode theory [3] were established in 1960s, there have been lots of work devoted to investigating reconnection. In collisional plasmas, resistivity is the only diffusive effect, which supports resistive magnetic reconnection. However, in space and high temperature plasmas, reconnection is always collisionless, so that other terms except resistivity term in generalized Ohm's Law [4], such as electron inertia and pressure tensor terms, become important.

Recently, the electron magnetohydrodynamics (EMHD) theory was developed to study fast collisionless reconnection conveniently [5, 6, 7, 8, 9, 10, 11], which describes plasma phenomena with a characteristic frequency in the whistler frequency regime and with spatial scale shorter than the ion skin depth scale. In this model, the ions can be assumed as an immobile neutralizing background, and the fluid dynamics is completely determined by electrons in EMHD frame. EMHD theory has been applied in many regimes, such as magnetic vortices [12, 13] and EMHD turbulence [14, 15]. In particular, EMHD theory has been employed to analyze reconnection [6, 7, 8, 9, 10, 11]. Recently, a series of experiments [16, 17] was carried out to investigate this phenomena in the parameter regime of EMHD.

It is known that electron inertia term and electron pressure tensor term both can break the frozen-flux constraint and cause reconnection. It has been shown that either electron inertia or pressure anisotropy can support collisionless reconnection electric field and cause fast reconnection [18, 19, 20]. So which is the dominant mechanism in collisionless reconnection? A series of papers was devoted to investigate the effects of electron inertia and pressure anisotropy [21, 22, 23, 24, 25]. It was shown that reconnection electric field is determined primarily by nongyrotropic pressure effects and less by electron inertia without guide field. Without magnetic guide field, the nongyrotropic pressure tensor is generated by the bounce motion of electrons in the reversal field region. But a large guide magnetic field will magnetized electrons and tends to generate gyrotropic distribution. Hesse *et. al.* [24] pointed out collisionless magnetic reconnection was still primarily provided by electron pressure nongyrotropy when electron motion is strongly affected by guide field. But Pritchett *et. al.* [23] showed that inertia-pressure dissipation is dominant. Hence, it still leaves a question whether the pressure-based dissipation or inertia-pressure dissipation is dominant if guide magnetic field is large?

In EMHD frame, guide field gradient also can generate electron shear flow. It is known that shear flow plays an important role at tearing mode in MHD frame. In EMHD frame, all current is carried by the motion of electron. In MHD, the direct connection between electron velocity and current density does not exist. Thus, the electron shear flow is generated by guide field gradient, meaning that the equilibrium transverse current density exists. Hence, it can be presupposed that guide field gradient will enhance tearing mode instability in EMHD frame, since tearing mode is driven by current density. Recently, the effect of electron velocity gradient in EMHD frame was investigated by simulation [26]. It was shown that both the tearing and the bending branches were driven by the electron velocity gradient in EMHD model.

In this article, following our two articles [9, 10](one can refer the detail), we will review the effects of electron pressure anisotropy and guide field gradient on tearing modes in EMHD.

1. BASIC EMHD EQUATIONS.

A set of EMHD equations [5, 6] is

$$\left(\frac{\partial \mathbf{v}_e}{\partial t} + \mathbf{v}_e \cdot \nabla \mathbf{v}_e\right) = -\frac{e}{m_e}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_e \times \mathbf{B}\right) - \frac{\nabla \cdot \mathbf{P}_e}{n_e\, m_e}, \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}, \tag{2}$$

$$\nabla \times \mathbf{B} = \frac{4\,\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = -\frac{4\,\pi\, e\, n_e}{c}\mathbf{v}_e + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \tag{3}$$

where m_e and $\mathbf{v}_e$ are the electron mass and velocities, respectively, n_e is the electron density, $\mathbf{J}$ is the current density, and $\mathbf{E}$, $\mathbf{B}$ are the electric and magnetic fields, respectively. $\nabla \cdot \mathbf{P}_e$ is the divergence of the electron pressure tensor. By deriving the Vlasov equation, the evolution of pressure tensor $\mathbf{P}_e$ in the center-of-mass system of the electron fluid can be obtained [24, 27]

$$\frac{\partial \mathbf{P}_e}{\partial t} = -\nabla \cdot (\mathbf{v}_e\, \mathbf{P}_e) - \mathbf{P}_e \cdot \nabla \mathbf{v}_e - [\mathbf{P}_e \cdot \nabla \mathbf{v}_e]^T - \frac{e}{m_e}\left(\mathbf{P}_e \times \mathbf{B} + [\mathbf{P}_e \times \mathbf{B}]^T\right), \tag{4}$$

where the electron heat flux tensor is neglected. The subscript T denotes the transpose matrix. Next, we study tearing mode in a two dimensional slab configuration with homogenous density, where the plasma parameters are independent of the coordinate z, and the dominant magnetic field is along $\mathbf{e}_z$. Consequently, the form of magnetic field can be written as $\mathbf{B} = \nabla\psi \times \mathbf{e}_z + B_z\, \mathbf{e}_z$, $B_{z0} \gg B_{y0}$ and the homogeneous density are assumed. Thus, based on Eqs.(1)-(4), we will analyze the dissipation mechanism of collisionless reconnection and the effects of guide field gradient on tearing modes, respectively.

2. The dissipation mechanism of collisionless reconnection

In this section, we will analyze the dissipation mechanism of collisionless reconnection. It is assumed that the diagonal elements of the pressure tensor are much larger than the non-diagonal components, namely $P_{eii0} \gg P_{eij0}$ $(i \neq j)$, which means that the electron distributions are nearly gyrotropic. Due to the profile of equilibrium magnetic field, $P_{eyy0} \simeq P_{exx0} = P_{\perp 0}$, $P_{ezz0} = P_{\| 0}$, $P_{exy0} = P_{eyx0} = P_{exz0} = P_{ezx0} = 0$ and $P_{eyz0} \neq 0$ can be assumed. Here, the parallel and perpendicular equilibrium pressures are anisotropic due to the dominant parallel magnetic field. The growth rate $\gamma \ll \omega_{ce}$ is also assumed, which is reasonable in EMHD frame, where $\omega_{ce} = e\,B_{z0}/(m_e\,c)$ is the electron gyrofrequency. Thus, by assuming two dimensional perturbations of the type $\tilde{f} = f_1\,\exp\left(i\,k\,y + \gamma\,t\right)$, the evolution of perturbative pressure tensor $\mathbf{P}_{e1}$ can be derived. Hence, based on Eqs.(1)-(4), one can obtain

$$
\begin{aligned}
&\gamma\,\left(\psi_1 - d_e^2\nabla^2\psi_1\right) - i\,k\,\frac{\mathrm{d}}{\mathrm{d}x}\left(\psi_0 - d_e^2\nabla^2\psi_0\right)B_{z1}\\
&= \rho^2\left[i\,k\,\frac{1}{P_0}\frac{\mathrm{d}P_{\perp 0}}{\mathrm{d}x}\nabla^2\psi_1 + \frac{i\,k}{d_e^2}\frac{1}{P_0}\frac{\mathrm{d}}{\mathrm{d}x}\left(P_{\|0} - P_{\perp 0}\right)\psi_1 + \frac{k^2}{\gamma\,d_e^2}\frac{1}{P_0}\frac{\mathrm{d}}{\mathrm{d}x}\left(P_{\|0} - P_{\perp 0}\right)\frac{\mathrm{d}\psi_0}{\mathrm{d}x}\,B_{z1}\right],
\end{aligned}
\tag{5}
$$

$$
\gamma\,\left(B_{z1} - d_e^2\nabla^2 B_{z1}\right) = i\,k\frac{\mathrm{d}^3\psi_0}{\mathrm{d}x^3}\psi_1 - i\,k\frac{\mathrm{d}\psi_0}{\mathrm{d}x}\nabla^2\psi_1 + i\,k\,\rho^2\left(\frac{1}{P_0}\frac{\mathrm{d}P_{\perp 0}}{\mathrm{d}x}\nabla^2 B_{z1} + \frac{1}{P_0}\frac{\mathrm{d}^2 P_{\perp 0}}{\mathrm{d}x^2}\frac{\partial B_{z1}}{\partial x}\right), \tag{6}
$$

where the variables have been normalized as: $\gamma \to \tau_w^{-1}\,\gamma$, $\mathbf{x} \to L\,\mathbf{x}$ and $\rho = v_{the}/\omega_{ce}$ is the thermal electron Larmor radius, $\tau_w = L^2/(\omega_{ce}\,d_e^2)$ is the characteristic time of whistler wave, $v_{the} = [P_0/(n\,m_e)]^{1/2}$ is the electron thermal velocity and $P_0 = n\,T_0$ is the reference value of pressure. One can see that there are two scales: electron inertia skin depth d_e and thermal electron Larmor radius ρ, which depends on the magnitudes of guide field and electron thermal velocity. The dissipation mechanism of reconnection strongly depends on the relative magnitude between these two scales. In the inner region, the familiar "constant-ψ" approximation can be applied for EMHD tearing mode [6], which implies the scaling order $d_e^2 \sim O(\delta)$, where δ is the singular layer width. Then by a series of derivations, the linear dispersion relation can be gotten as

$$
\hat{\gamma}^{1/4}\left[\hat{\gamma} - i\,k\,\hat{\rho}^2\,g_{\perp 0}\,\left(g_{\|0}/g_{\perp 0} - 1\right)\right]^{3/4}\left(\hat{\gamma} + i\,k\,\hat{\rho}^2\,g_{\perp 0}\right)^{-1/2} = 1, \tag{7}
$$

where $g_{\perp 0} = P_0^{-1}\,(\mathrm{d}P_{\perp 0}/\mathrm{d}x)_{x=0}$, $g_{\|0} = P_0^{-1}\,(\mathrm{d}P_{\|0}/\mathrm{d}x)_{x=0}$, $\hat{\gamma} = \gamma/\gamma_0$ and $\hat{\rho}^2 = \rho^2/(\gamma_0\,d_e^2)$. γ_0 is the growth rate without pressure gradient. $g_{\|0}$ and $g_{\perp 0}$ represent the effects of parallel pressure gradient and perpendicular pressure gradient, respectively. Due to fractional power appearing in the dispersion relation (7), there are four branches and the mode structure becomes complex, so that the physical branch must be picked up by analyzing the property of eigenfunctions. WKB analysis indicates that $Y(x) \sim exp(-x^2/(2\,\delta^2))$ for $|x| > |\delta|$. Thus, the reasonable roots of the dispersion relation (7) must satisfy the condition $Re(\delta^2) > 0$, in order that the corresponding eigenfunction can satisfy the boundary condition. Next, based on this principle, the roots of Eq.(7) will be picked up.

When $\hat{\rho}^2\,g_{\perp 0} \ll \hat{\gamma}$, namely the electron thermal Larmor radius is much smaller than the electron inertia skin depth, the electron inertia effect is dominant. Consequently, the reconnection electric field is sustained by electron inertia effect. By making Taylor expansion

$\hat{\gamma} = \hat{\gamma}_0 + \hat{\gamma}_1 + \hat{\gamma}_2 + \ldots$, one has

$$\begin{aligned} \hat{\gamma}_0 &= 1, \\ \hat{\gamma}_1 &= i\,k\,\hat{\rho}^2\,g_{\perp 0} + 3\,i\,k\,\hat{\rho}^2\,\left(g_{\parallel 0} - g_{\perp 0}\right)/2, \\ \hat{\gamma}_2 &= k^2\,\hat{\rho}^4\,g_{\perp 0}^2 + \frac{3}{2}k^2\,\hat{\rho}^4\,g_{\perp 0}\left(g_{\parallel 0} - g_{\perp 0}\right) + \frac{3}{8}k^2\,\hat{\rho}^4\,\left(g_{\parallel 0} - g_{\perp 0}\right)^2 . \end{aligned} \quad (8)$$

From the expression of $\hat{\gamma}_2$, if the condition $g_{\parallel 0}/g_{\perp 0} < (2\sqrt{3} - 3)/3$ is satisfied, $\hat{\gamma}_2 < 0$, namely the pressure gradient reduces the growth rate in this case. Here, we only consider the parallel and perpendicular pressure gradients have the same direction. While the condition is not satisfied, the pressure gradient enhances the growth rate. It can be explained roughly as follows: the second term of the right hand of Eq.(5) decreases reconnection electric field when $g_{\parallel 0} < g_{\perp 0}$; while the other terms of the right hand enhance reconnection electric field. Thus, if the value of $g_{\parallel 0}/g_{\perp 0}$ is small enough, namely $g_{\parallel 0}/g_{\perp 0} < (2\sqrt{3} - 3)/3$, the second term can be dominant, so that the pressure gradient would play a stable role in magnetic reconnection.

When the electron thermal Larmor radius becomes large, the variable $\hat{\gamma}_d = \hat{\gamma} - i\,k\,\hat{\rho}^2\,g_{\perp 0}\,(g_{\parallel 0}/g_{\perp 0} - 1)$ is introduced. Then Eq.(7) becomes

$$\hat{\gamma}_d^{3/4}\left[\hat{\gamma}_d + i\,k\,\hat{\rho}^2\,g_{\perp 0}\left(g_{\parallel 0}/g_{\perp 0} - 1\right)\right]^{1/4}\left(\hat{\gamma}_d + i\,k\,\hat{\rho}^2\,g_{\parallel 0}\right)^{-1/2} = 1. \quad (9)$$

When $k\,\hat{\rho}^2\,g_{\parallel 0} \gg |\hat{\gamma}_d|$ and $k\,\hat{\rho}^2\,g_{\perp 0}|(g_{\parallel 0}/g_{\perp 0} - 1)| \gg |\hat{\gamma}_d|$, meaning that electron thermal Larmor radius is much larger than the electron inertia skin depth, the pressure gradient effect dominates over electron inertia effect. Thus, pressure gradient sustains the reconnection electric field, and the pressure-based dissipation is dominant. Hence, by making Taylor expansion, one can obtain

$$\hat{\gamma}_d = \begin{cases} \exp\left(i\,\pi/6\right)\,\left(k\,\hat{\rho}^2\,g_{\perp 0}\right)^{1/3}\,\left(g_{\parallel 0}/g_{\perp 0}\right)^{2/3}\,\left(g_{\parallel 0}/g_{\perp 0} - 1\right)^{-1/3}, & g_{\parallel 0}/g_{\perp 0} > 1 \\ i\,\left(k\,\hat{\rho}^2\,g_{\perp 0}\right)^{1/3}\,\left(g_{\parallel 0}/g_{\perp 0}\right)^{2/3}\,\left(1 - g_{\parallel 0}/g_{\perp 0}\right)^{-1/3}, & g_{\parallel 0}/g_{\perp 0} < 1 \end{cases} \quad (10)$$

where only the physical branch is retained and the root satisfies the condition $Re(\delta^2) > 0$. It can be seen that the pressure gradient effects are completely different in the cases of $g_{\parallel 0} > g_{\perp 0}$ and $g_{\parallel 0} < g_{\perp 0}$. If $g_{\parallel 0} > g_{\perp 0}$, the unstable mode is driven by pressure gradient. If $g_{\parallel 0} < g_{\perp 0}$, tearing mode is suppressed completely. It can be known that the pressure tensor is the primary dissipative effect when the pressure gradient dominates over electron inertia. Then the behavior of tearing mode depends on the relative magnitude between the parallel and perpendicular pressure gradients. Actually, it depends on the competition of the three terms of the right hand of Eq.(5). As mentioned above, the effect of the second term of the right hand of Eq.(5) plays a stable role if $g_{\parallel 0} < g_{\perp 0}$. Hence, this term can be dominant when the electron thermal larmor radius is large enough, so that the tearing mode is suppressed. Next, one can see the effect of pressure gradient more explicitly in one special and simple case $g_{\perp 0} = g_{\parallel 0}$.

When $g_{\perp 0} = g_{\parallel 0}$, meaning that the parallel and perpendicular pressure gradients are identical, Eq.(7) can be reduced to

$$\hat{\gamma}\,\left(\hat{\gamma} + i\,k\,\hat{\rho}^2\,g_{\perp 0}\right)^{-1/2} = 1, \quad (11)$$

so that the growth rate can be obtain

$$\hat{\gamma} = \frac{1 + \sqrt{1 + 4\,i\,k\,\hat{\rho}^2\,g_{\perp 0}}}{2}. \quad (12)$$

When $k\,\hat{\rho}^2\,g_{\perp 0} \ll 1$, $\hat{\gamma} \simeq 1 + i\,\hat{\rho}^2\,g_{\perp 0} + \hat{\rho}^4\,g_{\perp 0}^2$, which is consistent with Eq.(8). When $k\,\hat{\rho}^2\,g_{\perp 0} \gg 1$, namely the pressure gradient is dominant, $\hat{\gamma} \simeq \exp(i\,\pi/4)\,(k\,\hat{\rho}^2\,g_{\perp 0})^{1/2}$, which satisfies $Re(\delta^2) > 0$. It can be distinctly seen that the unstable mode is driven by pressure gradient. The above analytical results also can be justified in *Fig.1.* The curve with $g_{\|0} = g_{\perp 0}$ is a separatrix. When $g_{\|0} \geq g_{\perp 0}$ (above and including the separatrix), the pressure gradient drives reconnection by supporting the reconnection electric field if it dominates over electron inertia. When $[(2\sqrt{3}-3)/3]\,g_{\perp 0} < g_{\|0} < g_{\perp 0}$ (below the separatrix), the growth rate of tearing mode is enhanced if $k\,\hat{\rho}^2\,g_{\perp 0}$ is small, while the growth rate decreases if $k\,\hat{\rho}^2\,g_{\perp 0}$ exceeds a definite value. The tearing mode will even be suppressed if $k\,\hat{\rho}^2\,g_{\perp 0}$ is large. When $g_{\|0} < [(2\sqrt{3}-3)/3]\,g_{\perp 0}$, the growth rate is reduced by pressure gradient, and can be reduced to zero as the pressure gradient increases. Hence, it can be known that the pressure gradient effects have great relations with the value of $g_{\|0}/g_{\perp 0}$, namely the relative magnitude between the parallel and perpendicular pressure gradients.

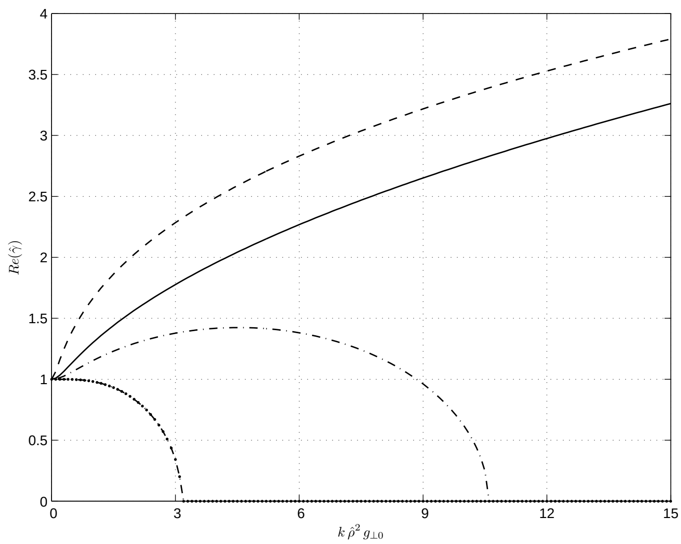

Fig. 1. : The growth rate $Re(\hat{\gamma})$ against the pressure gradient $k\,\hat{\rho}^2\,g_{\perp 0}$ with different values of $g_{\|0}/g_{\perp 0}$. The solid and dashed curves are plotted for $g_{\|0}/g_{\perp 0} = 1$ and $g_{\|0}/g_{\perp 0} = 4$, respectively. The dash dotted and dash dot dotted curves are plotted for $g_{\|0}/g_{\perp 0} = 0.5$ and $g_{\|0}/g_{\perp 0} = (2\sqrt{3}-3)/3$, respectively.

3. The effects of guide field gradient on tearing modes

In this section, the effects of guide field gradient on tearing modes will be analyzed. Here, the pressure anisotropy is neglected, namely $p_e = p_e(n_e)$ is assumed to be an appropriate closure relation for the electron pressure. The guide field $B_{z0} = B_{z0}(x)$ is assumed. Thus, by assuming two dimensional perturbations of the type $\tilde{f} = f_1\,\exp(i\,k\,y + \gamma\,t)$, Eq.(1)-(3) can be derived as

$$\gamma\left(\psi_1 - d_e^2\,\nabla^2\psi_1\right) = -i\,k\,\frac{\mathrm{d}B_{z0}}{\mathrm{d}x}\left(\psi_1 - d_e^2\nabla^2\psi_1\right) + i\,k\,\frac{\mathrm{d}}{\mathrm{d}x}\left(\psi_0 - d_e^2\frac{\mathrm{d}^2\psi_0}{\mathrm{d}x^2}\right)B_{z1}, \tag{13}$$

$$\gamma\,\left(B_{z1} - d_e^2\nabla^2 B_{z1}\right) = i\,k\,d_e^2\,\frac{\mathrm{d}B_{z0}}{\mathrm{d}x}\nabla^2 B_{z1} - i\,k\,d_e^2\,\frac{\mathrm{d}^3 B_{z0}}{\mathrm{d}x^3}\,B_{z1} + i\,k\,\left(\frac{\mathrm{d}^3\psi_0}{\mathrm{d}x^3}\psi_1 - \frac{\mathrm{d}\psi_0}{\mathrm{d}x}\nabla^2\psi_1\right), \tag{14}$$

where the parameters have been normalized: $x \to L\,x$,$t \to \tau_w t$, where $\tau_w = L^2/(\alpha\,B_0)$ is the characteristic time of whistler mode and L is the characteristic scale length of the equilibrium. Now, Eqs.(13) and (14) compose a set of basic equations for reconnection instability, including the gradient of guiding field.

Following the standard singular perturbation technique for resistive instabilities, the boundary-layer theory will be used, and the external and inner regions are separated. In the external region, the terms with respect to the small parameters γ and d_e^2 can be neglected. Then the external equation is the same as the case without guide field gradient $\mathrm{d}B_{z0}/\mathrm{d}x$. Hence, the matching condition between the solutions in external and inner regions does not change, namely the guide field gradient does not influence the solution in external region. In inner region, $\partial/\partial x \gg k$, and the small parameters γ and d_e^2 become important. For tearing mode in the electron magnetohydrodynamics (EMHD tearing mode) [6], the scaling order $d_e^2 \sim O(\delta)$ is assumed, where δ is the singular layer width. Here, the wave number $k \sim O(\delta^0)$ is taken. Consequently, the familiar 'constant-ψ' approximation is still valid. If $\mathrm{d}B_{z0}/\mathrm{d}x \sim const$, it can be easily justified that guide field gradient only contributes Doppler shift to EMHD tearing mode, which was investigated in **Ref.** [26]. Then $\mathrm{d}B_{z0}/\mathrm{d}x \sim R\,x$ can be expanded near the singular layer in the inner region. Making a transform

$$B_{z1} - \frac{R}{\mu_0}\psi_1(0) = \frac{i}{4\,k\,\mu_0\,\delta}\psi_1^{(0)}\,Y(X),$$

One can derive

$$\left(\frac{\gamma\,d_e^2}{2\,k\,\mu_0\,\delta^2} + i\,\frac{R}{\mu_0}\frac{d_e^2}{2\,\delta}X\right)^2\frac{\mathrm{d}^2Y}{\mathrm{d}X^2} = X\left(\gamma + \frac{1}{4}X\,Y\right). \tag{15}$$

where $\mu_0 = \mathrm{d}^2\psi_0(0)/\mathrm{d}x^2$. Next, we will investigated the effects of guide field gradient in two limits.

3.1. Small guide field gradient

In this section, the small guide field gradient $R/\mu_0 \ll \gamma/(k\,\mu_0\,\delta)$ will be discussed. Introducing the small parameter $\lambda = i(R/\mu_0)\,[d_e^2/(2\,\delta)]$, $Y(X)$ and γ can be expanded as

$$Y = \gamma_0\sum_{n=0}\lambda^n\,Y_n, \quad \gamma = \sum_{n=0}\lambda^n\,\gamma_n,$$

where $\gamma_0 = 2\,k\,\mu_0\delta^2/d_e^2$. Accordingly, the instability criterion Δ' also has an expansion in λ, $\Delta' = \Delta_0' + \lambda\,\Delta_1' + \lambda^2\,\Delta_2' + \cdots$. As was shown in the external solution (**??**), the effects of equilibrium guide field gradient can be neglected in the external region. Consequently, $\Delta_n' = 0$ for $n > 0$. By some derivations (one can refer detail in **Ref.** [10]), the linear growth rate for small guide field gradient can be obtained

$$\gamma = \gamma_0 + \lambda^2\,\gamma_2 = \gamma_0\left[1 + \frac{\pi}{8}\frac{k\,\mu_0\,d_e^2}{\gamma_0}\left(\frac{R}{\mu_0}\right)^2\right]. \tag{16}$$

where

$$\gamma_0 = 2\,k\,\mu_0\,C_0^{-2}\Delta'^2\,d_e^2, \tag{17}$$

$C_0 = 2^{3/2}\pi\,\Gamma(3/4)/\Gamma(1/4)$. $\gamma_1 = 0$ is also obtained. It can be easily seen that the linear growth rate is enhanced by guide field gradient in the small λ approximation, and has no oscillatory component due to the reasonable neglect of $\mathrm{d}^3\psi_0/\mathrm{d}x^3$. These results are independent of the gradient direction of guiding magnetic field.

3.2. Large guide field gradient

In this section, the large guide field gradient will be considered. For simplicity, the variables $\sigma = \gamma\, d_e^2/(2\,k\,\mu_0\,\delta^2)$, $\lambda = i\,R\,d_e^2/(2\,\mu_0\,\delta)$ are introduced. Then Eq.(15) can be rewritten as

$$(\sigma + \lambda\,X)^2\,\frac{\mathrm{d}^2 Y}{\mathrm{d}X^2} = X\left(\gamma + \frac{1}{4}X\,Y\right). \tag{18}$$

The case of large $|\lambda|$ indicates that the instability growth rate scales with a significant guide field gradient when $\Delta' > 0$. As $|\lambda|$ is large, the following new scaled variables are introduced: $X = \lambda\,\xi$, $\gamma = \lambda\,\hat{\gamma}$, then Eq.(18) becomes

$$\left(\sigma\lambda^{-1} + \xi\right)^2\frac{\mathrm{d}^2 Y}{\mathrm{d}\xi^2} = \xi\left(\hat{\gamma} + \frac{1}{4}\xi\,Y\right). \tag{19}$$

As $|\xi| \to \infty$, the asymptotic behavior of Y is described by the following equation,

$$\xi\frac{\mathrm{d}^2 Y}{\mathrm{d}\xi^2} - \frac{1}{4}\xi\,Y = \hat{\gamma}. \tag{20}$$

Then, by some derivation (one can refer detail in **Ref.** [10], the growth rate can be obtained

$$\gamma = \frac{1}{4\,\pi}k\,\mu_0\,d_e^2\,\Delta'\,\frac{R}{\mu_0} = \frac{1}{8\,\pi}\frac{C_0^2\,\gamma_0}{\Delta'}\frac{R}{\mu_0}, \tag{21}$$

where γ_0 is the growth rate of tearing mode without guide field gradient. It can be seen that the linear growth rate $\gamma \propto \gamma_0\,R/\mu_0$ in the large $|\lambda|$, which is consistent with the scaling orders in Eq.(15). It can be seen that guide field gradient, namely electron shear flow, enhances tearing mode instability effectively. This is different from shear flow at tearing mode for large flow shear in MHD model. In MHD model, it was shown that tearing mode is stabilized when the flow shear is larger than the magnetic field shear at the magnetic null plane [28]. In EMHD frame, electron shear flow and the gradient of equilibrium current density exist simultaneously, which can drive bending instability [26]. Consequently, it is reasonable that electron shear flow can drive tearing mode instability which is also driven by current density.

4. conclusions

We have review the process of tearing modes in electron magnetohydrodynamics from our two papers. The dissipation mechanisms of reconnection and the effects of pressure gradient on tearing mode are analyzed. It is found that the conditions either pressure-based dissipation or inertia-based dissipation dominates depend on the relative magnitude between electron thermal Larmor radius and electron inertia skin depth. It also can be concluded that the dissipation mechanisms have a great relation with the guide field, since electron thermal Larmor radius depends on the magnitude of guide field. Moreover, the effects of pressure gradient depend on the relative magnitude between parallel and perpendicular pressure gradient.

The effect of guide field gradient at tearing mode instability in EMHD frame has been analyzed in small and large guide field gradient limits, respectively. In the small guide field gradient limit, it was found that guide field gradient increases growth rate of tearing mode, and does not arise an oscillating mode. In the large guide field gradient limit, tearing mode instability is enhanced dramatically by guide field gradient. Since the guide field gradient and the gradient of current density are equivalent in EMHD frame, and can drive bending instability which is similar to Kelvin-Helmholtz instability. In this case, the growth rate is proportional to guide field gradient.

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