Sources of intrinsic rotation in the low flow ordering

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Abstract. A low flow δf gyrokinetic formulation to obtain intrinsic rotation profiles is presented. The momentum conservation equation in the low flow ordering contains new terms, neglected in previous first principles formulations, that may explain the intrinsic rotation observed in tokamaks. The intrinsic rotation profile depends on the ion and electron temperature profiles, the up-down symmetry and the type of heating.

1 Introduction

Experimental observations have shown that tokamak plasmas rotate spontaneously without momentum input [1]. This intrinsic rotation has been the object of recent work [1, 2]. It is important because rotation in ITER [3] will be mostly intrinsic, as the projected momentum input from neutral beams is small.

The origin of the intrinsic rotation is still unclear. The momentum pinch due to the Coriolis drift [4] has been argued to transport momentum generated in the edge. It has also been argued that up-down asymmetry generates intrinsic rotation [5, 6]. However, neither of these explanations are able to account for all experimental observations. The up-down asymmetry is only large in the edge, generating rotation in that region that then needs to be transported inwards by the Coriolis pinch. Thus, intrinsic rotation in the core could only be explained by the pinch. The pinch of momentum is not sufficient because it does not allow the toroidal rotation to change sign in the core as is observed experimentally [7]. The new model presented in this article, implementable in δf flux tube simulations, based on the low flow ordering of Ref. [6], includes self-consistently higher order contributions. As a result, new effects appear that depend on the gradients of the background profiles of density and temperature and on the heating mechanisms.

In the remainder of this article we present the model, developed originally in [6], in a form more suitable for δf flux tube simulation. In Section 2 we give the complete model, and in Section 3 we discuss its implications for intrinsic rotation.

2 Transport of toroidal angular momentum

The derivation of the transport of toroidal angular momentum in the low flow regime, including both turbulence and neoclassical effects, is described in detail in Ref. [6]. To simplify the derivation, the extra expansion parameter $B_p/B \ll 1$ was employed, with B the total magnetic field and B_p its poloidal component. In this section, we review the results of Ref. [6], recast them in a more convenient form and add a collisional term and a term that depends on the heating mechanisms that were previously neglected.

We assume that the turbulence is electrostatic and that the magnetic field is axisymmetric, i.e., $\mathbf{B} = I\nabla\zeta + \nabla\zeta \times \nabla\psi$, where ψ is the poloidal magnetic flux, ζ is the toroidal angle, and we use a poloidal angle θ as our third spatial coordinate. With an axisymmetric magnetic field, in steady state and in the absence of momentum input, the equation that determines the rotation profile is $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot \overleftrightarrow{\mathbf{P}}_i \cdot\nabla\psi\rangle_\psi\rangle_{\mathrm{T}} = 0$, where $\overleftrightarrow{\mathbf{P}}_i= \int d^3v'\, f_i M\mathbf{v}'\mathbf{v}'$ is the ion stress tensor, R is the major radius, $\hat{\boldsymbol{\zeta}}$ is the unit vector in the toroidal direction, $\langle\ldots\rangle_\psi = (V')^{-1}\int d\theta\, d\zeta\,(...)/(\mathbf{B}\cdot\nabla\theta)$ is the flux surface average, $V' = \int d\theta\, d\zeta\,(\mathbf{B}\cdot\nabla\theta)^{-1}$ is the the flux surface area, and $\langle\ldots\rangle_{\mathrm{T}}$ is the coarse grain or "transport" average over the time and length scales of the turbulence, much shorter than the transport time scale $\delta_i^{-2}a/v_{ti}$ and the minor radius a. Here $\delta_i = \rho_i/a \ll 1$ is the ion gyroradius ρ_i over the minor radius a, and v_{ti} is the ion thermal speed. Note that we use the prime in $\mathbf{v}'$ to indicate that the velocity is measured in the laboratory frame. Later we will find the equations in a convenient rotating frame where the velocity is $\mathbf{v} = \mathbf{v}' - R\Omega_\zeta\hat{\boldsymbol{\zeta}}$.

In Ref. [6] we derived a method to calculate $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot \overleftrightarrow{\mathbf{P}}_i \cdot\nabla\psi\rangle_\psi\rangle_{\mathrm{T}}$ to order $(B/B_p)\delta_i^3 p_i R|\nabla\psi|$, with p_i the ion pressure. We present the method again in different form to make it easier to compare with previous work in the high flow regime [8, 9]. In addition, instead of using the simplified ion Fokker-Planck equation $df_i/dt \equiv \partial f_i/\partial t + \mathbf{v}'\cdot\nabla f_i + (Ze/M)(-\nabla\phi + c^{-1}\mathbf{v}'\times\mathbf{B})\cdot\nabla_{v'}f_i = C_{ii}\{f_i\}$ of Ref. [6], where C_{ii} is the ion-ion collision operator, ϕ is the electrostatic potential, Ze and M are the ion charge and mass, and e and c are the electron charge magnitude and the speed of light, in this article we use the more complete equation

$$\frac{df_i}{dt} \equiv \frac{\partial f_i}{\partial t} + \mathbf{v}'\cdot\nabla f_i + \frac{Ze}{M}\left(-\nabla\phi + \frac{1}{c}\mathbf{v}'\times\mathbf{B}\right)\cdot\nabla_{v'}f_i = C_{ii}\{f_i\} + C_{ie}\{f_i\} + \mathcal{S}^{\mathrm{ht}}, \qquad (1)$$

where $\mathcal{S}^{\mathrm{ht}} \sim \delta_i^2 f_i v_{ti}/a$ is a source that models the different heating mechanisms, and $C_{ie}\{f_e\} = [(\mathbf{F}_{ei} - n_e m\nu_{ei}\mathbf{V}_i)/n_i M]\cdot\nabla_{v'}f_i + (n_e m/n_i M)\nu_{ei}\nabla_{v'}\cdot[(T_e/M)\nabla_{v'}f_i + \mathbf{v}'f_i]$ is the ion-electron collision operator, with $\mathbf{F}_{ei} = m\int d^3v\, C_{ei}\mathbf{v}$ the electron-ion friction force, C_{ei} the electron-ion collision operator, ν_{ei} the electron-ion collision frequency, n_i and $\mathbf{V}_i$ the ion density and average velocity, n_e and T_e the electron density and temperature, and m the electron mass. Applying the procedure in Ref. [6] to Eq. (1) we find two additional terms that were not considered in Ref. [6].

In subsection 2.1 we explain how we split the distribution function and the electrostatic potential into different pieces, and we present the equations to self-consistently obtain them. In subsection 2.2 we evaluate $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot \overleftrightarrow{\mathbf{P}}_i \cdot\nabla\psi\rangle_\psi\rangle_{\mathrm{T}}$ employing the pieces of the distribution function and the potential obtained in subsection 2.1. Before presenting all the results, we emphasize that our results and order of magnitude estimates are valid for $B_p/B \ll 1$ and for collisionality in the range $\delta_i^2 \ll qR\nu_{ii}/v_{ti} \lesssim 1$ [6], where ν_{ii} is the ion-ion collision frequency and q the safety factor.

2.1 Distribution function and electrostatic potential

The electrostatic potential is composed to the order of interest by the pieces in Table I [6]. The axisymmetric, long wavelength pieces $\phi_0(\psi,t)$, $\phi_1^{\mathrm{nc}}(\psi,\theta,t)$ and $\phi_2^{\mathrm{nc}}(\psi,\theta,t)$ are the zeroth, first and second order equilibrium pieces of the potential. The lowest order component ϕ_0 is a flux surface function. The corrections ϕ_1^{nc} and ϕ_2^{nc} give the electric field parallel to the flux surface, established to force quasineutrality at long wavelengths (the superscript nc refers to neoclassical, even though turbulence affects its final value). We need not

TABLE I: PIECES OF THE POTENTIAL.

Potential	**Size**	**Length scales**	**Time scales**
$\phi_0(\psi, t)$	T_e/e	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$\phi_1^{\rm nc}(\psi, \theta, t)$	$(B/B_p)\delta_i T_e/e$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$\phi_2^{\rm nc}(\psi, \theta, t)$	$(B/B_p)^2\delta_i^2 T_e/e$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$\phi^{\rm tb}(\mathbf{r}, t)$	$\phi_1^{\rm tb} \sim \delta_i T_e/e$ $\phi_2^{\rm tb} \sim (B/B_p)\delta_i^2 T_e/e$	$k_\perp \rho_i \sim 1$ $k_{\vert\vert} a \sim 1$	$\partial/\partial t \sim v_{ti}/a$

calculate $\phi_2^{\rm nc}$ because it will not appear in the final expression for $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot \overleftrightarrow{\mathbf{P}}_i \cdot\nabla\psi\rangle_\psi\rangle_{\rm T}$. The piece $\phi^{\rm tb}(\mathbf{r}, t)$ is turbulent and includes both axisymmetric components (zonal flow) and non-axisymmetric fluctuations. It is small in δ_i but it has strong perpendicular gradients, i.e., $k_\perp\rho_i \sim 1$. Its parallel gradient is small, i.e., $k_{||}a \sim 1$. The function $\phi^{\rm tb}$ is calculated to order $(B/B_p)\delta_i^2 T_e/e$, i.e., $\phi^{\rm tb} = \phi_1^{\rm tb} + \phi_2^{\rm tb}$ with $\phi_1^{\rm tb} \sim \delta_i T_e/e$ and $\phi_2^{\rm tb} \sim (B/B_p)\delta_i^2 T_e/e$, and both $\phi_1^{\rm tb}$ and $\phi_2^{\rm tb}$ are calculated. It is more convenient to keep both pieces together as $\phi^{\rm tb}$ as we do hereafter.

To write the distribution function it will be useful to consider the reference frame that rotates with toroidal angular velocity $\Omega_\zeta = -c\,\partial_\psi\phi_0 - (c/Zen_i)\partial_\psi p_i$, where $n_i(\psi, t)$ and $p_i(\psi, t)$ are the lowest order ion density and pressure. In this new reference frame it is easier to compare with previous formulations [8, 9]. In Ref. [6] we used as gyrokinetic variables the gyrocenter position $\mathbf{R} = \mathbf{r} + \mathbf{R}_1 + \mathbf{R}_2 + \ldots$, the gyrokinetic kinetic energy $E = E_0 + E_1 + E_2 + \ldots$ and the magnetic moment $\mu = \mu_0 + \mu_1 + \ldots$, with $E_0 = (v')^2/2$, $\mu_0 = (v'_\perp)^2/2B$, $\mathbf{R}_1 = \Omega_i^{-1}\mathbf{v}' \times \hat{\mathbf{b}}$ and the rest of the parameters defined in [10]. Here $\Omega_i = ZeB/Mc$ is the ion gyrofrequency. To change to the new reference frame, where the velocity is $\mathbf{v} = \mathbf{v}' - R\Omega_\zeta\hat{\boldsymbol{\zeta}}$, the distribution function has to be written as a function of the new gyrokinetic variables $\mathbf{R}$, ε and μ. The new kinetic energy variable ε is related to the old kinetic energy variable by $E = \varepsilon + (I/B)\Omega_\zeta u$, where $u = \pm\sqrt{2[\varepsilon - \mu B - (I/B)^2\Omega_\zeta^2/2]}$ is the parallel velocity in the rotating frame. It is easy to check that $u' = \pm\sqrt{2(E - \mu B)} = u + (I/B)\Omega_\zeta$. In what follows we rewrite the results in Ref. [6] using the new gyrokinetic kinetic energy ε.

Using the gyrokinetic variables $\mathbf{R}$, ε and μ, the different pieces of the ion distribution function are given in Table II [6]. The functions f_{Mi}, $H_{i1}^{\rm nc}$, $H_{i2}^{\rm nc}$, $H_{i2}^{\rm tb}$ and $H_{i2}^{\rm ht}$ are axisymmetric long wavelength contributions. The Maxwellian $f_{Mi}(\psi(\mathbf{R}), \varepsilon)$ is uniform in a flux surface. The first and second order corrections $H_{i1}^{\rm nc}$ and $H_{i2}^{\rm nc}$ are neoclassical corrections, and they are not the functions $F_{i1}^{\rm nc}$ and $F_{i2}^{\rm nc}$ in Ref. [6] because we are now working in the rotating frame. The function $H_{i2}^{\rm tb}$ is an axisymmetric piece of the distribution function that originates from collisions acting on the ions transported by turbulent fluctuations into a given flux surface [6]. The function $H_{i2}^{\rm ht}$ that was not included in Ref. [6] depends on the heating mechanism. The function $f_i^{\rm tb}$ is the turbulent contribution. It will be determined self-consistently up to order $(B/B_p)\delta_i^2 f_{Mi}$, i.e., $f_i^{\rm tb} = f_{i1}^{\rm tb} + f_{i2}^{\rm tb}$ with $f_{i1}^{\rm tb} \sim \delta_i f_{Mi}$ and $f_{i2}^{\rm tb} \sim (B/B_p)\delta_i^2 f_{Mi}$, and both $f_{i1}^{\rm tb}$ and $f_{i2}^{\rm tb}$ are self-consistently determined. It is more convenient to combine both pieces of the turbulent distribution function into one function $f_i^{\rm tb}$.

The electron distribution function is very similar to the ion distribution function. It will have its own gyrokinetic variables that can be easily deduced from the ion counterparts.

TABLE II: PIECES OF THE ION DISTRIBUTION FUNCTION.

Distrib. function	**Size**	**Length scales**	**Time scales**
$f_{Mi}(\psi(\mathbf{R}), \varepsilon, t)$	f_{Mi}	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$H_{i1}^{\rm nc}(\psi(\mathbf{R}), \theta(\mathbf{R}), \varepsilon, \mu, t)$	$(B/B_p)\delta_i f_{Mi}$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$H_{i2}^{\rm nc}(\psi(\mathbf{R}), \theta(\mathbf{R}), \varepsilon, \mu, t)$	$(B/B_p)^2\delta_i^2 f_{Mi}$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$H_{i2}^{\rm tb}(\psi(\mathbf{R}), \theta(\mathbf{R}), \varepsilon, \mu, t)$	$(B/B_p)(v_{ti}/qR\nu_{ii})\delta_i^2 f_{Mi}$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$H_{i2}^{\rm ht}(\psi(\mathbf{R}), \theta(\mathbf{R}), \varepsilon, \mu, t)$	$(B/B_p)(v_{ti}/qR\nu_{ii})\mathcal{S}^{\rm ht}a/v_{ti}$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$f_i^{\rm tb}(\mathbf{R}, \varepsilon, \mu, t)$	$f_{i1}^{\rm tb} \sim \delta_i f_{Mi}$ $f_{i2}^{\rm tb} \sim (B/B_p)\delta_i^2 f_{Mi}$	$k_\perp\rho_i \sim 1$ $k_{\vert\vert}a \sim 1$	$\partial/\partial t \sim v_{ti}/a$

TABLE III: PIECES OF THE ELECTRON DISTRIBUTION FUNCTION.

Distrib. function	**Size**	**Length scales**	**Time scales**
$f_{Me}(\psi(\mathbf{R}), \varepsilon, t)$	f_{Me}	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$H_{e1}^{\rm nc}(\psi(\mathbf{R}), \theta(\mathbf{R}), \varepsilon, \mu, t)$	$(B/B_p)\delta_i f_{Me}$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$f_e^{\rm tb}(\mathbf{R}, \varepsilon, \mu, t)$	$f_{e1}^{\rm tb} \sim \delta_i f_{Me}$ $f_{e2}^{\rm tb} \sim (B/B_p)\delta_i^2 f_{Me}$	$k_\perp\rho_i \sim 1$ $k_{\vert\vert}a \sim 1$	$\partial/\partial t \sim v_{ti}/a$

To the order of interest in this calculation, the electron distribution function is determined by the pieces in Table III. The long wavelength, axisymmetric pieces f_{Me} and $H_{e1}^{\rm nc}$ are the lowest order Maxwellian and the first order neoclassical correction. The second order long wavelength neoclassical correction is not needed for transport of momentum. The piece $f_e^{\rm tb}$ is the short wavelength, turbulent component that will be self-consistently calculated to order $(B/B_p)\delta_i^2 f_{Me}$.

We know proceed to describe how to find the different pieces of the distribution function and the potential. We use the equations in Ref. [6] but we change to the new gyrokinetic kinetic energy ε.

2.1.1 First order neoclassical distribution function and potential

The equation for $H_{i1}^{\rm nc}$ is

$$u\hat{\mathbf{b}} \cdot \nabla_{\mathbf{R}} \left[H_{i1}^{\rm nc} + \frac{Ze\phi_1^{\rm nc}}{T_i} f_{Mi} + \left(\frac{M\varepsilon}{T_i} - \frac{5}{2} \right) \frac{Iuf_{Mi}}{\Omega_i T_i} \frac{\partial T_i}{\partial \psi} \right] - C_{ii}^{(\ell)} \{H_{i1}^{\rm nc}\} = 0. \tag{2}$$

The correction $H_{i1}^{\rm nc}$ gives the parallel component of the velocity [11, 12] $V_{i||}^{\rm nc} = \int d^3v\, v_{||} H_{i1}^{\rm nc} = (kcIB/Ze\langle B^2\rangle_\psi)(\partial T_i/\partial\psi)$, where k is a constant that depends on the collisionality and the magnetic geometry. Interestingly, the density perturbation due to $H_{i1}^{\rm nc}$ is small for $qR\nu_{ii}/v_{ti} \ll 1$, i.e., $\int d^3v\, H_{i1}^{\rm nc} \sim (B/B_p)(qR\nu_{ii}/v_{ti})\delta_i n_i \ll (B/B_p)\delta_i n_i$ [6]. This will be important when determining $\phi_1^{\rm nc}$ below.

The lowest order solution for $H_{e1}^{\rm nc}$ is the Maxwell-Boltzmann response $(e\phi_1^{\rm nc}/T_e) f_{Me} \sim (B/B_p)(qR\nu_{ii}/v_{ti})\delta_i f_{Me}$. The rest of the terms are small because $\delta_e = \rho_e/a \ll \delta_i$, where ρ_e is the electron gyroradius.

Finally the poloidal variation of the potential is determined by quasineutrality,

$$Z \int d^3v\, H_{i1}^{\rm nc} + Z \int d^3v\, H_{i2}^{\rm tb} = \frac{e\phi_1^{\rm nc}}{T_e} n_e, \tag{3}$$

giving that $e\phi_1^{\rm nc}/T_e \sim (B/B_p)(qR\nu_{ii}/v_{ti})\delta_i$. We have included the density $\int d^3v\, H_{i2}^{\rm tb} \sim (B/B_p)(v_{ti}/qR\nu_{ii})\delta_i^2 n_i$ because it becomes important for $qR\nu_{ii}/v_{ti} \lesssim (f_i^{\rm tb}/f_{Mi})\sqrt{a/\rho_i} \ll 1$ with $f_i^{\rm tb}/f_{Mi} \sim \rho_i/a$.

2.1.2 Turbulent distribution function and potential

The turbulent piece of the ion distribution function is obtained using the gyrokinetic equation

$$\frac{Df_i^{\rm tb}}{Dt} + \left(u\hat{\mathbf{b}} + \mathbf{v}_M + \mathbf{v}_C + \mathbf{v}_E^{\rm tb}\right)\cdot\nabla_{\mathbf{R}} f_i^{\rm tb} - \left\langle C_{ii}^{(\ell)}\left\{f_i(\mathbf{R},\varepsilon,\mu,t) - f_{Mi}(\psi(\mathbf{r}),\varepsilon_0)\right\}\right\rangle$$
$$= -\mathbf{v}_E^{\rm tb}\cdot\nabla_{\mathbf{R}}\psi\left[\frac{1}{n_i}\frac{\partial n_i}{\partial\psi} + \left(\frac{M\varepsilon}{T_i} - \frac{3}{2}\right)\frac{1}{T_i}\frac{\partial T_i}{\partial\psi} + \frac{MIu}{BT_i}\frac{\partial\Omega_\zeta}{\partial\psi}\right] f_{Mi} - \mathbf{v}_E^{\rm tb}\cdot\nabla_{\mathbf{R}} H_{i1}^{\rm nc}$$
$$-\frac{Zef_{Mi}}{T_i}\left(u\hat{\mathbf{b}} + \mathbf{v}_M + \mathbf{v}_C\right)\cdot\nabla_{\mathbf{R}}\langle\phi^{\rm tb}\rangle + \frac{Ze}{M}\frac{\partial H_{i1}^{\rm nc}}{\partial\varepsilon}\left(u\hat{\mathbf{b}} + \mathbf{v}_M\right)\cdot\nabla_{\mathbf{R}}\langle\phi^{\rm tb}\rangle, \qquad (4)$$

where $D/Dt = \partial/\partial t + R\Omega_\zeta\hat{\boldsymbol{\zeta}}\cdot\nabla_{\mathbf{R}} \simeq \partial/\partial t + (I/B)\Omega_\zeta\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}$ is the time derivative in the rotating frame, $u = \pm\sqrt{2[\varepsilon - \mu B - (I/B)^2\Omega_\zeta^2/2]} \simeq \pm\sqrt{2(\varepsilon - \mu B)}$ is the parallel velocity in the rotating frame, $\mathbf{v}_M = (\mu/\Omega_i)\hat{\mathbf{b}}\times\nabla_{\mathbf{R}}B + (u^2/\Omega_i)\hat{\mathbf{b}}\times(\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\hat{\mathbf{b}})$ are the ∇B and curvature drifts, $\mathbf{v}_C = (2u\Omega_\zeta/\Omega_i)\hat{\mathbf{b}}\times[(\nabla R\times\hat{\boldsymbol{\zeta}})\times\hat{\mathbf{b}}] \simeq (2uI\Omega_\zeta/B\Omega_i)\hat{\mathbf{b}}\times(\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\hat{\mathbf{b}})$ is the Coriolis drift, $\mathbf{v}_E^{\rm tb} = -(c/B)\nabla_{\mathbf{R}}\langle\phi^{\rm tb}\rangle\times\hat{\mathbf{b}}$ is the turbulent $\mathbf{E}\times\mathbf{B}$ drift and $C_{ii}^{(\ell)}$ is the linearized ion-ion collision operator. Here $\langle\ldots\rangle$ is the gyroaverage holding $\mathbf{R}$, ε, μ and t fixed. In the collision operator we use $f_i(\mathbf{R},\varepsilon,\mu,t) - f_{Mi}(\psi(\mathbf{r}),\varepsilon_0) \simeq f_{ig}^{\rm tb} + [Ze(\phi^{\rm tb} - \langle\phi^{\rm tb}\rangle)/M][-(Mf_{Mi,0}/T_i) + (\partial H_{i1,0}^{\rm nc}/\partial\varepsilon_0) + B^{-1}(\partial H_{i1,0}^{\rm nc}/\partial\mu_0)]$. To obtain this expression we have Taylor expanded $\mathbf{R} = \mathbf{R}_g + \ldots$, $\varepsilon = \varepsilon_0 + E_1 + \ldots$ and $\mu = \mu_0 + \mu_1 + \ldots$ around $\mathbf{R}_g = \mathbf{r} + \Omega_i^{-1}\mathbf{v}\times\hat{\mathbf{b}}$, $\varepsilon_0 = v^2/2$ and $\mu_0 = v_\perp^2/2B$, and we have only kept terms that have short wavelengths. The subscript g in $f_{ig}^{\rm tb} = f_i^{\rm tb}(\mathbf{R}_g,\varepsilon_0,\mu_0,t)$ indicates that we have replaced the variables $\mathbf{R}$, ε and μ by $\mathbf{R}_g$, ε_0 and μ_0; similarly, the subscript 0 in $f_{Mi,0} = f_{Mi}(\psi(\mathbf{r}),\varepsilon_0)$ and $H_{i1,0}^{\rm nc} = H_{i1}^{\rm nc}(\psi(\mathbf{r}),\theta(\mathbf{r}),\varepsilon_0,\mu_0)$ indicates that we have replaced the variables $\mathbf{R}$, ε and μ by $\mathbf{r}$, ε_0 and μ_0.

The equation for electrons is equivalent to the one for the ions, giving

$$\frac{Df_e^{\rm tb}}{Dt} + \left(u\hat{\mathbf{b}} + \mathbf{v}_M + \mathbf{v}_E^{\rm tb}\right)\cdot\nabla_{\mathbf{R}} f_e^{\rm tb} - \left\langle C_e^{(\ell)}\left\{f_e(\mathbf{R},\varepsilon,\mu,t) - f_{Me}(\psi(\mathbf{r}),\varepsilon_0)\right\}\right\rangle$$
$$= -\mathbf{v}_E^{\rm tb}\cdot\nabla_{\mathbf{R}}\psi\left[\frac{1}{n_e}\frac{\partial n_e}{\partial\psi} + \left(\frac{M\varepsilon}{T_e} - \frac{3}{2}\right)\frac{1}{T_e}\frac{\partial T_e}{\partial\psi}\right] f_{Me} + \frac{ef_{Me}}{T_e}\left(u\hat{\mathbf{b}} + \mathbf{v}_M\right)\cdot\nabla_{\mathbf{R}}\langle\phi^{\rm tb}\rangle, \qquad (5)$$

where $C_e^{(\ell)} = C_{ee}^{(\ell)} + C_{ei}^{(\ell)}$ are the linearized electron-electron and the electron-ion collision operators. If we can neglect the effect of the trapped electrons, the solution to this equation is simply the adiabatic response $f_e^{\rm tb} \simeq (e\langle\phi^{\rm tb}\rangle/T_e)f_{Me}$.

Finally, the electrostatic potential $\phi^{\rm tb}$ is obtained from the quasineutrality equation

$$Z\int d^3v\,\frac{Ze(\phi^{\rm tb} - \langle\phi^{\rm tb}\rangle)}{M}\left[-\frac{Mf_{Mi,0}}{T_i} + \left(\frac{\partial H_{i1,0}^{\rm nc}}{\partial\varepsilon_0} + \frac{1}{B}\frac{\partial H_{i1,0}^{\rm nc}}{\partial\mu_0}\right)\right] + Z\int d^3v\, f_{ig}^{\rm tb} = \int d^3v\, f_{eg}^{\rm tb}. \qquad (6)$$

TABLE IV: CONTRIBUTIONS TO TRANSPORT OF MOMENTUM.

Π	**Size** $[(B/B_p)\delta_i^3 p_i R\lvert\nabla\psi\rvert]$	**Dependences**
$\Pi_{-1}^{\rm tb}$	$(B_p/B)\Delta_{ud}\delta_i^{-1}$ for $\Delta_{ud} \gtrsim (B/B_p)\delta_i$ 1 for $\Delta_{ud} \lesssim (B/B_p)\delta_i$	$\partial_\psi \Omega_\zeta, \Omega_\zeta, \Delta_{ud}, \partial_\psi T_i, \partial_\psi n_e, \partial_\psi T_e, \partial_\psi^2 T_i$
$\Pi_0^{\rm tb}$	1	$\partial_\psi T_i, \partial_\psi n_e, \partial_\psi T_e, \partial_\psi^2 T_i, \partial_\psi^2 n_e, \partial_\psi^2 T_e$
$\Pi_{-1}^{\rm nc}$	$\Delta_{ud}(qR\nu_{ii}/v_{ti})\delta_i^{-1}$ for $\Delta_{ud} \gtrsim (B/B_p)\delta_i$ $(B/B_p)(qR\nu_{ii}/v_{ti})$ for $\Delta_{ud} \lesssim (B/B_p)\delta_i$	$\partial_\psi \Omega_\zeta, \Delta_{ud}, \partial_\psi T_i, \partial_\psi n_e, \partial_\psi^2 T_i, \mathbf{F}_{ei}$
$\Pi_0^{\rm nc}$	$(B/B_p)(qR\nu_{ii}/v_{ti})$	$\partial_\psi T_i, \partial_\psi n_e, \partial_\psi^2 T_i$
$\Pi^{\rm ht}$	$\delta_i^{-2}(\mathcal{S}^{\rm ht} a/v_{ti} f_{Mi})$	Heating
Π^{ie}	$(B_p/B)(qR\nu_{ii}/v_{ti})\delta_i^{-2}\sqrt{m/M}$	$T_i - T_e$

2.1.3 Second order, long wavelength distribution function and potential

The long wavelength pieces $H_{i2}^{\rm nc}$, $H_{i2}^{\rm tb}$ and $H_{i2}^{\rm ht}$ are given by

$$u\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}} H_{i2}^{\alpha} - C_{ii}^{(\ell)}\{H_{i2}^{\alpha}\} = \mathcal{S}^{\alpha} - \left\langle \int d^3v\, \mathcal{S}^{\alpha} + \left(\frac{2M\varepsilon}{3T_i} - 1\right)\int d^3v\, \mathcal{S}^{\alpha}\left(\frac{M\varepsilon}{T_i} - \frac{3}{2}\right)\right\rangle_{\psi} \frac{f_{Mi}}{n_i}, \quad (7)$$

where $\alpha =$ nc, tb, ht, and

$$\mathcal{S}^{\rm nc} = -\frac{MIuf_{Mi}}{BT_i}\frac{\partial\Omega_\zeta}{\partial\psi}\mathbf{v}_M\cdot\nabla_{\mathbf{R}}\psi - \left(\mathbf{v}_C - \frac{c}{B}\nabla_{\mathbf{R}}\phi_1^{\rm nc}\times\hat{\mathbf{b}}\right)\cdot\nabla_{\mathbf{R}}\psi\left(\frac{M\varepsilon}{T_i} - \frac{5}{2}\right)\frac{f_{Mi}}{T_i}\frac{\partial T_i}{\partial\psi}$$
$$-\mathbf{v}_M\cdot\nabla_{\mathbf{R}} H_{i1}^{\rm nc} + \frac{I}{n_i M\Omega_i}\frac{\partial p_i}{\partial\psi}\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}} H_{i1}^{\rm nc} - \frac{Zef_{Mi}}{T_i}\left(u\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\phi_2^{\rm nc} + \mathbf{v}_M\cdot\nabla_{\mathbf{R}}\phi_1^{\rm nc}\right)$$
$$+\frac{Ze}{M}\left(u\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\phi_1^{\rm nc} - \frac{1}{Zen_i}\frac{\partial p_i}{\partial\psi}\mathbf{v}_M\cdot\nabla_{\mathbf{R}}\psi\right)\frac{\partial H_{i1}^{\rm nc}}{\partial\varepsilon} + C_{ii}^{(n\ell)}\{H_{i1}^{\rm nc}, H_{i1}^{\rm nc}\}$$
$$-\frac{uf_{Mi}}{p_i}\hat{\mathbf{b}}\cdot(\mathbf{F}_{ei} - n_e m\nu_{ei}\mathbf{V}_i) + \frac{n_e m\nu_{ei}}{n_i M}\langle C_B\{H_{i1}^{\rm nc}\}\rangle, \quad (8)$$

where the ion-electron collisions have been added to the result in Ref. [6] and $C_B\{f_i\} = \nabla_v\cdot[(T_e/M)\nabla_v f_i + \mathbf{v}f_i]$ is the Brownian motion operator, and

$$\mathcal{S}^{\rm tb} = -\frac{|u|}{B}\nabla_{\mathbf{R}}\cdot\left(\frac{B}{|u|}\left\langle f_i^{\rm tb}\mathbf{v}_E^{\rm tb}\right\rangle_{\rm T}\right) + \frac{Ze}{M}\frac{|u|}{B}\frac{\partial}{\partial\varepsilon}\left(\frac{B}{|u|}\left\langle f_i^{\rm tb}\left(u\hat{\mathbf{b}} + \mathbf{v}_M\right)\cdot\nabla_{\mathbf{R}}\langle\phi^{\rm tb}\rangle\right\rangle_{\rm T}\right). \quad (9)$$

2.2 Calculation of the momentum transport

We obtain an equation for $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot\overleftrightarrow{\mathbf{P}}_i\cdot\nabla\psi\rangle_\psi\rangle_{\rm T}$ similar to Eq. (39) of Ref. [6] by employing the same procedure that was used in that reference with the more complete Fokker-Planck equation (1). The final result is as in Eq. (39) of Ref. [6] plus the new terms

$$-\frac{M^2c}{2Ze}\left\langle\left\langle\int d^3v'\, C_{ie}\{f_i\}R^2(\mathbf{v}'\cdot\hat{\boldsymbol{\zeta}})^2 + \int d^3v'\,\mathcal{S}^{\rm ht}(\mathbf{r},\mathbf{v}')R^2(\mathbf{v}'\cdot\hat{\boldsymbol{\zeta}})^2\right\rangle_\psi\right\rangle_{\rm T}. \quad (10)$$

With the expression (39) in Ref. [6] and these new terms, we find

$$\langle\langle R\hat{\boldsymbol{\zeta}}\cdot\overleftrightarrow{\mathbf{P}}_i\cdot\nabla\psi\rangle_\psi\rangle_{\rm T} = \Pi_{-1}^{\rm tb} + \Pi_0^{\rm tb} + \Pi_{-1}^{\rm nc} + \Pi_0^{\rm nc} + \Pi^{\rm ht} + \Pi^{ie} + \frac{Mc\langle R^2\rangle_\psi}{2Ze}\frac{\partial p_i}{\partial t}, \quad (11)$$

with

$$\Pi_{-1}^{\rm tb} = -\left\langle\left\langle \frac{c}{B}(\nabla\phi^{\rm tb}\times\hat{\mathbf{b}})\cdot\nabla\psi \int d^3v\, f_{ig}^{\rm tb}\left(\frac{IMv_{||}}{B} + MR\Omega_\zeta\right)\right\rangle_\psi\right\rangle_{\rm T}, \tag{12}$$

$$\Pi_0^{\rm tb} = -\frac{M^2c}{2Ze}\frac{1}{V'}\frac{\partial}{\partial\psi}V'\left\langle\left\langle \frac{c}{B}(\nabla\phi^{\rm tb}\times\hat{\mathbf{b}})\cdot\nabla\psi \int d^3v\, f_{ig}^{\rm tb}\frac{I^2v_{||}^2}{B^2}\right\rangle_{\rm T}\right\rangle_\psi$$
$$+\left\langle\left\langle \frac{cI}{B}\hat{\mathbf{b}}\cdot\nabla\phi^{\rm tb}\int d^3v\, f_{ig}^{\rm tb}\frac{IMv_{||}}{B}\right\rangle_\psi\right\rangle_{\rm T} - \frac{M^2c}{2Ze}\left\langle\int d^3v\, C_{ii}^{(\ell)}\{H_{i2,0}^{\rm tb}\}\frac{I^2v_{||}^2}{B^2}\right\rangle_\psi, \tag{13}$$

$$\Pi_{-1}^{\rm nc} = -\frac{M^2c}{2Ze}\left\langle\int d^3v\,\left(C_{ii}^{(\ell)}\{H_{i1,0}^{\rm nc}+H_{i2,0}^{\rm nc}\} + C_{ii}^{(n\ell)}\{H_{i1,0}^{\rm nc}, H_{i1,0}^{\rm nc}\}\right)\frac{I^2v_{||}^2}{B^2}\right\rangle_\psi, \tag{14}$$

$$\Pi_0^{\rm nc} = -\frac{M^3c^2}{6Z^2e^2}\frac{1}{V'}\frac{\partial}{\partial\psi}V'\left\langle\int d^3v\, C_{ii}^{(\ell)}\{H_{i1,0}^{\rm nc}\}\frac{I^3v_{||}^3}{B^3}\right\rangle_\psi - \frac{n_emMc\nu_{ei}}{2Zen_i}\left\langle\int d^3v\, C_B\{H_{i1,0}^{\rm nc}\}\frac{I^2v_{||}^2}{B^2}\right\rangle_\psi, \tag{15}$$

$$\Pi^{\rm ht} = -\frac{M^2c}{2Ze}\left\langle\int d^3v\, C_{ii}^{(\ell)}\{H_{i2,0}^{\rm ht}\}\frac{I^2v_{||}^2}{B^2}\right\rangle_\psi - \frac{M^2c}{2Ze}\left\langle\int d^3v\, \mathcal{S}^{\rm ht}\frac{I^2v_{||}^2}{B^2}\right\rangle_\psi \tag{16}$$

and

$$\Pi^{ie} = \frac{n_imc\langle R^2\rangle_\psi\nu_{ei}}{Ze}(T_i - T_e). \tag{17}$$

Recall that the subscript g indicates that $\mathbf{R}$, ε and μ have been replaced by $\mathbf{R}_g$, ε_0 and μ_0, and the subscript 0 that they have been replaced by $\mathbf{r}$, ε_0 and μ_0. In Table IV we summarize the size of all these contributions compared to the reference size $(B/B_p)\delta_i^3 p_i R|\nabla\psi|$, and we write what they depend on. To obtain these dependences, we simply use Eqs. (2), (3), (4), (5), (6), (7) and (17). Most of the size estimates are taken from Ref. [6], except for $\Pi^{\rm ht}$ and Π^{ie} that are trivially found from the results here. We use Δ_{ud} to denote a measure of the flux surface up-down asymmetry. It ranges from zero for perfect up-down symmetry to one for extreme asymmetry. Notice that for extreme up-down asymmetry, $\Pi_{-1}^{\rm tb}$ and $\Pi_{-1}^{\rm nc}$ clearly dominate. The contribution Π^{ie} is formally very large for $qR\nu_{ii}/v_{ti} \sim 1$, but since the ion energy conservation equation requires that $(T_i - T_e)/T_i \sim (B/B_p)(v_{ti}/qR\nu_{ii})\delta_i^2\sqrt{M/m}$, it will be comparable to the rest of the terms.

3 Discussion

We finish by showing how this new formalism gives a plausible model for intrinsic rotation. Until now, models have only considered the contribution $\Pi_{-1}^{\rm tb}$, with $f_i^{\rm tb}$ and $\phi^{\rm tb}$ obtained by employing Eqs. (4) and (6) without the terms that contain $H_{i1}^{\rm nc}$. This is acceptable for $R\Omega_\zeta \sim v_{ti}$ or high up-down asymmetry $\Delta_{ud} \sim 1$. As a result, $\Pi_{-1}^{\rm tb}(\partial_\psi\Omega_\zeta, \Omega_\zeta) \simeq -\nu^{\rm tb}\partial_\psi\Omega_\zeta - \Gamma^{\rm tb}\Omega_\zeta + \Pi_{ud}^{\rm tb}$, where to obtain this last expression we have linearized around $\partial_\psi\Omega_\zeta = 0$ and $\Omega_\zeta = 0$ for $R\Omega_\zeta/v_{ti} \ll 1$. Here $\nu^{\rm tb}$ is the turbulent diffusivity, $\Gamma^{\rm tb}$ is the turbulent pinch of momentum and $\Pi_{ud}^{\rm tb} \sim \Delta_{ud}\delta_i^2 p_i R|\nabla\psi|$ is the value of $\Pi_{-1}^{\rm tb}$ at $\Omega_\zeta = 0$ and $\partial_\psi\Omega_\zeta = 0$, and is zero for perfect up-down asymmetry when Eqs. (4) and (6) are solved without the terms that contain $H_{i1}^{\rm nc}$. Notice then that imposing $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot \overleftrightarrow{\mathbf{P}}_i \cdot\nabla\psi\rangle_\psi\rangle_{\rm T} \simeq \Pi^{\rm tb} = -\nu^{\rm tb}\partial_\psi\Omega_\zeta - \Gamma^{\rm tb}\Omega_\zeta + \Pi_{ud}^{\rm tb} = 0$ gives intrinsic rotation only for up-down asymmetry or if momentum is pinched into the core from the edge.

The complete model described in this article includes contributions that have not been considered before. On the one hand, the gyrokinetic equations (4) and (6) have new terms with $H_{i1}^{\rm nc}$, giving $\Pi_{-1}^{\rm tb} \simeq -\nu^{\rm tb}\partial_\psi\Omega_\zeta - \Gamma^{\rm tb}\Omega_\zeta + \Pi_{ud}^{\rm tb} + \Pi_{-1,0}^{\rm tb}$, where $\Pi_{-1,0}^{\rm tb} \sim (B/B_p)\delta_i^3 p_i R|\nabla\psi|$ is a new contribution due to the new terms in the gyrokinetic equation. On the other hand, there are the new terms $\Pi_{-1}^{\rm nc}$, $\Pi_0^{\rm nc}$, $\Pi^{\rm ht}$ and Π^{ie}. As we did for $\Pi_{-1}^{\rm tb}$, we can linearize $\Pi_{-1}^{\rm nc}(\partial_\psi\Omega_\zeta)$ around $\partial_\psi\Omega_\zeta = 0$ to find $\Pi_{-1}^{\rm nc} \simeq -\nu^{\rm nc}\partial_\psi\Omega_\zeta + \Pi_{ud}^{\rm nc} + \Pi_{-1,0}^{\rm nc}$, where $\Pi_{ud}^{\rm nc} \sim \Delta_{ud}(B/B_p)(qR\nu_{ii}/v_{ti})\delta_i^2 p_i R|\nabla\psi|$ and $\Pi_{-1,0}^{\rm nc} \sim (B/B_p)^2(qR\nu_{ii}/v_{ti})\delta_i^3 p_i R|\nabla\psi|$. Combining all these results and imposing that $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot \overleftrightarrow{\mathbf{P}}_i \cdot\nabla\psi\rangle_\psi\rangle_{\rm T} = 0$, we obtain

$$\Omega_\zeta = -\int_\psi^{\psi_a} d\psi' \left.\frac{\Pi^{\rm int}}{\nu^{\rm tb}+\nu^{\rm nc}}\right|_{\psi=\psi'} \exp\left(\int_\psi^{\psi'} d\psi'' \left.\frac{\Gamma^{\rm tb}}{\nu^{\rm tb}+\nu^{\rm nc}}\right|_{\psi=\psi''}\right) + \Omega_\zeta|_{\psi=\psi_a} \exp\left(\int_\psi^{\psi_a} d\psi' \left.\frac{\Gamma^{\rm tb}}{\nu^{\rm tb}+\nu^{\rm nc}}\right|_{\psi=\psi'}\right), \qquad (18)$$

where ψ_a is the poloidal flux at the edge, $\Omega_\zeta|_{\psi=\psi_a}$ is the rotation velocity in the edge and $\Pi^{\rm int} = \Pi_{ud}^{\rm tb} + \Pi_{-1,0}^{\rm tb} + \Pi_0^{\rm tb} + \Pi_{ud}^{\rm nc} + \Pi_{-1,0}^{\rm nc} + \Pi_0^{\rm nc} + \Pi^{\rm ht} + \Pi^{ie}$. Notice that this equation gives a rotation profile that depends on $\Pi^{\rm int}$ that in turn depends on the gradients of the temperatures, the geometry and the heating mechanism. The typical size of the rotation is $\Omega_\zeta \sim (B/B_p)\delta_i v_{ti}/R$, being larger, $O(\Delta_{ud} v_{ti}/R)$, if $\Delta_{ud} \gtrsim (B/B_p)\delta_i$.

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