Magnetic X-points, edge instabilities, and the H-mode edge

L.E. Sugiyama[1], H.R. Strauss[2], and the M3D Team

1) Massachusetts Institute of Technology, Cambridge MA, U.S.A
2) HRS Fusion, West Orange NJ, U.S.A.

E-mail contact of main author: sugiyama@mit.edu

Abstract. A new picture of the edge of D-shaped toroidal fusion plasmas emerges from recent large scale nonlinear MHD simulations of Edge Localized Modes (ELMs), edge instabilities, and applied nonaxsiymmetric fields. At high resolution and realistic or near-realistic resistivities, the resulting multi-stage ELM matches many experimental observations. A freely moving magnetic boundary surface possessing one or more X-points, surrounded by vacuum, undergoes asymptotic surface splitting when perturbed, similar to a two-degree of freedom Hamiltonian dynamical system with a hyperbolic saddle point. The resulting stochastic magnetic tangle influences many aspects of the ELM. A ballooning-type instability in the plasma edge can self-consistently generate a tangle and interact coherently with it to drive a rapidly growing mode. Through the tangle, sufficiently strong outboard ballooning can destabilize the inboard side of the torus, after a delay. Significant losses of density and energy can occur directly to the divertors from near the X-points. The instability can propagate deeply into the plasma, through an interchange rather than tearing mechanism. The nonlinear instability differs significantly from linear eigenmodes, even with free boundary. Strong nonlinear toroidal harmonic beating reduces the macroscopic ELM to moderate mode numbers, with poloidal and toroidal asymmetry. Toroidal rotation and two-fluid effects may be important below strong ideal MHD instability and can limit the coupling of edge instabilities to interior, large-scale modes. Tangles can be driven by weaker perturbations and provide a natural, weakly stochastic magnetic field over the plasma edge that affects stability and confinement.

1. Introduction

Recent nonlinear simulations of Edge Localized Modes (ELMs)[1, 2] and applied nonaxisymmetric fields[3] in tokamaks and spherical torii show that the edge region of D-shaped, magnetically confined fusion plasmas has a characteristic behavior due to the interaction of the magnetic X-points with small perturbations of the plasma. The extended MHD code M3D[4] solves the full MHD or extended MHD equations for the plasma/vacuum/bounding-wall system, keeping a full spectrum of toroidal harmonics. An improved code and computers allow higher spatial resolution and realistic or near-realistic values of plasma resistivity. At these levels, a freely moving plasma boundary with one or more magnetic X-points gives an intrinsically nonlinear and stochastic character to the dynamics of the plasma edge. The toroidal magnetic field behaves similarly to a Hamiltonian dynamical system with a hyperbolic saddle point[5] in two degrees of freedom. Under small perturbations, the magnetic boundary surface (plasma separatrix) splits into two different asymptotic limiting surfaces that are approached by the field lines as they are traced infinitely in opposite directions. The X-point remains a fixed point. The two limiting surfaces (or manifolds) intersect multiple times and, together with their connecting field lines, generate a stochastic magnetic tangle. The simulations show that, despite the superficial stochasticity, transverse field-aligned instabilities such as ELM ballooning modes can couple to the self-generated field to produce a mostly coherent, growing plasma response. The characteristic magnetic tangle structure also allows the plasma instability to penetrate deeply into the plasma interior. Partial preservation of the original field by the tangle allows a relatively rapid relaxation back to the original plasma shape once the ballooning drive saturates.

2. Nonlinear ELM

Sufficiently high spatial and temporal resolution allows simulation at lower plasma resistivities that match or nearly match experimental values. This eliminates a major source of uncertainty in the ELM dynamics, since simulations of large Type I ELMs in DIII-D have shown[1] that resistivity influences the instability down to actual values.

Figure 1 shows the time evolution of an MHD ELM in DIII-D discharge 126006, time t = 3500[8]. The simulation (details in [1]) used central $\eta = S^{-1} = 3 \times 10^{-8}$ with $(T_e/T_{eo})^{-3/2}$ dependence, approximately 5× the actual resistivity. The ion viscosity was $\mu_\perp = 6 \times 10^{-6}$, and density and thermal conductivtities $D_n = \kappa_\perp = 10^{-5}$, $\kappa_\| = 3.5$. Upwind advection contributed additional diffusivity. No hyper-resistivity or hyper-viscosity was used in MHD. The MHD "vacuum" had $\eta = 10^{-3}$, $n_o/n = 0.1$, and $T = J_\phi = 0$. The simulation used 19441 triangle vertices in a poloidal plane, with 72 planes. It took approximately 200 hours on 432 cpus on the Cray XT-4 at NERSC, up to approximately $330\tau_A$, where $\tau_A = 0.43\mu$sec.

The partly stochastic ELM that results from the magnetic tangle proceeds in several stages that have many similarities to experimental observations[1, 2]. Ballooning-type instabilities near the plasma edge can provide a transverse perturbation of the separatrix that drives a tangle-like magnetic perturbation in MHD and two-fluids[1]. The tangle then continues to interact naturally with a nonlinear ballooning or interchange-like plasma instability Given unstable ballooning near the midplane with a spectrum of toroidal mode numbers, the harmonics rapidly coalesce into more moderate mode numbers (Fig. 1, t = 78.6). Ballooning fingers begin to grow rapidly well off the midplane, followed by a fast, explosive outburst over much of the outboard side (t = 78–103). Plasma can be lost in blobs directly to the divertors from near the X-points (t = 103–145.6). Some density can also be transported over the top of the plasma to the inboard side, in a layer near the separatrix. The outboard ballooning growth rate quickly saturates as the local edge density gradient drops, although the perturbation continues to grow. Particle density is more strongly affected than the temperature[1], as for an resonant magnetic perturbation (RMP)[3]. The tangle also helps the outboard ballooning grow inward into the plasma interior in alternating lower and higher density regions. Both interior and inboard growth continue after the main outboard ballooning growth rate saturates.

The inward-moving instability is interchange-like, rather than magnetic island formation due to tearing ([1], Fig. 9). The magnetic field stochasticizes over the affected region, taking the characteristic form of 'bubbles' or 'voids' associated with an idealized Hamiltonian tangle.

The external, originally unconfined field lines are little affected despite the large ballooning plasma fingers. The tangle perturbation arguments do not apply to unconfined field lines and leave them unchanged. As the outboard ballooning grows, the interior disturbance begins to affect the inboard side of the plasma, even as the outboard growth rate adecays. At sufficient amplitude, loops of the tangle naturally extend inward from near the X-points and wrap around the initially stable inboard side of the plasma edge, starting from the separatrix and expanding inward. In large ELMs, such as DIII-D 119690[1], plasma density perturbations can form

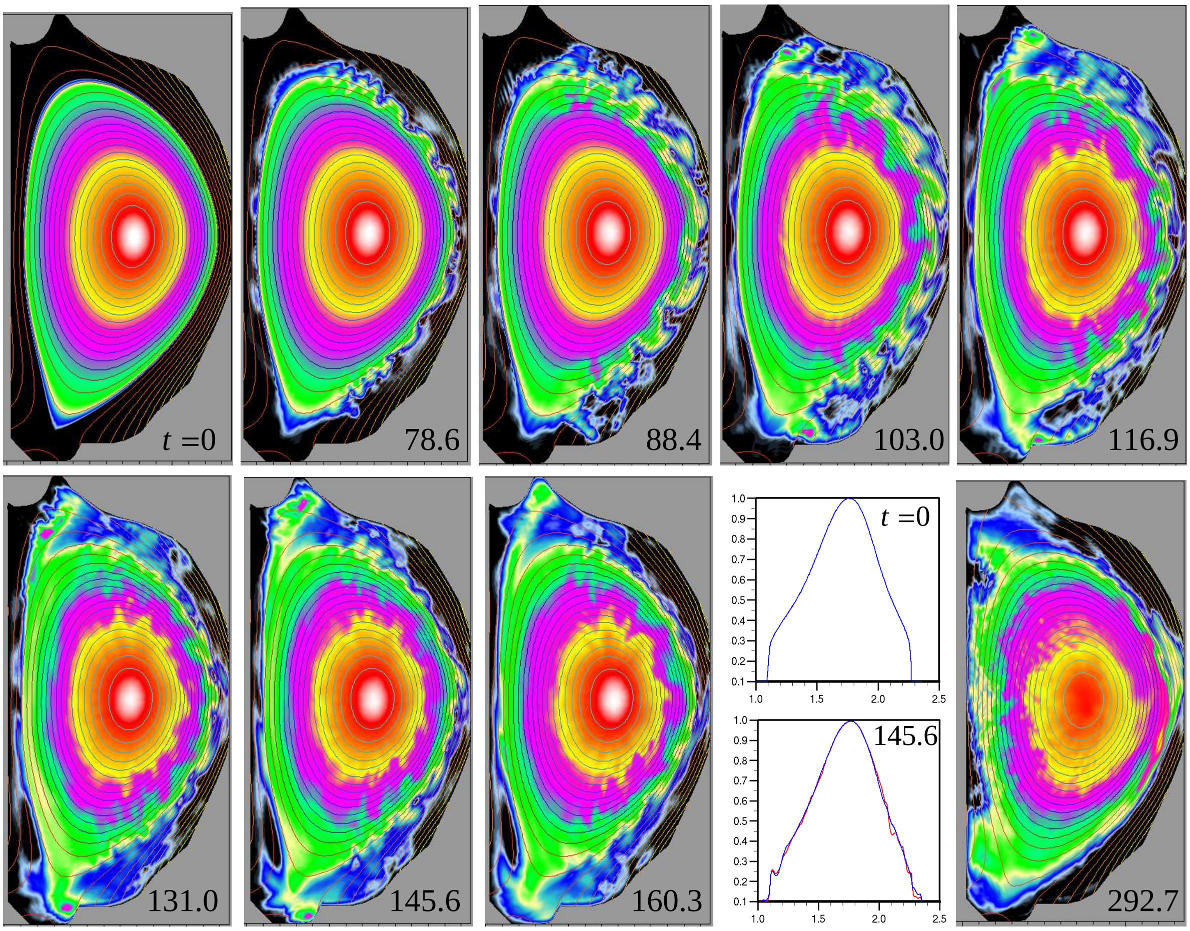


FIG. 1. MHD ELM in DIII-D discharge 126006, no equilibrium rotation. Density contours show major changes. Underlying curves show poloidal magnetic flux surfaces. Initial state at $t = 0$*. Black region shows surrounding vacuum and wall shape. Ballooning outburst appears just before* $t = 78.6$ *and rapidly reaches the outboard wall; followed by X-point losses and inboard disturbances as the outboard side clears. Midplane density profiles at* $t = 0$ *and 145.6 show the main ELM crash primarily affects the edge region (blue is toroidal average, red is actual). Last time 292.7 shows the later development of a large* $n = 1, 2$ *internal mode that will eventually lock to the outboard wall, as well as a single, rapid pulse of inboard instability driven by the original ELM.*

evolving ridges along the inner boundary. Magentic loops and density fingers can extend from both top and bottom X-points. Eventually, a strong instability can be triggered where the tangle loops intersect around the inboard midplane. Blobs of cold plasma density can propagate along the inboard edge of the plasma to the divertors (Fig. 1, $t = 292.7$). Decreasing pulses of activity and plasma loss can occur on both inboard and outboard sides. Poloidal rotation of interior plasma surfaces and the edge can be driven at longer times, even in MHD. Barring interior modes, the plasma profiles remain close to axisymmetry over most of the plasma.

Similar behavior, with stages of varying intensity depending on the plasma configuration and conditions, is observed in DIII-D and NSTX ELMs. Qualitatively similar, but more muted response over a narrower edge region, appears for rapidly applied nonaxisymmetric RMPs[3]. Weaker plasma edge instabilities that do not result in large ELMs have smaller outer-edge perturbations with a simpler, singly peaked radial density loss structure, generally more concentrated around the horizontal midplane.

3. Linear and nonlinear response

Instabilities of a freely moving plasma boundary with X-point cannot be described by conventional linearized plasma perturbation theory, even when they involve small perturbations. The MHD free-energy analysis of Bernstein, Frieman, et al.[9], implicitly relies on the existence of well-defined flux tubes, by assuming a relatively uniform local response over a flux tube and setting the perturbation boundary conditions along flux tubes. Asymptotic field splitting due to an X-point perturbation and the resulting magnetic stochasticity destroy flux tube coherence locally. Globally, the perturbation becomes large near the X-point and along the far plasma boundary, although over small regions. Although magnetic puncture plots are superficially stochastic, the perturbation of most field lines over one circuit around the torus remains small and an effective average structure still exists, but is difficult to describe analytically.

For transverse instabilities that disturb the plasma edge, such as ballooning/peeling modes, the driven tangle reduces the nonlinear growth rate because only the "unstable" limiting manifold of the perturbed field lines responds to the cross-field plasma motion. The "stable" manifold barely moves from the equilibrium. The response is further constrained by the continuity of the magnetic field lines, since the poloidal spacing of the unstable loops along separatrix near the X-point is determined by the field line spacing in the equilibrium configuration, independent of the midplane perturbation. Growth rate reduction was shown for a large ELM in Ref. [1], Table II, by modifying the boundary condition on the exterior "unstable" tangle loops.

Direct computation of linearized small perturbations with free boundary demonstrate that the linear eigenmodes give quite different results compared to the early nonlinear ELM. Not all differences are due to the tangle. The mode extends beyond the plasma to the bounding surface. Equilibrium flux contours intersect the partially conducting wall, where the equilibrium quantities are fixed. The plasma/vacuum interface may involve sharp variations.

A further source of nonlinear difference arises when the early nonlinear ELM is started from a multi-harmonic initial perturbation. Typically, linear MHD ballooning eigenmodes increase in growth rate with toroidal harmonic, up to relatively large values $\simeq 40$, as in Fig. 3. for DIII-D case 126006 of Fig. 1. When a full spectrum of toroidal modes is perturbed, the toroidal harmonics beat nonlinearly at quite small amplitude, to produce a consolidated instability with relatively moderate mode numbers and a lower-n envelope. Beating of the n and $n \pm 1$ modes appears particularly strong, resulting in a significant $n = 1$ component in the envelope. The resulting ELM at macroscopic size is asymmetric toroidally and poloidally, as clearly seen in the experimentally observed filaments at the plasma edge. For spherical torii, the asymmetry can be especially large. Figure 2 shows a generic ELM in NSTX, at higher than actual resistivity, compared to the DIII-D case. (Further details of a DIII-D ELM appear in Ref. [1]). For NSTX, the basic $n = 3$ and 1 structure and greater midplane concentration may explain why the $n = 3$ applied fields produced by the external midplane field coils typically destabilize ELMs, rather than stabilize them as in DIII-D. (This picture, however, neglects toroidal rotation.)

For DIII-D 126006 with no equilibrium rotation, the maximum nonlinear MHD growth rate was much smaller than the linear growth rates. It reached 0.13 during the ELM outburst, compared to 0.4 in Fig. 3.. The dominant nonlinear harmonics were $n = 10$ and 13, out of $|n| \leq 23$ in the simulation. (The growth rate is defined from the square root of the kinetic energy averaged over the simulation volume.)

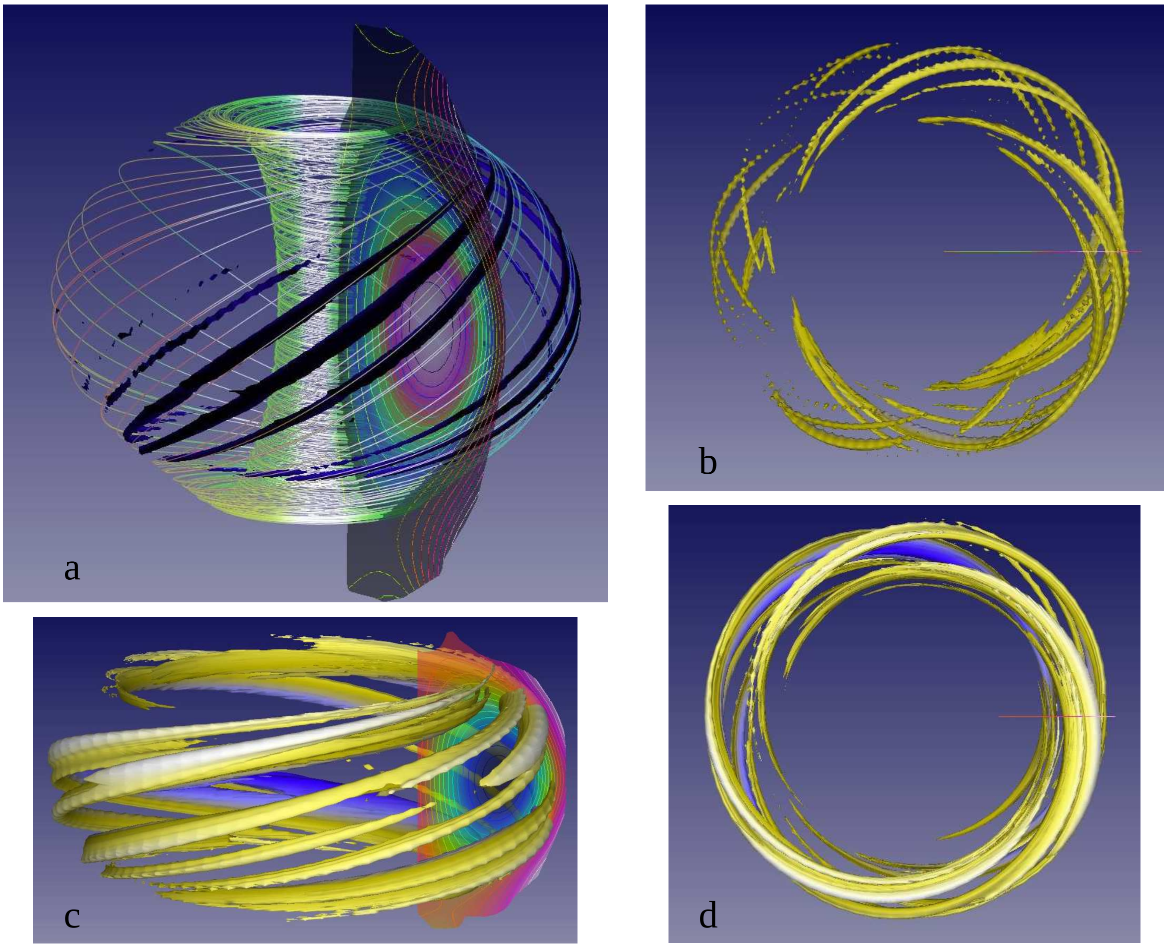


FIG. 2. Nonlinear ELM in the NSTX spherical torus (top) shows greater concentration near the midplane and stronger toroidal and poloidal asymmetry than in a tokamak (bottom). a) NSTX helical filaments of perturbed temperature $\tilde{T}$ near the plasma edge, in black, over white magnetic field lines, and b) top view of torus shows dominant $n = 3$ and $n = 1$ components in the perturbed poloidal magnetic flux $\tilde{\psi}$, similar to $\tilde{T}$. c) and d) Corresponding views of $\tilde{\psi}$ for DIII-D 126006 ELM at $t = 88.4$ in Fig. 1.

The tangle may contribute to the nonlinear harmonic consolidation, by constraining the unstable magnetic loops near the X-point to follow the equilibrium field line spacing, regardless of the driving perturbation. For each plasma configuration this may favor certain mode numbers, a circumstance that may be experimentally testable. Keeping higher harmonics beyond a certain range does not significantly affect the nonlinear ELM, despite their higher linear growth rates[1]. The initial very small perturbation tends to have nearly maximum mode numbers, but this changes rapidly as the ELM grows.

The DIII-D 126006 case represents[8] a time when the applied RMP field has completely stabilized the ELM in the experiment. The nonaxisymmetric RMP field itself was not applied in the simulation, but the equilibrium reconstruction included its effects on the edge density and temperature profiles and bootstrap current. In MHD at 5× the actual resistivity, a strong ELM appeared. It had smaller growth rate than some other DIII-D Type I ELMs, but grew rapidly to hit the outboard wall and divertors, Fig. 1. (The level of resistivity enhancement should have little accelerant effect.) Toroidal rotation, with the experimentally measured profile, did not greatly change the nonlinear ELM, but slightly accelerated it and amplified the later low-n interior instability.

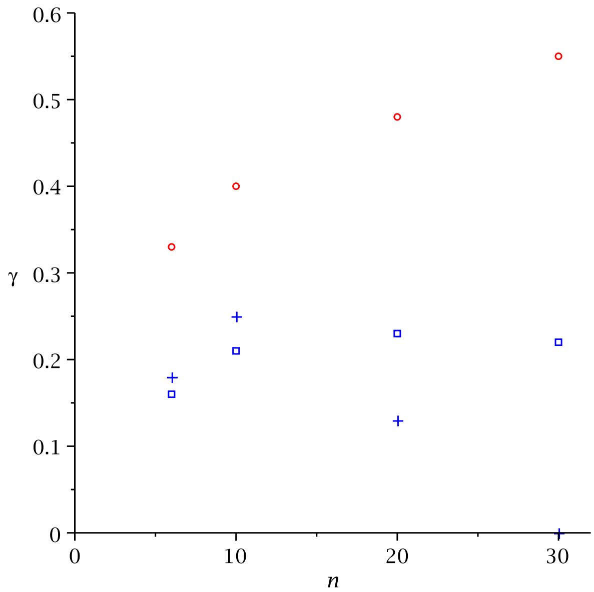
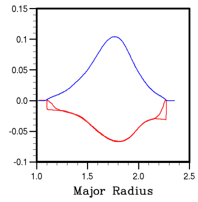


FIG. 3. Linear MHD growth rates for a range of toroidal mode numbers n, in units τ_A^{-1}, for a free-boundary plasma, DIII-D ELM discharge 126006 at 5× actual central resistivity. Red circles for stationary equilibrium, blue symbols for toroidal rotation. Blue squares use the experimental rotation profile, crosses a modified profile with constant angular velocity near the plasma edge, over normalized flux $\hat{\psi}$ = 0.7–1.0.



Midplane profiles: both versions of toroidal rotation in red, plasma pressure in blue).

The question of whether magnetic stochasiticity is required for the RMP stabilization is difficult to test directly by MHD simulation. Applying an RMP to the 126006 case further changes the edge profiles. For a rapidly applied RMP, the nonlinear edge profile changes, primarily reduction of the density gradient, and magnetic stochasticization of an edge layer occur fast enough to stabilize the ELM[3]. The experimental turn-on times are much longer and difficult to simulate in MHD. Instead, other stabilizing contributions are tested.

Equilibrium toroidal rotation is strongly stabilizing for the linear eigenmode. Blue squares in Fig. 3. show that rotation with the experimental profile reduces the growth rate by half at moderate $n = 10$ and gives roughly flat growth rates to $n \lesssim 30$. The experimental velocity falls off over the outer region of the plasma wider than the edge density pedestal, to a small finite edge value (inset figure). For this case, rotation rather than rotational shear yields stronger stabilization. The blue crosses show the same central profile, modified to have constant angular rotation over the outer plasma from $\hat{\psi} \simeq 0.7$ to the separatrix $\hat{\psi} = 1.0$, including the pedetal and ballooning region. The velocity drops to zero outside the separatrix. While the growth rate is slightly higher at low harmonics $n \leq 10$, it falls rapidly to near zero at $n = 30$. The strong effect may result from the extension of the mode outside the plasma, particularly the magnetic field, resulting in strong rotational shear and drag across the separatrix that exceeds the mode width at high mode number.

Nonlinearly, the simulations suggest that the combination of toroidal rotation and two-fluid effects is much more strongly stabilizing for plasma edge instabilities than either effect alone, in cases removed from strong ideal MHD instability. Combined rotation and two-fluid stabilization may play an important role in the stabilization of ELMs and other, non-ELMing edge instabilities. Toroidal rotation alone has little stabilizing effect on MHD unstable ELMs, including the 126006 case[1]. Instead, it tends to increase the ELM amplitude and the appearance of later, low-n interior modes. In contrast, for the externally applied RMP, MHD rotation greatly reduces the internal penetration and layer width of magnetic stochasticization in simulation[3]. Two-fluid effects alone could be either stabilizing or destabilizing linearly, as expected theoretically, but had nonlinearly had relatively weak effects on most MHD unstable edge modes.

The combination of rotation and two-fluid effects was also found to strongly limit the nonlinear coupling of edge instabilities to more slowly growing large scale, low-mode-number internal modes. Experimentally, low-*n* internal modes often follow ELMs in DIII-D discharges with low toroidal rotation speeds, apparently triggered by the ELM, and they frequently grow large enough to lock to the wall and cause serious loss of confinement. Such a mode was observed in the 126006 plasma[8]. One also appeared in the ELM simulation with no equilibrium rotation after the primary ELM crash, Fig. 1. The growing outboard midplane bulge eventually locked the instability-driven poloidal rotation to the wall, even without a full resistive wall. Toroidal rotation alone proved insufficient to suppress these interior modes in MHD simulation and instead increased their amplitude and growth rate. Two-fluid effects also were only weakly stabilizing. The effectiveness of the combined processes on reducing the interior mode was seen consistently in the simulation of ELMs in different experiments and also for weaker, non-ELMing electromagnetic instabilities, such as the Edge Harmonic Oscillation (EHO). It remains unclear whether the main mechanism is decoupling of edge to interior or direct suppression of the interior mode. Numerical difficulties with the two-fluid model limit conclusions on complete suppression.

4. Discussion

Due to the finite time response and other physical constraints on the plasma evolution, the magnetic response to an ELM instability is non-Hamiltonian. The ELM magnetic tangle therefore differs from an idealized Hamiltonian tangle[6]. In particular, the unstable tangle loops need not have equal areas inside and outside the separatrix — simulation suggests that the outer loops are typically smaller except immediately near the X-point. Nevertheless, the basic tangle mechanism is robust, since it arises from the simple perturbation properties of an X-point in 2D. The ELM tangle also retains key properties that reflect the equilibrium field line spacing and configuration[1, 3]. Theoretically, field lines on internal closed flux surfaces near the separatrix become enchained, expanding the affected region as the mode grows. The external originally unconfined field lines are unconstrained. Interior tangle field lines remain locally nearly aligned with the equilibrium field lines over many toroidal circuits. Eventually, many are lost from the vicinity of the X-points. The interior field can still respond to slower growing, low-mode-number instabilities, such as magnetic “islands” and sawteeth, although these no longer have the clean magnetic forms that they possess in axisymmetric backgrounds.

The results suggest that magnetic field stochasticity at a low level should be an important property of the outer part of D-shaped plasmas, in H- or L-mode, since edge perturbations exist in both regimes. Stochasticity could help explain a number of phenomena, while allowing large scale interior MHD processes to proceed with only minor modification. It is consistent with the existence of a steep edge pressure gradient supported by a minimum threshold level of heating power to enter H-mode. It may also contribute to the higher levels of thermal transport commonly observed in the outer plasma, that are difficult to explain by turbulent transport in axisymmetry.

The 2D tangle picture oversimplifies the plasma response. In higher dimensions, asymptotic surface splitting and stochasticity are general features of the perturbation of invariant manifolds. Recent developments in fluid dynamics[10] relate local measures of surface splitting or spreading in the fluid motion to the overall turbulence. Magnetic plasmas have greater sources of variation, since they possess two vorticities, the current density $\mathbf{J} = \nabla\times\mathbf{B}$ and the usual fluid velocity vorticity $\mathbf{w} = \nabla\times\mathbf{v}$ The fluid techniques may help diagnose and characterize the tangle stochasticity.

5. Conclusions

Recent higher resolution, large scale numerical simulations using an improved version of the extended MHD code M3D[1, 2] show that, for a freely moving plasma boundary with magnetic X-points, transverse plasma instabilities can self-consistently generate a stochastic magnetic tangle. The characteristic tangle structure strongly influences the stability and dynamics of edge instabilities in magnetically confined fusion plasmas. The resulting multi-stage ELM is consistent with many experimental observations. The importance of the equilibrium field configuration to the tangle leads to important geometric differences between ELMs in different plasma configurations, including tokamaks and spherical torii. Large differences exist between the nonlinear ELM behavior and stability and linear eigenmode predictions for free boundary plasmas, not all due to the tangle. Similar effects are seen for weaker, non-ELMing edge oscillations. The results suggest a new picture of natural, weakly stochastic plasma edge that could have major implications for stability and confinement in fusion plasmas.

Acknowledgements. Work supported in part by the U.S. Department of Energy Fusion Energy Sciences and SciDAC programs. Computations performed at the National Energy Research Scientific Computing Center.

References

[1] SUGIYAMA, L.E., and STRAUSS, H.R., “Magnetic X-points, edge localized modes, and stochasticity,” Phys. Plasmas **17** (2010) 062505.

[2] SUGIYAMA, L.E., et al., “Magnetic X-points, stochasticity, and Edge Localized Modes,” J. Phys.: Conf. Series **180** (2009) 012080.

[3] STRAUSS, H.R., et al., “Extended MHD simulation of resonant magnetic perturbations, Nucl. Fusion **49** (2009) 055025.

[4] PARK, W., et al., Plasma Simulation Studies Using Multi-Level Simulation Models, Phys. Plasmas **6** (1999) 1796; SUGIYAMA, L.E. and PARK, W., A nonlinear two-fluid model for toroidal plasmas, Phys. Plasmas **7** (2000) 4644.

[5] ROEDER, R.K.W., et al., “Explicit calculations of homoclinic tangles in tokamaks,” Phys. Plasmas **10** (2003) 3796.

[6] LICHTENBERG, A.J., and LIEBERMAN, M.A., Regular and Chaotic Dynamics, 2nd ed., Springer-Verlag, New York (1992).

[7] EVANS, T.E., et al., “Suppression of large edge localized modes with edge resonant magnetic fields in high confinement DIII-D plasmas,” Nucl. Fusion **45** (2005) 595.

[8] EVANS, T.E., et al., “RMP ELM suppression in DIII-D plasmas with ITER similar shapes and collisionalities,” Nucl. Fusion **48** 024002 (2008).

[9] BERNSTEIN, I.B., et al., “An energy principle for hydromagnetic stability problems,” Proc. Royal Soc. London Ser. A, **244** (1958) 17.

[10] LEKIEN, F., et al., “Lagrangian coherent structures in n-dimensional systems,” J. Math. Phys. **48** (2007) 065404; MATHUR, M., et al., “Uncovering the Lagrangian Skeleton of Turbulence,” Phys. Rev. Lett. **98** (2007) 144502.