3D-MAPTOR Code for Computation of Magnetic Fields in Tokamaks

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Abstract. A 3-D code has been developed in order to simulate the magnetic field lines in tokamaks, in two versions. In the first one, the toroidal magnetic field can be obtained from the individual fields of circular coils arranged around the torus, or alternatively, as a ripple-less field. The poloidal field is provided by a given toroidal current density profile. In an upgraded version, rectangular toroidal field coils and D-shaped plasma cross sections have been included, in order to aid in the design of spherical tokamaks. Proposing initial conditions for magnetic filed lines, they are integrated along the toroidal angle coordinate, and Poincaré maps can be obtained at any desired cross section plane along the toroidal coordinate. The evolution of the field lines is also monitored from above, so the ripple due to the toroidal field coils can be appreciated. The effects of loss of axisymmetry, either originated by ripples, or by additional external coils, such as an inner coil with tilted circular loops, can therefore be studied. This is useful for the study of breaking-up of external surfaces, as in the case of ergodic divertors. The code can also be used in order to reconstruct the evolution of the plasma column, using the experimental signals of tokamak discharges. In the latter case, the results have been compared with tomographic results of the ISTTOK tokamak.

1. Introduction

It is well known that toroidal magnetic fields can be described in terms of a Hamiltonian formulation, in which the magnetic flux plays the role of the Hamiltonian, and the toroidal angle coordinate plays the role of time, in analogy with the mechanical problem [1]. Thus, it is only natural to apply the techniques and results of Hamiltonian mechanics to the study of toroidal magnetic fields in fusion confinement devices, such as tokamaks and stellerators.

To a certain extent, the success in designing a magnetic confinement device, rests in the capacity of producing a configuration, such that there is a symmetry which allows the existence of closed and sturdy magnetic field surfaces. If such symmetry is broken, either by instabilities within the plasma, or by engineering defects in the design and construction of the system, the surfaces break up, leading to the loss of confinement. It is the merit of the magnetic confinement fusion program that such good devices, with strong symmetry properties have been designed and operated successfully, through the past few years.

The purpose of this work is to develop a three dimensional code with the capability of reproducing the behaviour of the magnetic field lines in tokamaks with circular cross section, and in a latter stage, with rectangular field coils. The code may be useful for design purposes, allowing an estimate of the necessary currents, when a given number of coils with given size and aspect ratio are chosen. Being three dimensional, it allows studies of the effect of the ripple on the magnetic field surfaces, and to establish when they may be broken due to the loss of axisymmetry. On the other hand, when the currents from a given device, in a given shot are known, this code allows a reconstruction of the behaviour of the plasma column, and the determination of the existence of magnetic surfaces. While it is not possible in this approach to know if a given configuration is in MHD equilibrium, this may be turned into an advantage in the sense that it can still describe real life situations of discharges that happen when such equilibrium does not exist. This will be clearly illustrated.

Some results using this code have been previously reported in Refs. 2-4.

2. The Model

The toroidal magnetic field is computed from the individual fields of circular coils arranged around the torus, or alternatively, as a ripple-less field. The inductance of the toroidal field coils is obtained in terms of elliptic functions, as shown in Ref. [5]. The case of the ripple-less field is assumed to be given by a magnetic field which goes as $1/r$, where r is the radial coordinate in cylindrical coordinates. The ripple-less version can be used when the it is necessary to study the behaviour of magnetic surfaces due to a different loss of symmetry, such as in the case of the ergodic divertor. In such a case it may be useful to start from an unperturbed axisymmetric state. The poloidal field is provided by a given toroidal current density profile, which can be chosen at will.

Fig. 1 shows the coordinates chosen for the determination of the magnetic fields due to a set of circular toroidal field coils.

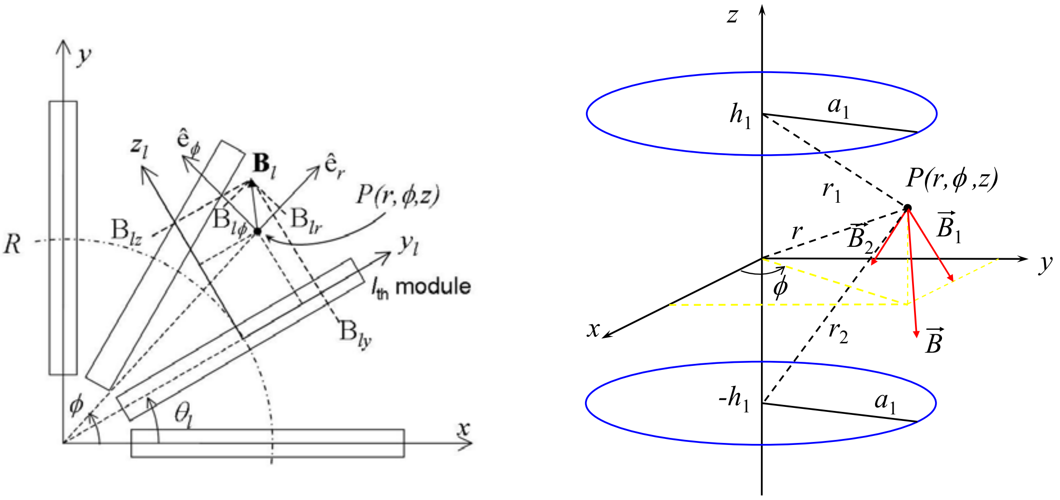


FIG. 1 Coordinates used for the determination of the magnetic field due to a set of circular magnetic field coils.

The magnetic field created by a current density distribution **J** is given by the Biot-Savart law. A given profile for the current density must be chosen, such as

$$J_\phi = 4J_o\left(1-\frac{r^2}{a^2}\right) \quad , \tag{1}$$

for instance, which we have commonly used.

Proposing initial conditions for magnetic filed lines, the equation

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dx}{B_x} = \frac{ds}{B} \tag{2}$$

is integrated along the toroidal angle coordinate, taking into account the full set of plasma and coil currents, and Poincaré maps can be obtained at any desired cross section plane along the toroidal coordinate. The evolution of the field lines is also monitored from above, so the ripple due to the toroidal field coils can be appreciated.

In order to obtain the magnetic field surfaces, a set of initial conditions is provided on the equatorial plane, at an angle which we define to be 0, and the path of the field line is obtained by integration, using a fourth order Runge-Kutta method.

3. Breaking of Magnetic Field Lines due to Loss of Axisymmetry in a Ripple-Less Unperturbed System

In this case we start with a ripple-less magnetic field toroidal system, and then break the axisymmetry by introducing a set of tilted coils in the inboard side. Both the tilting of the coil and its current can be varied, in order to study the appearance of magnetic islands and the breaking up of the magnetic field surfaces.

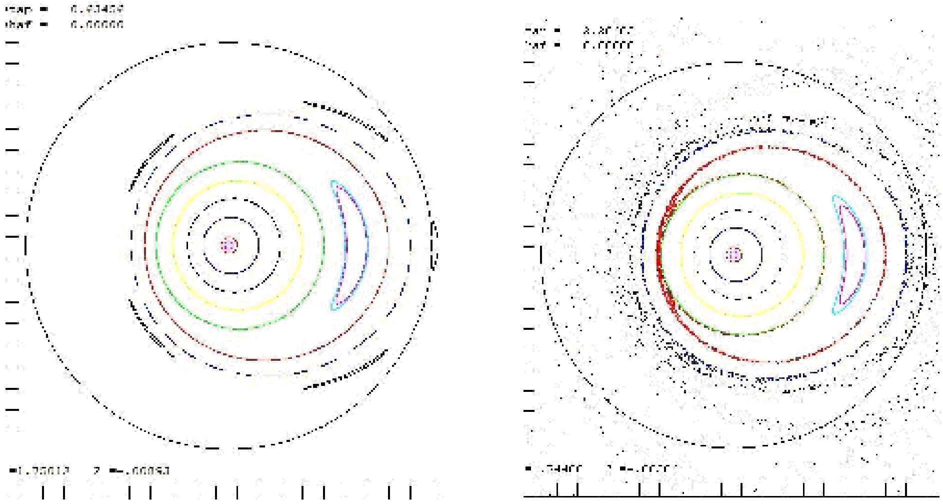


FIG 2. Magnetic surfaces perturbed by an inner coil on the inboard side, tilted 10 degrees, and with a 5.00 kA and 5.75 kA currents. The axis of symmetry is on the left.

The three dimensional capability of the code is illustrated in Fig. 3, in which the evolution of the islands is followed along the toroidal angle.

4. Reconstruction of the Behaviour of a Plasma Column

In a given experiment, the 3D-MAPTOR code may be useful to study the behaviour of the magnetic field for a given set of operational parameters, even if the plasma is not necessarily in equilibrium. Two such cases have been studied. The first one concerns the NOVILLO tokamak, which operated at the Instituto Nacional de Investigaciones Nucleares in Mexico, a

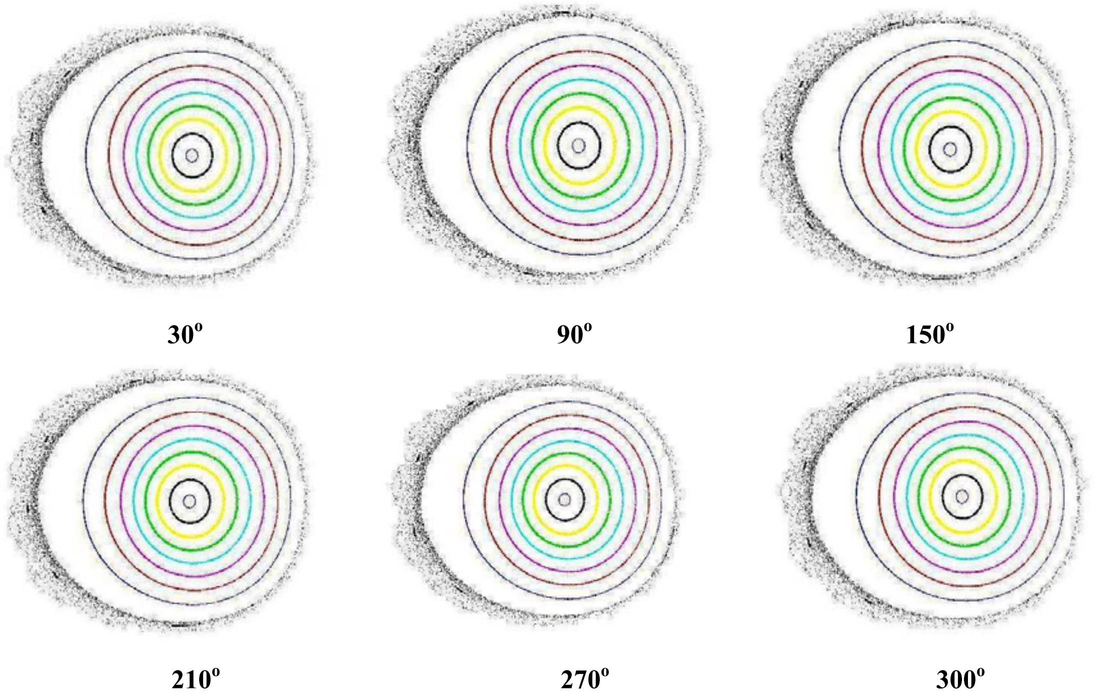
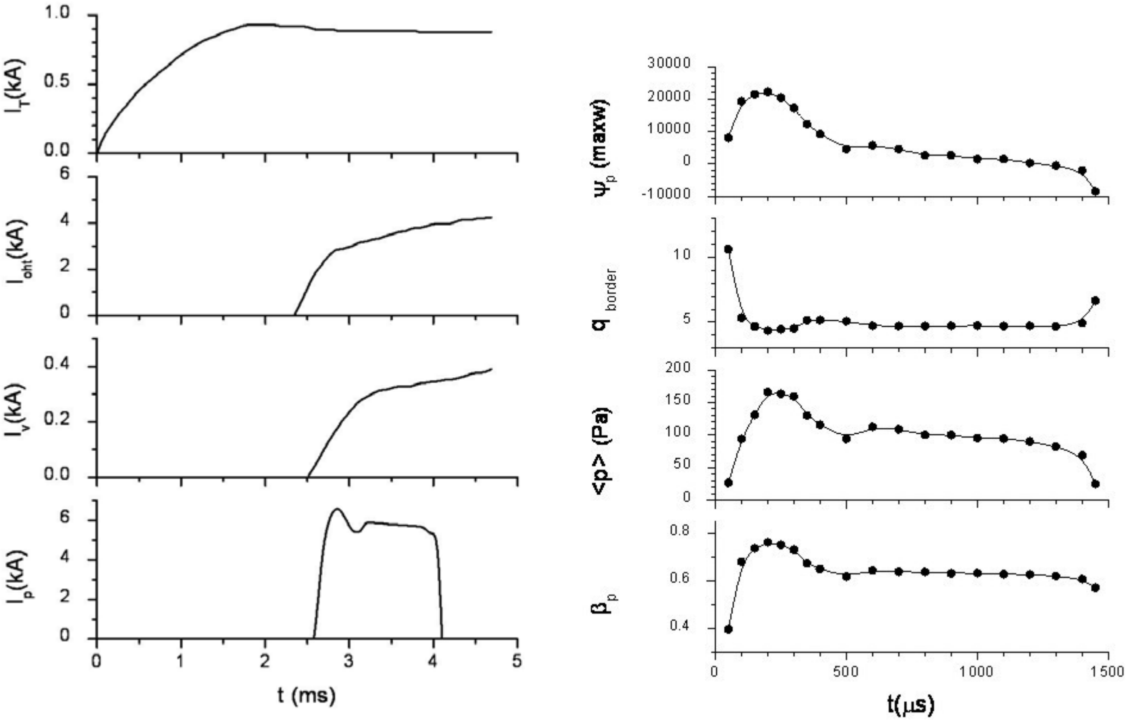
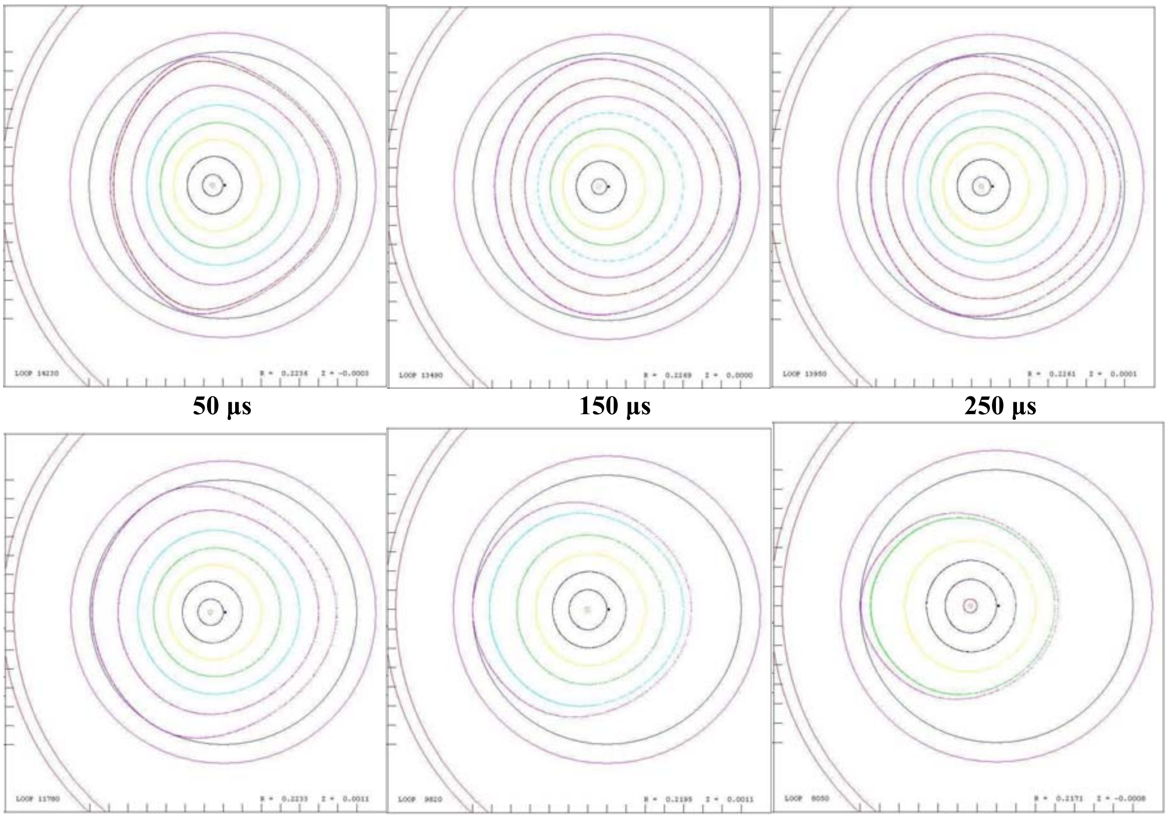


FIG. 3. Evolution of magnetic islands along the toroidal angle.



(a) **(b)**

FIGURE 4. ***(a)*** *Set of signals used for the reconstruction of the magnetic field surfaces in a plasma discharge lasting approximately 1500 μs. These are, from top to bottom, the toroidal field coils current, the ohmic heating coil current, the vertical field coils current, and the plasma current.(b) Temporal evolution of the poloidal flux ψ_p and the safety factor q, both at the outer surface, the average pressure, and beta poloidal β_p.*



50 µs 150 µs 250 µs

350 µs 450 µs 1400 µs

FIG. 5 Evolution of the plasma column in the Novillo tokamak. The discharge starts on the outboard side, and shortly crashes into the inboard side, where it remains until the end of the discharge at 1400 µs.

few years ago, and the second to the ISTTOK tokamak at the Instituto Superior Técnico in Portugal.

4.1 Reconstruction of a Shot at the Novillo Tokamak

Doubts had been cast on whether magnetic field surfaces actually existed in the first place in this machine. Looking at the records, we chose a particular shot, for which all the relevant information to feed the code was provided. Signals are shown in Fig. 4 a. for the toroidal field coils current, the ohmic heating coil current, the vertical field coils current, and the plasma current. The reconstruction of the behaviour of the plasma column is shown in Fig. 5. Four constant circles can be appreciated in them. The outer two are the toroidal field coil at 0^o. The inner two represent the vacuum chamber and the limiter. The outer magnetic field surface is defined as the one which touches the limiter. It can be clearly seen how the column is centered at 50 µs, then it moves towards the outer board by 150 µs, when the plasma current reaches its maximum, but the vertical field is too strong, and by 350 µs it has been thrown inward. The column contracts as time increases, and extinguishes by 1450 µs when the plasma current falls. This result is consistent with qualitative observations of the plasma column position made with magnetic probes.

Further information can be obtained from the magnetic field surfaces, such as the evolution of the profiles of the poloidal flux $\psi_p(r)$, which can be readily integrated and the safety factor $q(r)$. Fig. 4 b shows the evolution in time of the poloidal flux and the safety factor at the outer surface, the average pressure $<p>$, and beta poloidal β_p.

4.2 Reconstruction of a Shot at the ISTTOK tokamak

Knowing the currents involved in ISTTOK shot no. 16465, a comparison is made between the 3D-MAPTOR reconstruction and tomographic reconstruction developed by the ISTTOK team for the purpose of a real-time plasma position control system [4]. The current of the toroidal field coils is constant at 6 kA for the time span of interest, which lasts 50 ms. The rest of the relevant currents are shown in Fig. 6. The device is operated in AC mode, so there is a change of polarity around 27 ms. It must be appreciated that the 3D-MAPTOR maps the magnetic field, while the tomographic reconstruction is based on the visible spectrum. Therefore, it is interesting to note how the plasma behaves during the change of polarity, when the magnetic field surfaces are lost.

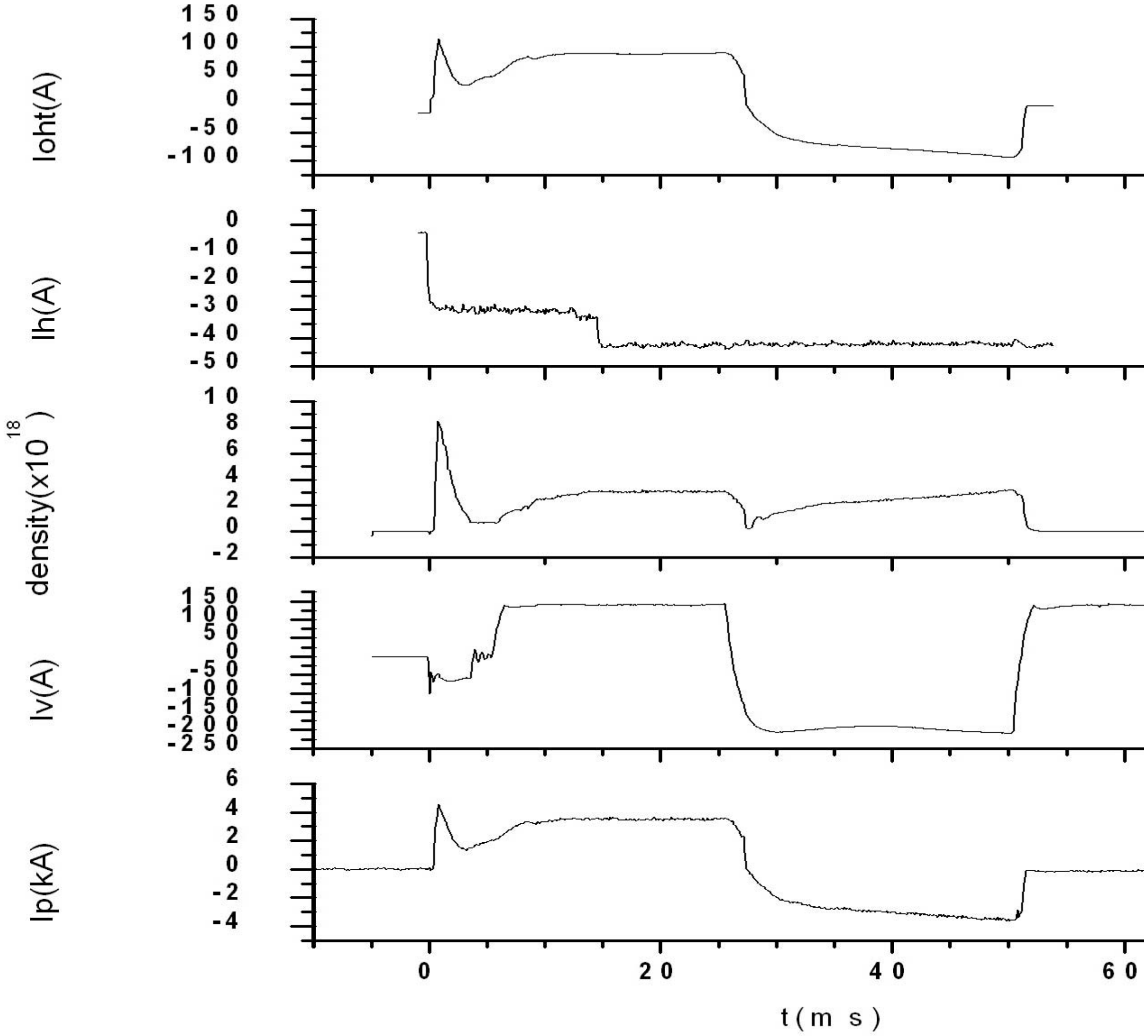
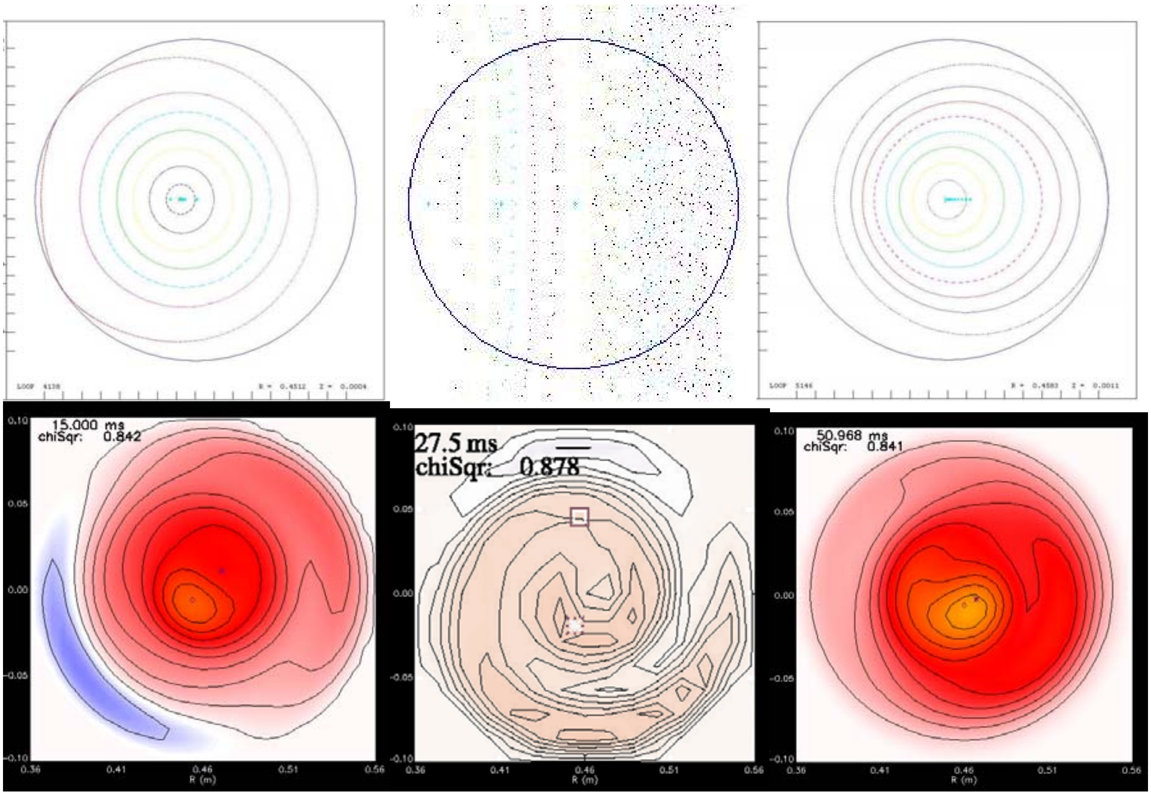


FIG. 6. Current signals of shot no.16465. From top to bottom; Ohmic heating current (note the change in polarity around 37 ms, horizontal coils current, density, vertical coils current and plasma current.

The reconstruction for this discharge is shown in Fig. 7. for three different times.



15 ms **27.5 ms** **51 ms**

FIG. 7. Comparison between the magnetic field, as reconstructed with the 3D-MAPTOR, and the visible tomographic reconstruction for shot no. 16465 of the ISTTOK tokamak.

5. Use of the 3D-MAPTOR in the Design of a Spherical Tokamak

As an illustration of the use of the code in the design of new machines, we show an example in which we explore the magnetic field of a spherical tokamak with 12 rectangular coils. In this case a D-shaped current density profile is used in order to simulate the plasma

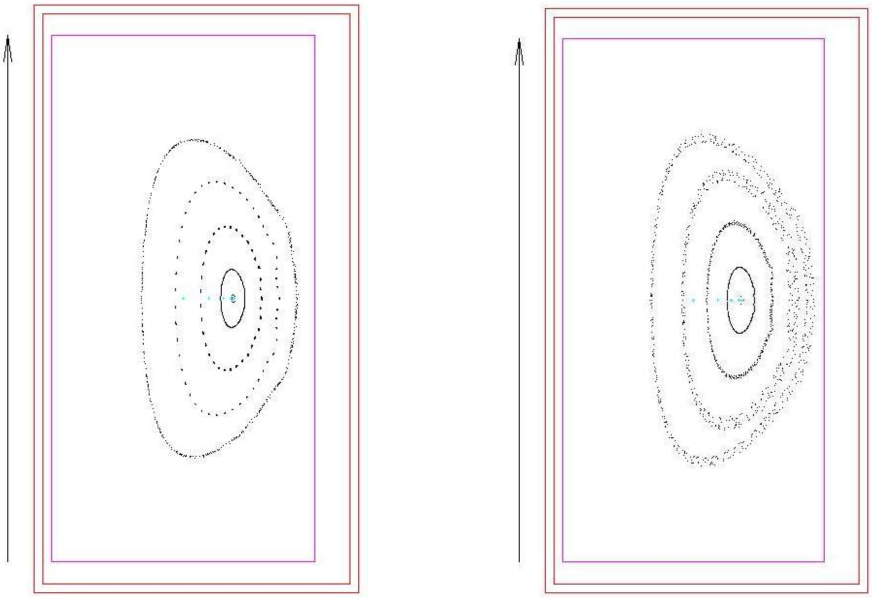


FIG. 8. Breaking of magnetic field surfaces due to ripple effect in the design of a spherical tokamk with 12 coils.

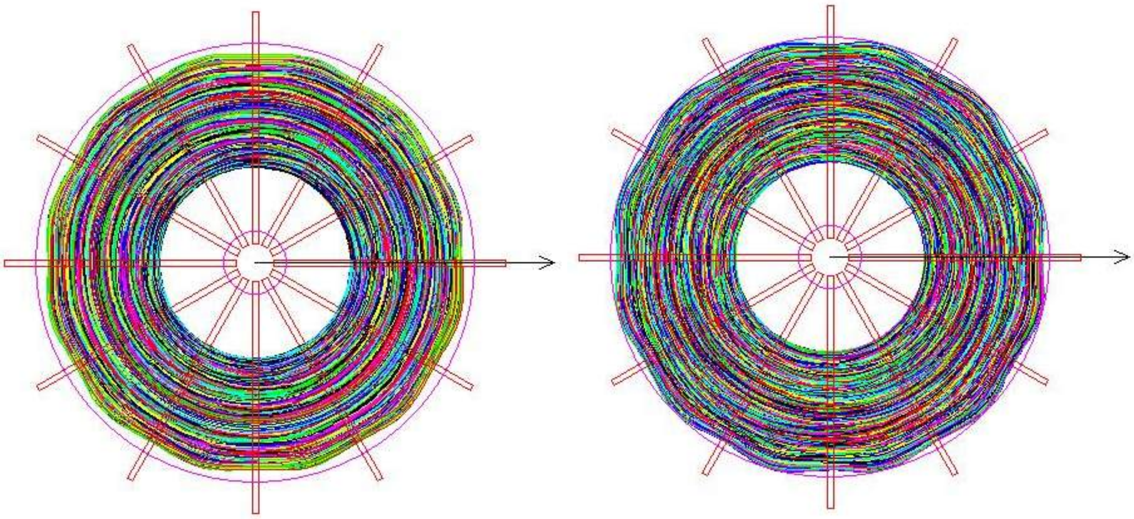


FIG. 9. The tracing of the magnetic field as seen from above for the surfaces mapped in Fig. 8. The size of the ripple can be clearly appreciated.

The main purpose here is to study the effect of the ripple on the existence of the magnetic field surfaces. Fig. 8 shows the outcome for two different plasma current values. The tracing of the magnetic field as seen from above is shown in Fig. 9.

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