Intrinsic Toroidal and Poloidal Flow Generation in the Background of ITG Turbulence

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Abstract. The generation of intrinsic toroidal and poloidal flows in the background of ion temperature gradient driven microturbulence has been studied using quasilinear theory. It is shown that the dynamics of mean toroidal and poloidal flows is coupled. The radial fluxes of toroidal and poloidal momenta are calculated. The polarization drift driven toroidal momentum flux is found to be independent of mean $E \times B$ flow shear and can become comparable to $E \times B$ shear driven flux in weak pressure gradient region and hence complements the $E \times B$ shear induced $\langle k_\parallel \rangle$ symmetry breaking mechanism [O. D. Gurcan *etal.*, Phys. Plasmas 14, 042306 (2007)] of intrinsic rotation.

1. Introduction

The intrinsic rotation[1], which is a manifestation of anomalous momentum transport in many present day tokamaks, is an important unresolved issue since its first observation in early 1990s [2]. Observation of toroidal rotation and its shear in determining the power threshold for L-H transition[4], in stabilizing resistive wall modes (RWMs) and "Rice scaling" prediction of ~ 250 km/s of intrinsic rotation speed of ITER plasma which appears to be sufficient to stabilize RWMs[3], has accelerated the theoretical and experimental studies on intrinsic rotation. Mean field generation by turbulent Reynolds stresses is now well studied. For adiabatic electron response the mean parallel momentum flux is solely determined by the Reynolds stress which in the spirit of mean field electrodynamics[5], is shown to be decomposed as[6] $\langle \tilde{v}_r \tilde{v}_\parallel \rangle \approx -\chi_\parallel \frac{\partial \langle v_\parallel \rangle}{\partial r} + V_c \langle v_\parallel \rangle + S$, where the first term is diffusive, the second term is convective (or pinch) and the third term known as residual stress is independent of mean parallel flow and its shear and is the most crucial component in generating intrinsic rotation. Momentum diffusivity $\chi_\phi \sim \chi_i$(thermal diffusivity) is claimed in Refs.[2, 8, 9], but some recent work[10] show departure from this. Turbulent momentum pinch has been treated in Refs.[11] and residual stress has been shown to arise due to asymmetry of fluctuation spectra induced by mean radial electric field shear[6, 9], which has been confirmed by perturbation experiments carried out on JT60U [12], due to up-down symmetry breaking of equilibrium magnetic topology[13] and from parallel nonlinearity in gyrokinetic framework [14].

However so far to the best of our knowledge little attention has been paid to simultaneous mean poloidal flow generation. Again in the same spirit of mean field electrodynamics we can write the poloidal turbulent Reynolds stress as $\langle \tilde{v}_r \tilde{v}_y \rangle \approx -\chi_y \frac{\partial \langle v_y \rangle}{\partial r} + V_r \langle v_y \rangle + S_y$, Here the residual stress term S_y is independent of mean poloidal flow $\langle v_y \rangle$ and its shear $\langle v_y' \rangle$ but may depend on mean toroidal flow $\langle v_\parallel \rangle$ and its shear $\langle v'_\parallel \rangle$. In other words, the off-diagonal part for one type of stress may serve as the diagonal part for the other, hence providing a coupling between toroidal and poloidal flow generation mechanisms. This is the prime focus of the present paper. In this article we have performed a systematic calculation of toroidal and poloidal Reynolds stresses for electrostatic ITG turbulence with adiabatic electron response, within the fluid theoretical framework, and found that toroidal and poloidal flow generation mechanisms and their evolutions are naturally coupled. We also find that the polarization drift can generate parallel residual stress even in the absence of a "seed" poloidal flow. While turbulent $E \times B$ drift induced poloidal stress requires assymetric eigen spectrum with radial outgoing wave nature[15], polarization drift found not to drive poloidal stress at all.

2. Flow Generation and Amplification: Momentum Conservation

We separate the temporal and perpendicular spatial scales into a set of "fast" variables,$(\vec{x}_\perp, t)$, associated with the microturbulence and a set of "slow" variables $(\vec{X}_\perp, T)$, typical of slowly evolving equilibrium profiles[16]. Write $\vec{\nabla}_\perp \rightarrow \vec{\nabla}^{(0)}_\perp + \epsilon\vec{\nabla}^{(1)}_\perp$, $\frac{\partial}{\partial t} \rightarrow \epsilon\frac{\partial}{\partial t} + \epsilon^2\frac{\partial}{\partial T}$, $\nabla_\parallel \rightarrow \epsilon\nabla_\parallel$ where $\epsilon \sim (\rho_s/L_n) \sim (\omega/\omega_{ci}) \sim (k_\parallel/k_y) \ll 1$ in drift wave ordering. Here $\vec{\nabla}^{(0)}_\perp$ corresponds to a derivative with respect to $\vec{x}_\perp$ and $\vec{\nabla}^{(1)}_\perp$ corresponds to a derivative with respect to $\vec{X}_\perp$. Furthermore, for any field f and it's fluctuation $\delta f(x, t, X, T)$, we may define a space-time average over the fast scales such that $\langle \delta f(x, t, X, T) \rangle = 0$, but functions of only slow variables are left unaltered, i.e., $\langle f(X, T) \rangle = f(X, T)$. Similarly, averages over the fast scales annihilate derivatives of fast variables as well as derivatives along magnetic field lines but commute with slow derivatives, i.e., $\langle \vec{\nabla}^{(0)}_\perp f \rangle = \langle \nabla_\parallel f \rangle = 0$, but $\langle \vec{\nabla}^{(1)}_\perp f \rangle = \vec{\nabla}^{(1)}_\perp \langle f \rangle$. We, thus, obtain the evolution equation for mean toroidal and poloidal flows from standard ion momentum equation upto order ϵ^4 as

$$\frac{\partial \langle V_\parallel \rangle}{\partial T} + \frac{\partial}{\partial X}\langle \delta v_{Ex} \delta v_\parallel \rangle + \frac{\partial}{\partial X}\langle \delta v_{polx} \delta v_\parallel \rangle = 0 \tag{1}$$

and

$$\frac{\partial \langle V_y \rangle}{\partial T} + \frac{\partial}{\partial X}\langle \delta v_{Ex} \delta v_{Ey} \rangle + \frac{\partial}{\partial X}\langle \delta v_{polx} \delta v_{Ey} \rangle = 0 \tag{2}$$

which are coupled through the radial force balance equation

$$\langle V_y \rangle = -\frac{E_x}{B_z} + \frac{1}{n_0 e B_z}\frac{\mathrm{d}P_0}{\mathrm{d}X} + \frac{\langle V_z \rangle B_y}{B_z} \tag{3}$$

where δv_E is fluctuating $E \times B$ drift velocity and δv_{pol} is the fluctuating polarization drift velocity. Note that, though the third term in the above mean toroidal and poloidal momentum equation is nominally one order higher it will be shown in Section 5 that under certain conditions the second and third terms can become comparable.

3. Radial Eigenmode Analysis

In a sheared slab configuration of magnetic field $\vec{B} = B(\hat{z} + (x - x_0)/L_s\hat{y})$ in the neighborhood of a rational surface at x_0, and for fluctuations localized on a particular rational surface at

$x = x_0$, expanding the mean ion flow velocity $\vec{V}_{i0}$ as $\vec{V}_{i0}(x) = \vec{V}_{i0}(x_0) + (x - x_0)\left(\frac{\partial \vec{V}_{i0}}{\partial x}\right) + ...$ and taking adiabatic electron response $\delta n_e/n_0 = e\delta\phi/T_e$. the perturbed linearized continuity, momentum and pressure equation for ions describing fluid ITG instability is

$$(\frac{\partial}{\partial t} + x\hat{V}'_{E0}\nabla_y)(1 - \nabla_\perp^2)\phi + (1 + K\nabla_\perp^2)\nabla_y\phi + \nabla_\parallel v = 0 \quad (4)$$

$$(\frac{\partial}{\partial t} + x\hat{V}'_{E0}\nabla_y)v - \hat{V}'_{\parallel 0}\nabla_y\phi + \nabla_\parallel(p + \phi) = 0 \quad (5)$$

$$(\frac{\partial}{\partial t} + x\hat{V}'_{E0}\nabla_y)p + K\nabla_y\phi + \Gamma\nabla_\parallel v = 0 \quad (6)$$

where the normalizations are $x = (x - x_0)/\rho_s$, $y = y/\rho_s$, $z = z/L_n$, $t = tc_s/L_n$, $\phi = (e\delta\phi/T_e)(L_n/\rho_s)$, $n_i = (\delta n_i/n_0)(L_n/\rho_s)$, $v = (\delta v_{\parallel i}/c_s)(L_n/\rho_s)$, $p = (\tau_i\delta p_i/P_{i0})(L_n/\rho_s)$, $L_n\nabla_\parallel \equiv \nabla_\parallel = \frac{\partial}{\partial z} + xs\frac{\partial}{\partial y}$ and the nondimensional parameters are $\eta_i = L_n/L_T$, $K = \tau_i(1+\eta_i) = \tau_i\alpha_i$, $\Gamma = \gamma\tau_i$, $s = L_n/L_s$,$\tau_i = T_i/T_e$, $\hat{V}'_{E0} = (L_n/c_s)V'_{E0}$, $\hat{V}'_{\parallel 0} = (L_n/c_s)V'_{\parallel 0}$, $\rho_s = c_s/\omega_{ci}$. Now considering the perturbation of the form $f = f_k(x)exp(ik_y y - i\omega t)$, where k_y and ω are normalized as $k_y = k_y\rho_s, \omega = \omega/(c_s/L_n)$, and assuming that the flow shear frequency is much smaller than the mode frequency, the above set of Eqs.(4-6) form an eigenvalue problem in x direction in ϕ_k

$$\frac{\mathrm{d}^2\phi_k}{\mathrm{d}x^2} + \left(A_1 + A_2 x + A_3 x^2\right)\phi_k = 0 \quad (7)$$

where

$$A_1 = \frac{k_y - \omega}{\tau_i\alpha_i k_y + \omega} - k_y^2, A_2 = \frac{k_y}{\tau_i\alpha_i k_y + \omega}\left(\hat{V}'_{E0} - k_y s\frac{\hat{V}'_{\parallel 0}}{\omega}\right), A_3 = \left(\frac{k_y s}{\omega}\right)^2 \quad (8)$$

The solution of equation(7) for the most dominant $l = 0$ mode is shifted off the mode rational surface as shown bellow

$$\phi_k = \phi_0 \exp\left[-\frac{1}{2}i\sqrt{A_3}\left(x + \frac{A_2}{2A_3}\right)^2\right] \quad (9)$$

and the corresponding eigenmode dispersion relation is

$$\omega^2\left(1 + k_y^2\right) + \omega k_y\left(-1 + k_y^2\tau_i\alpha_i + is\right) + is\tau_i\alpha_i k_y^2 = -\frac{\omega k_y^2\left[\frac{\omega}{sk_y}\hat{V}'_{E0} - \hat{V}'_{\parallel 0}\right]^2}{4\left(\tau_i\alpha_i k_y + \omega\right)} \quad (10)$$

In the limit $\hat{V}'_{E0} = \hat{V}'_{\parallel 0} = 0$ and $s \ll 1$ and for long wavelength mode satisfying $|k_y^2\tau_i\alpha_i| \ll 1$ we get a purely a growing mode $\omega = is\tau_i\alpha_i k_y$, called the slow mode
When $\hat{V}'_{E0} = \hat{V}'_{\parallel 0} = 0$ and under the assumption that $|1 - k_y^2\tau_i\alpha_i| \lesssim s \ll 1$ the fastest growing mode occurs at $k_y^2 = (\tau_i\alpha_i)^{-1}$ which is given by $\omega = (-1 + i)\sqrt{\frac{s}{2(1+k_y^2)}}$ The corresponding slow and fast eigenfunctions are

$$\phi_k = \phi_{0ks}\exp\left[-\frac{1}{2}\left(\frac{x - \xi_{ks}}{\Delta_{ks}}\right)^2\right]\exp\left[i\frac{\hat{V}'_{\parallel 0}}{2\tau_i\alpha_i}x\right], \quad \Delta_{ks} = \sqrt{\tau_i\alpha_i} \quad (11)$$

where the mode shift from the mode rational surface is given by $\xi_k = \tau_i \alpha_i \hat{V}'_{E0}/2$. Fast growing eigenfunction turns out to be

$$\phi_k = \phi_{0kf} \exp\left[-\frac{1}{2}\left(\frac{x-\xi_{kf}}{\Delta_{kf}}\right)^2\right] \exp\left[\frac{i}{2}\left(\frac{x+\xi_{kf}}{\Delta_{kf}}\right)^2\right] \tag{12}$$

where $\xi_{kf} = \tau_i \alpha_i \hat{V}'_{E0}/2s\,(\tau_i\alpha_i + 1)$ is the mode shift off the rational surface and $\Delta_{kf}^{-2} = \sqrt{s(\tau_i\alpha_i+1)/2(\tau_i\alpha_i)^2}$ where Δ_{kf} is half-width of the mode. In the following we will be using the slow and fast mode frequencies in the absence of flow for flux calculations.

4. Momentum Flux

In this section we calculate the toroidal and poloidal quasilinear momentum fluxes due to $E \times B$ drift and polarization drift assuming the flow shear frequency much smaller than the mode frequency.

4.1. $E \times B$ ***Flux***

From linear responses for $\delta v_{\|,k}$ from Eq.(5) and for δv_{Ex} we obtain a quasilinear toroidal Reynolds stress

$$\langle \delta v_{Ex} \delta v_{\|} \rangle = Re\left(\frac{c_s\rho_s}{L_n}\right)^2 \sum_{\vec{k}} i\frac{k_y^2}{\omega}\left[-\hat{V}'_{\|0} + \frac{k_{\|}}{k_y}\left[1 - \frac{\omega_{*pi}}{\omega}\right]\right]|\phi_k|^2 \tag{13}$$

where $\omega_{*pi} = -\tau_i\alpha_i k_y$, $k_x = -i\partial ln\phi_k/\partial x$ and $Re(f)$ means real part of (f). The first term is diffusive and the second term, being independent of $V_{\|}$ and $V'_{\|}$, is the non-diffusive residual flux. Since $k_{\|} = k_y s x$ so, $\langle k_{\|}/k_y \rangle$ survives when scalar potential ϕ_k has odd parity about a mode-rational surface and poloidal stress is

$$\langle \delta v_{Ex} \delta v_y \rangle = -Re \sum_{\vec{k}} \left(\frac{c_s\rho_s}{L_n}\right)^2 \left[1 - \frac{\omega_{*pi}}{\omega}\right] k_y k_x |\phi_k|^2 \tag{14}$$

where the first term is due to $E \times B$ drift and the second term is due to ion diamagnetic drift. It is obvious that poloidal flux survives only if $k_x \neq 0$. In case of standing eigenmodes k_x is imaginary and hence mean flow can not be generated. If k_x is linear in x, as happens to be for electrostatic drift waves, another turbulence characteristic necessary for poloidal flow generation is that of radial asymmetry of fluctuation spectrum about mode-rational surface. This is analogous to $\langle k_{\|} \rangle$ symmetry breaking and hence may be termed as $\langle k_x \rangle$ symmetry breaking. In the following we calculate toroidal and poloidal stresses explicitly for slow and fast modes making use of $\sum_{\vec{k}} \equiv \sum_{k_y} \int_{-\infty}^{+\infty} \mathrm{d}x k_y s$.

Slow mode: Using slow mode frequency and eq.(11) in eq.(13) gives

$$\langle \delta v_{Ex} \delta v_{\|} \rangle = \sum_{k_y} \left(\frac{c_s\rho_s}{L_n}\right)^2 k_y^2 \left[-\hat{V}'_{\|0} + s\xi_{ks}\right] \sqrt{\pi}/\Delta_{ks} |\phi_{0ks}|^2 \tag{15}$$

and eq.(14) gives

$$\langle \delta v_{Ex} \delta v_y \rangle = -\sum_{k_y} \left(\frac{c_s \rho_s}{L_n} \right)^2 k_y^2 s \left(\hat{V}'_{\|0} / 2\Delta_{ks} \right) |\phi_{0ks}|^2 \tag{16}$$

which is a purely non-diffusive flux, and hence is capable of producing intrinsic mean poloidal rotation.

Fast mode: The fast mode frequency and eq.(12) in (13) gives

$$\langle \delta v_{Ex} \delta v_{\|} \rangle = \sum_{k_y} \left(\frac{c_s \rho_s}{L_n} \right)^2 \frac{k_y^3 s}{2\gamma_f} \left[-\hat{V}'_{\|0} + (1 + \omega_{*pi}/\gamma_f) s \xi_{kf} \right] \Delta_{kf} \sqrt{\pi} |\phi_{0kf}|^2 \tag{17}$$

Poloidal stress due to $E \times B$ drift for fast mode is

$$\langle \delta v_{Ex} \delta v_y \rangle = -\sum_{k_y} \left(\frac{c_s \rho_s}{L_n} \right)^2 k_y^2 s \left[1 + \frac{\omega_{*pi}}{2\gamma_f} \right] \left[2 \frac{\xi_{kf}}{\Delta_{kf}} \right] \sqrt{\pi} |\phi_{0kf}|^2 \tag{18}$$

From the above analysis of $E \times B$ flux a few points may be noted. The residual toroidal flux is proportional to mean electric field shear, and hence to mean pressure gradient, and so this mechanism of flow generation is active in the high pressure gradient region, typical of the edge region of H-mode plasmas.

4.2. ***Polarization Drift Driven Flux***

From linear responses for $\delta v_{\|,k}$ from equations(5) and (6) and $\delta v_{polx} = -c_s \left(\rho_s / L_n \right)^2 \partial_t \partial_x \phi$ we obtain a quasilinear form of toroidal stress

$$\langle \delta v_{polx} \delta v_{\|} \rangle = -c_s^2 \left(\frac{\rho_s}{L_n} \right)^3 \frac{\partial}{\partial T} \langle \frac{\partial \phi}{\partial x} \delta v_{\|} \rangle + c_s^2 \left(\frac{\rho_s}{L_n} \right)^3 Re \sum_{\vec{k}} \left[\hat{V}'_{\|0} k_x^* k_y - k_x^* k_{\|} \left[1 - \frac{\omega_{*pi}}{\omega} \right] \right] |\phi_k|^2 \tag{19}$$

and poloidal stress

$$\langle \delta v_{polx} \delta v_y \rangle = -c_s^2 \left(\frac{\rho_s}{L_n} \right)^3 \frac{\partial}{\partial T} \left[\frac{1}{2} \langle \left(\frac{\partial \phi}{\partial x} \right)^2 \rangle + \langle \frac{\partial \phi}{\partial x} \frac{\partial p}{\partial x} \rangle \right] \tag{20}$$

Eq.(19) reveals that the time assymptotic polarization drift induced residual stress survives when the spectral average $\langle k_{\|} k_x \rangle \neq 0$ which is always satisfied because $k_{\|} \propto x$ and $k_x \propto x$ in general, and due to the shifted gaussian structure of the fluctuation spectrum. Eq.(20) suggests that the time assymptotic poloidal momentum flux due to polarization drift vanishes. Moreover the time derivatives in equations(19) and (20) are of ϵ^4 order and so they are not considered in our further discussions. Now we calculate toroidal stresses due to polarization drift explicitly for slow and fast modes.

Slow mode: From (11) and (19) we obtain toroidal stress

$$\langle \delta v_{polx} \delta v_{\|} \rangle = \sum_{k_y} c_s^2 \left(\frac{\rho_s}{L_n} \right)^3 k_y^2 s \left[\frac{\hat{V}'_{\|0} k_y}{2\tau_i \alpha_i} \hat{V}'_{\|0} \Delta_{ks} \sqrt{\pi} + \frac{\Delta_{ks}}{2} \sqrt{\pi} \right] |\phi_{0ks}|^2 + \epsilon^4 term \tag{21}$$

where it is clearly seen that the diffusive component of flux is modulated by the propagating wave nature of the eigenmode. The term independent of $\hat{V}'_{\|0}$ and ξ_{ks} is a "seedless" residual stress contribution to the total toroidal momentum flux.

Fast mode: Similarly from (12) and (19) we obtain toroidal stress

$$\langle \delta v_{polx} \delta v_{\|} \rangle = \sum_{k_y} c_s^2 \left(\frac{\rho_s}{L_n}\right)^3 (k_y s)^2 \left[2\frac{\xi_{kf}}{s\Delta_{kf}}\hat{V}'_{\|0} - \left(1+\frac{\omega_{*pi}}{\gamma_f}\right)\frac{\Delta_{kf}}{2}\right]\sqrt{\pi}|\phi_{0kf}|^2 + \epsilon^5 term \quad (22)$$

The diffusive flux shown in the above expression is of ϵ^4 order and hence will not be considered in momentum flux budget. We strongly emphasize that in the limit of vanishing "seed flows" the polarization drift driven toroidal momentum fluxes are residual in nature which we term here as "seedless" residual flux which, being independent of mean radial electric field shear, is likely to be active in wide parameter regime and may complement the toroidal flow generation mechanism in weak mean electric field shear region or flat pressure profile region.

5. Coupled Toroidal and Poloidal Flow Equations

Using expressions for toroidal and poloidal stresses obtained in the previous section we obtain coupled toroidal and poloidal flow equations, from eqs.(1,2) as

$$\frac{\partial\langle V_{\|}\rangle}{\partial T} + \frac{\partial}{\partial X}\left[-\left(\chi^{\|}_{\|E} + \chi^{\|}_{\|pol}\right)\frac{\partial V_{\|}}{\partial X} - \chi^{\|}_{y}\frac{\partial V_y}{\partial X} + S_{\|E} + S_{\|pol}\right] = 0 \quad (23)$$

and

$$\frac{\partial\langle V_{y}\rangle}{\partial T} + \frac{\partial}{\partial X}\left[-\left(\chi^{y}_{\|E} + \chi^{y}_{\|pol}\right)\frac{\partial V_{\|}}{\partial X} - \chi^{y}_{y}\frac{\partial V_y}{\partial X} + S_{yE} + S_{ypol}\right] + \nu_{neo}V_y = 0 \quad (24)$$

where the various transport coefficients $\chi_{\|}$, χ_y and residual stresses $S_{\|}$, and S_y for slow and fast modes are obtained in Appendix A. Note that a neoclassical damping term $\nu_{neo}V_y$ is included in the poloidal flow eq.(24) for saturation of flow. The coupling of toroidal and poloidal flow dynamics is appreciable in the following limits. a) For slow mode: when $k_y \approx s(B_y/B)(\tau_i\alpha_i)/2$ in eq.(A.1) and $V'_y \sim V'_{\|}(B_y/B) \sim V'_{*pi}$ in eq.(23). b) For fast mode: when $(1+\frac{\omega_{*pi}}{\gamma_f})(B_y/B)(\tau_i\alpha_i)/(2(1+\tau_i\alpha_i)) \sim 1$ in eq.(A.4) and $V'_y \sim V'_{\|}(B_y/B) \sim V'_{*pi}$ in (23). Similarly from poloidal flow eq.(24) and the expressions following eq.(A.9), for fast mode, the flow coupling is appreciable when $V'_y \sim V'_{\|}(B_y/B) \sim V'_{*pi}$.

We next determine under what condition is the contribution due to polarization drift important. Estimating the magnitude of radial electric field shear by diamagnetic term in the radial force balance equation i.e, $\hat{V}'_{E0} \approx (\rho_s/L_n)\tau_i\alpha_i$, we get from residual component in eq.(15) and "seedless" residual component in eq.(21) and from residual component in eq.(17) and "seedless" residual component in eq.(22), respectively

$$\left|\frac{S_{\|,pol,slow}}{S_{\|,E,slow}}\right| \approx \frac{1}{\tau_i\alpha_i}, \qquad and \qquad \left|\frac{S_{\|,pol,fast}}{S_{\|,E,fast}}\right| \approx \sqrt{2}s\left(\frac{s}{\tau_i\alpha_i}\right)^{1/2} for \quad \tau_i\alpha_i \gg 1 \quad (25)$$

It immediately follows from eq.(25) that the polarization drift driven toroidal momentum flux increasingly gains relative importance in weak temperature gradient region where radial electric field shear driven flux becomes vanishingly small. Above comparison also shows that the polarization driven flux is more active for slow mode.

6. Conclusion

The principal results of this paper are as follows. A coupled set of equations for evolution of mean toroidal and poloidal flows have been derived (see eqs.(23) and (24)). Next a novel polarization drift driven non-diffusive residual component to the radial flux of toroidal momentum is obtained. Though nominally it is higher order in expansion in $\epsilon \sim \omega/\omega_{ci} \sim \rho_s/L_n$, detailed analysis in Section 5 shows that the polarization drift driven flux can become comparable to mean electric field shear driven flux in the weak pressure gradient region and hence complements the $E \times B$ shear driven flow generation mechanism in that region. This mechanism for toroidal flow generation does not require a mean electric field shear and it is due to $\langle k_\parallel k_x \rangle \neq 0$ which is fundamentally different from $\langle k_\parallel \rangle$ symmetry breaking mechanism. Polarization drift is found not to drive any poloidal momentum flux.

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Appendix A. Derivation of $\chi_\parallel$, χ_y, $S_\parallel$ and S_y in Eqns.(23) and (24)

From the exact expressions for momentum fluxes in Section 4 and using the radial force balance equation in the form $\frac{\partial V_{Ey}}{\partial X} = \frac{\partial V_y}{\partial X} - \frac{\partial V_{*y}}{\partial X} - \frac{\partial}{\partial X}\left(\frac{B_y}{B}V_\parallel\right)$ various transport coefficients and residual stresses appearing in Equation (23) are obtained as follows. For **slow mode**

$$\chi_{\parallel E}^{\parallel} = \sum_{k_y}\left(\frac{c_s\rho_s}{L_n}\right)^2 k_y^2 \frac{\sqrt{\pi}}{\Delta_{ks}}\frac{L_n}{c_s}|\phi_{0ks}|^2 + \sum_{k_y}\left(\frac{c_s\rho_s}{L_n}\right)^2 k_y s\frac{B_y}{B}\frac{\tau_i\alpha_i}{2}\frac{\sqrt{\pi}}{\Delta_{ks}}\frac{L_n}{c_s}|\phi_{0ks}|^2 \tag{A.1}$$

$$\chi_{\parallel pol}^{\parallel} = \sum_{k_y} c_s^2\left(\frac{\rho_s}{L_n}\right)^3 k_y^2 s\hat{V}_{\parallel 0}'\frac{\sqrt{\pi}}{2\Delta_{ks}}\frac{L_n}{c_s}|\phi_{0ks}|^2,\ \chi_y^{\parallel} = -\sum_{k_y}\left(\frac{c_s\rho_s}{L_n}\right)^2 k_y s\frac{\tau_i\alpha_i}{2}\frac{\sqrt{\pi}}{\Delta_{ks}}\frac{L_n}{c_s}|\phi_{0ks}|^2 \tag{A.2}$$

$$S_{\parallel E} = -\sum_{k_y}\left(\frac{c_s\rho_s}{L_n}\right)^2 k_y^2 s\frac{\tau_i\alpha_i}{2}\frac{\sqrt{\pi}}{\Delta_{ks}}V_{*pi}'\frac{L_n}{c_s}|\phi_{0ks}|^2,\ S_{\parallel pol} = \sum_{k_y} c_s^2\left(\frac{\rho_s}{L_n}\right)^3 k_y^2 s\Delta_{ks}\frac{\sqrt{\pi}}{2}|\phi_{0ks}|^2 \tag{A.3}$$

and for **fast mode**

$$\chi_{\parallel E}^{\parallel} = \sum_{k_y}\left(\frac{c_s\rho_s}{L_n}\right)^2 \frac{k_y^3 s}{2\gamma_f}\left[1+\left(1+\frac{\omega_{*pi}}{\gamma_f}\right)\frac{B_y}{B}\frac{\tau_i\alpha_i}{2\left(1+\tau_i\alpha_i\right)}\right]\sqrt{\pi}\Delta_{kf}\frac{L_n}{c_s}|\phi_{0ks}|^2 \tag{A.4}$$

$$\chi_y^{\parallel} = -\sum_{k_y} \left(\frac{c_s \rho_s}{L_n}\right)^2 \frac{k_y^3 s}{2\gamma_f} \left[1 + \frac{\omega_{*pi}}{\gamma_f}\right] \frac{\tau_i \alpha_i}{2\left(1+\tau_i \alpha_i\right)} \sqrt{\pi} \Delta_{kf} \frac{L_n}{c_s} |\phi_{0ks}|^2, \chi_{\parallel pol}^{\parallel} = 0 \tag{A.5}$$

$$S_{\parallel E} = -\sum_{k_y} \left(\frac{c_s \rho_s}{L_n}\right)^2 \frac{k_y^3 s}{2\gamma_f} \left[1 + \frac{\omega_{*pi}}{\gamma_f}\right] \frac{\tau_i \alpha_i}{2\left(1+\tau_i \alpha_i\right)} V'_{*pi} \sqrt{\pi} \Delta_{kf} \frac{L_n}{c_s} |\phi_{0ks}|^2 \tag{A.6}$$

$$S_{\parallel pol} = -\sum_{k_y} c_s^2 \left(\frac{\rho_s}{L_n}\right)^3 (k_y s)^2 \left(1 + \omega_{*pi}/\gamma_f\right) \Delta_{kf} \frac{\sqrt{\pi}}{2} |\phi_{0ks}|^2 \tag{A.7}$$

Similarly transport coefficients and residual stresses appearing in Equation (24) are obtained as follows. For **slow mode**

$$\chi_{\parallel E}^{y} = -\sum_{k_y} \left(\frac{c_s \rho_s}{L_n}\right)^2 k_y s \frac{\sqrt{\pi}}{2\Delta_{ks}} \frac{L_n}{c_s} |\phi_{0ks}|^2, S_{ypol} = 0, \chi_y^y = 0, S_{yE} = 0, \chi_{\parallel pol}^{y} = 0 \tag{A.8}$$

and for **fast mode**

$$\chi_{\parallel E}^{y} = \sum_{k_y} \left(\frac{c_s \rho_s}{L_n}\right)^2 k_y^2 s \left[1 + \frac{\omega_{*pi}}{2\gamma_f}\right] \frac{B_y}{B} \frac{\tau_i \alpha_i}{2s\left(1+\tau_i \alpha_i\right)} \frac{2\sqrt{\pi}}{\Delta_{kf}} \frac{L_n}{c_s} |\phi_{0ks}|^2 \tag{A.9}$$

$$\chi_{\parallel pol}^{y} = 0, \chi_y = -\sum_{k_y} \left(\frac{c_s \rho_s}{L_n}\right)^2 k_y^2 s \left[1 + \frac{\omega_{*pi}}{2\gamma_f}\right] \frac{\tau_i \alpha_i}{2s\left(1+\tau_i \alpha_i\right)} \frac{2\sqrt{\pi}}{\Delta_{kf}} \frac{L_n}{c_s} |\phi_{0ks}|^2 \tag{A.10}$$

$$S_{yE} = \sum_{k_y} \left(\frac{c_s \rho_s}{L_n}\right)^2 k_y^2 s \left[1 + \frac{\omega_{*pi}}{2\gamma_f}\right] \frac{\tau_i \alpha_i}{2s\left(1+\tau_i \alpha_i\right)} V'_{*pi} \frac{2\sqrt{\pi}}{\Delta_{kf}} \frac{L_n}{c_s} |\phi_{0ks}|^2, \quad S_{ypol} = 0 \tag{A.11}$$

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