Developments in the Theory of Tokamak Flow Self-Organization

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Abstract. The structure of the toroidal and poloidal momentum fluxes are discussed with an eye toward elucidating mechanisms through which microturbulence drives mean flows. Particular emphasis is placed on discussing the role of off-diagonal contributions to the momentum flux in determining toroidal and poloidal rotation profiles.

1. Introduction

Toroidal rotation in Tokamak plasmas has recently garnered significant attention due to the prominent role these flows play in determining plasma stability, as well as their influence on the L to H power threshold [1, 2, 3, 4, 5, 6, 7]. Extensive experimental as well as numerical studies have unambiguously demonstrated the critical role in which microturbulence plays in determining toroidal flow profiles [8, 9]. Similarly, increasing evidence, both experimental as well as numerical, has indicated that microturbulence may play an important role in determining poloidal rotation profiles in numerous confinement regimes [10, 11, 12, 13, 14]. Our focus within this analysis will be on the identification of constraints on turbulent stresses, as well as an elucidation of the current state of the mean field theory of turbulent flow generation in strongly magnetized plasmas. In this regard, it is useful to consider explicit expressions for the relevant turbulent stresses, which can be most concisely expressed in terms of their parallel and perpendicular components, namely:

$$\Pi_{yr} = m_i \frac{c}{B} \left\langle \delta E_y \delta \left(n_i u_y \right) \right\rangle , \tag{1a}$$

$$\Pi_{\parallel r} = m_i \frac{c}{B} \left[\left\langle \delta E_y \delta \left(n_i u_\parallel \right) \right\rangle + \left\langle \delta E_\parallel \delta \left(n_i u_y \right) \right\rangle \right] . \tag{1b}$$

Here Π_{yr} (note $\hat{e}_y = \hat{b} \times \hat{e}_r$) is given simply by the perpendicular Reynolds stress, whereas the parallel stress can be seen to be composed of two contributions. The former of these is the parallel Reynolds stress, whereas the latter has been shown to be explicitly linked to polarization charge. This latter stress, which we will refer to as a parallel polarization stress, is necessary in order to ensure the symmetry of the stress tensor with respect to the interchange of its indices (i.e. $\Pi_{\parallel r} = \Pi_{r\parallel}$). These stresses, in conjunction with an appropriate edge boundary condition as well as collisional stresses, can be shown to determine the mean flow profile of an axisymmetric large aspect ratio plasma. It will thus be instructive to derive criteria under which significant nondiffusive contributions from these stresses are present.

In the following, we will discuss our current understanding of the turbulent stresses noted in Eq. (1) with particular emphasis on the physical mechanisms underlying nondiffusive momentum transport contributions, and thus on mechanisms through which microturbulence may drive mean toroidal and poloidal flows. In section 2, the theory of nondiffusive toroidal momentum transport is reviewed. Both the physical origin, as well as general scaling trends of residual stress and pinch terms are discussed. Section 3 provides a discussion of turbulence driven poloidal flows, where emphasis is placed on determining regimes in which turbulence may drive significant deviations from neoclassical predictions of poloidal rotation. Section 4 consists of a brief summary.

2. Summary of Toroidal Turbulent Momentum Flux

This section provides a discussion of off-diagonal contributions to the toroidal momentum flux. Namely, the toroidal stress can generally be decomposed as [15]

$$\Pi_{r\varphi} = -\chi_\varphi \frac{\partial \overline{v}_\varphi}{\partial r} + V\overline{v}_\varphi + \Pi_{r\varphi}|_{resid}\,, \tag{2}$$

where χ_φ is the turbulent momentum diffusivity induced by $E \times B$ scattering, V is the convective or "pinch" term, and $\Pi_{r\varphi}|_{resid}$ is the residual stress. Note that the pinch, which is strongly suggested by several experiments [3, 4] reflects a ∇T_i, ∇n, or ∇T_e driven flux of momentum, in proportion to the existing mean flow. The pinch can be either turbulent equipartition (i.e. a consequence of $\nabla \cdot \delta \mathbf{v}_E \neq 0$ in toroidal geometry) or thermoelectric. Examples of recent work include Refs. [16, 17, 18, 19, 20].

The residual stress in contrast, which is required to address the phenomenon of intrinsic rotation, is a consequence of ∇P and ∇T driven stresses which act in the absence of a pre-existing flow. This stress, which is conceptually different from a "pinch", is a consequence of wave-mean flow momentum exchange [21]. The residual stress requires some form of symmetry breaking. Examples discussed within the literature include $E \times B$ shear [22], fluctuation intensity gradients [23], plasma current [24], plasma current poloidal asymmetry [25], or polarization stresses arising within a sheared magnetic topology [26]. These examples can be recognized as generic traits of a magnetized plasma, suggesting that the residual stress is a far more general concept than the simple paradigm of $E \times B$ shear induced symmetry breaking.

Table 1: Residual stress contributions, their associated symmetry breaking mechanisms and scaling forms

Turbulent Stress/Spectral Moment	Symmetry Breaking	ρ_s/L_{resid}
Parallel Reynolds Stress	$\hat{s}I_p \langle V_E \rangle'$	$(\Delta\omega/\omega)(\omega_{ci}/\omega)(\delta x/L_s)$
$\langle k_y k_\parallel \lvert\delta\phi_k\rvert^2 \rangle$	$\hat{s}I_p \partial \langle N \rangle / \partial r$	$(\Delta\omega/\omega)(\omega_{ci}/\omega)(\Delta^2/L_s L_I)$
	for CDDW, I_p	
Mean Radial Current	$I_p \partial \langle N \rangle / \partial r$	$(B_\theta/B)(\tau_c \omega_{ci})(\rho_s^2/L_n L_I)$
$(B_\theta/B)\langle k_y k_r \lvert\delta\phi_k\rvert^2 \rangle$	$I_p \langle V_E \rangle'$	$(B_\theta/B)\,\tau_c \langle V_E \rangle'$
Parallel Polarization Stress	$\hat{s}I_p$	$\mu\Delta^2(\rho_s/L_s)$
$\langle k_r k_\parallel \lvert\delta\phi_k\rvert^2 \rangle$		

2.1 Theory of Residual Stress

In the above, general expressions for the turbulent parallel and perpendicular stresses were recorded [Eq. (1)]. Within this subsection our primary interest will be to determine the turbulent toroidal stress, which can be written as a projection of the parallel and perpendicular stresses noted above. Hence, it is clear that three turbulent stresses are capable of affecting toroidal rotation, namely the parallel Reynolds stress, the parallel polarization stress, as well as the toroidal projection of the perpendicular Reynolds stress. It will be convenient to discuss residual stress contributions from these three stresses in turn. For convenience we have summarized the various residual stress mechanisms discussed below in Table 1.

i.) Parallel Reynolds stress

The parallel Reynolds stress requires $\left\langle k_y k_{\|} \right\rangle \neq 0$ (here after referred to as $k_{\|}$ symmetry breaking), which can be recognized as a consequence of un-balanced toroidally counter-propagating acoustic wave populations. Broken symmetry can be produced by a spectral shift, as for the oft-quoted case of $E \times B$ shear, or, by a spectral envelope intensity gradient, or by toroidal current. Intensity profile gradients are ubiquitous, since fluctuation levels are usually observed to increase with radius. Intensity profiles are intrinsic to confinement, since at constant heat flux, negative temperature profile curvature (i.e. as for a pedestal) requires a decreasing intensity profile, while positive profile curvature (i.e. as for an L-mode edge) implies a rising intensity profile.

Proceeding in more detail, $k_{\|}$ symmetry breaking can result from: a.) electric field shear, b) current or c) intensity gradients.

Regarding a), the essential physics is that $\langle V_E \rangle'$ converts poloidal flow shear into toroidal flow shear via asymmetry in wave-particle momentum deposition [22]. The latter results from the tendency of finite $\langle V_E \rangle'$ – working together with generic drift-acoustic coupling – to produce a shifted spectral envelope, so $\left\langle k_{\|} \right\rangle \neq 0$. Here $\left\langle k_{\|} \right\rangle = k_\theta \delta x / L_s$, where the spectral shift $\delta x \sim \langle V_E \rangle'$. The broken symmetry and $\left\langle k_{\|} \right\rangle \neq 0$ result in a directional imbalance of acoustic wave population and thus in the profile of momentum dissipation by ion Landau damping.

Regarding b), asymmetric spectral shifts may be produced by current, as for the well known case of resistivity gradient driven turbulence or rippling modes. These models, and others like them, are intrinsically asymmetric about resonant surfaces. For current driven drift waves (CDDW), $\left\langle k_{\|} \right\rangle \sim v_d / c_s$, where v_d is the mean electron drift velocity [24].

Regarding c), it is not difficult to see that *any* intensity gradient results in effective $k_{\|}$ symmetry breaking. This follows from the fact that, (schematically) [23]

$$\begin{aligned} m_i \frac{c}{B} \left\langle \delta E_y \delta \left(n_i u_y \right) \right\rangle \Big|_{resid} &\sim \left\langle k_y k_{\|} \left| \delta \phi_k \right|^2 \right\rangle \\ &\approx \left\langle k_y^2 \frac{r - r_0}{L_s} \left\{ \left| \delta \phi_k \left(r_0 \right) \right|^2 + \left(r - r_0 \right) \frac{\partial}{\partial r} \left| \delta \phi_k \left(r_0 \right) \right|^2 + \ldots \right\} \right\rangle \\ &\approx \left\langle k_y^2 \frac{\left(r - r_0 \right)^2}{L_s} \frac{\partial}{\partial r} \left| \delta \phi_k \left(r_0 \right) \right|^2 \right\rangle . \end{aligned} \tag{3}$$

Here, $\left\langle \left(r - r_0 \right)^2 \right\rangle \rightarrow \langle \Delta^2 \rangle$, where Δ is the spectral width. Note that an intensity gradient is a very general phenomenon, which will occur whenever the temperature gradient has finite curvature, for constant heat flux. This follows from $Q = -\chi_i \partial \langle T_i \rangle / \partial r$, where $\chi_i = \chi_{i,turb} + \chi_{i,nc}$, so fixed heat flux gives $\partial Q / \partial r = 0$ and

$$\frac{\partial \chi_{i,turb} / \partial r}{\chi_i} = - \frac{\partial^2 \langle T \rangle / \partial r^2}{\partial \langle T \rangle / \partial r} - \frac{\partial \chi_{i,nc} / \partial r}{\chi_i} . \tag{4}$$

Since $\chi_{i,turb} \sim |\delta\phi_k|^2$, the connection between intensity gradients and profile curvature follows. Intensity gradients are especially likely to occur near transport barriers, where electric field shear can be strong as well. Thus, there will likely be strong overlap and mutual reinforcement between these two effects. In particular, since the electric field shearing rate has the form:

$$\langle V_E \rangle' = -\frac{c}{B}\frac{P_i''}{nZe} + \frac{c}{B}\frac{P_i' n'}{n^2 Ze} - v_\varphi' \frac{B_\theta}{B} + v_\theta' \frac{B_\varphi}{B},$$

we see that *both* $\langle V_E \rangle'$ and intensity gradient I' are ultimately related to the temperature profile curvature.

ii.) Parallel polarization stress

This stress is explicitly linked to polarization charge, and emerges via the parallel nonlinearity within the gyrokinetic formulation. In order to demonstrate this link more explicitly it is useful to note the relation (written here in simplified geometry) [26]

$$\left\langle \sum_s q_s \int d^3\bar{v} \delta F_s \hat{b} \cdot \nabla J_0(k_\perp \rho_\perp) \delta\phi \right\rangle = \frac{\partial}{\partial x} \left\langle m_i \frac{c}{B} \delta E_\parallel \delta \left(n_i u_y \right) \right\rangle, \tag{5}$$

where J_0 is a Bessel function and $\int d^3\bar{v} \equiv 2\pi \int d\mu dv_\parallel B$. The left hand side of this expression can be recognized as the $v_\parallel$ moment of the gyrokinetic parallel nonlinearity summed over all particle species, whereas the right hand side is the divergence of the second stress in Eq. (1b) above. The relevant spectral moment of this stress can be identified via a straightforward quasilinear analysis, yielding

$$m_i \frac{c}{B} \left\langle \delta E_\parallel \delta \left(n_i u_y \right) \right\rangle = \sum_k v_{gr} \frac{k_\parallel}{\omega_k} E_k \sim \left\langle k_r k_\parallel \right\rangle, \tag{6}$$

where E_k is the energy density of the fluctuation spectrum and v_{gr} is the radial group velocity. We note that while, typically, the spectral average of the parallel phase velocity is approximately zero (i.e. $\left\langle k_\parallel / \omega_k \right\rangle \approx 0$), its radial distribution about a rational surface is asymmetric in the presence of magnetic shear [i.e. $k_\parallel \left(-x\right) = -k_\parallel \left(x\right)$]. Similarly, the radial group velocity is also asymmetric [i.e. $v_{gr}(-x) = -v_{gr}(x)$] [27, 28], such that a straightforward eigenmode analysis demonstrates that the spectral moment given by Eq. (6) will generally be nonvanishing in a sheared magnetic topology within the small amplitude limit considered here. Note that the sign of the magnetic shear and plasma current ultimately determine the direction of the mean flow driven by this stress.

iii.) Mean radial current

As noted above, the perpendicular Reynolds stress possesses a finite toroidal projection, and is thus capable of driving toroidal flows. The relevant spectral moment for this stress can be identified via a straightforward quasilinear analysis, yielding

$$m_i \frac{c}{B} \left\langle \delta E_y \delta \left(n_i u_y \right) \right\rangle = \sum_k v_{gr} \frac{k_y}{\omega_k} E_k \sim \left\langle k_r k_y \right\rangle. \tag{7}$$

We note that k_y / ω_k is typically a positive or negative definite quantity for ITG or TEM modes, and thus this stress will vanish in the absence of a finite spectrally averaged radial group velocity, i.e. $\langle v_{gr} \rangle \neq 0$ [29]. Alternatively, this stress can be seen to be linked to a $J \times B$ torque via the relation

$$\frac{B_\theta}{B} \frac{\partial}{\partial x} \left[m_i \frac{c}{B} \left\langle \delta E_y \delta \left(n_i u_y \right) \right\rangle \right] = -\frac{\left\langle J_r^{(pol)} \right\rangle B_\theta}{c}, \tag{8}$$

where $J_r^{(pol)}$ is a radial polarization current (see Ref. [29], or Ref. [30] for its gyrokinetic equivalent). We note that a finite Reynolds stress requires either a diffusive flux of poloidal wave momentum or modulation instability feedback amplifier based on $E \times B$ shear-induced wave refraction, as is the case for zonal flow growth. These mechanisms work via spectral symmetry breaking in either real space ($\partial \langle N \rangle / \partial r \neq 0$) or spectral space ($\partial \langle N \rangle / \partial k_r \neq 0$, often induced by $E \times B$ shear). Here N refers to the wave action density. Due to the fragility of $k_\parallel$ symmetry breaking mechanisms, and the likely modest contribution from the parallel polarization stress, $\left\langle J_r^{(pol)} \right\rangle$-driven stresses correspond to an attractive candidate for driving intrinsic rotation.

2.2 Theory of Parallel Velocity Pinch

While residual stress is required to drive intrinsic rotation, a momentum pinch will contribute to rotation profile formation and peaking. The turbulent angular momentum flux carried by ions resonant with ITG turbulence has been calculated via quasilinear theory using a lab frame, phase space conserving gyrokinetic equation. Results near ITG marginality indicate that the *inward* turbulent equipartition (TEP) momentum pinch emerges as *the* robust pinch process. Regarding thermoelectric effects, results for typical parameters characteristic of the near marginal regime indicate that the ion ∇T-driven momentum flux is usually inward, while the ∇n-driven momentum flux is usually outward. Thus, these two fluxes tend to negate each other, leaving the TEP pinch as the robust survivor. Note that since tokamak plasma dynamics is not Galilean invariant (i.e. pinches are curvature driven), the issue of Galilean invariance constraints on the momentum pinch is moot.

Proceeding in more detail, we can in general write the momentum pinch velocity V as:

$$V = V_{TEP} + V_{Th}. \tag{9}$$

The TEP part is a consequence of compressibility of the $E \times B$ velocity in toroidal geometry (i.e. $\nabla \cdot \delta \mathbf{v}_E \neq 0$) and thus represents the irreducible minimum of turbulent inward pinches. Note, however, that the TEP momentum pinch is rather modest in that:

$$RV_{TEP}/\chi_\phi = -\alpha, \tag{10}$$

where $\alpha \approx 3$ for the definitions in the present analysis, or $\alpha \approx 4$ if the transport coefficients in Eq. (2) are defined with respect to the toroidal angular frequency [18]. In addition to the basic and unavoidable TEP pinch, the momentum convection velocity also contains thermoelectric contributions, driven by ∇T and ∇n (note, here we discuss ITG only). Thus,

$$V_{Th} = \sum_k \left|\delta v_{r,k}\right|^2 \tau_{c,k} \left\{ g_1(k) \frac{\nabla T_i}{T_i} + g_2(k) \frac{\nabla n}{n} \right\}. \tag{11}$$

Here, $g_1(k)$ and $g_2(k)$ can each be positive or negative and reflect complex parameter dependencies. Note that superficially, $RV_{Th}/\chi_\phi \sim \mathcal{O}(1/\varepsilon)$ so the thermoelectric pinch may *appear* more robust than the TEP pinch. However, reality is not so simple. Detailed gyrokinetic analysis reveals that close to ITG threshold (i.e. the relevant regime for stiff profiles), the ∇T-driven thermoelectric flow is *inward* (i.e. a pinch) but the ∇n-driven thermoelectric flux is *outward*. Thus, the two tend to compete with one another and cancel, so the TEP pinch is, in fact, the most robust momentum pinch process. The complex interplay and dependencies are summarized in Table 2.

Table 2: Analytic predictions on momentum pinch. Note that the ∇T_i-driven momentum pinch is similar to the ∇T_i-driven particle pinch. See Ref. [20] for relevant references and notation.

For ITG: V_{pinch}/χ_φ	∇n driven	∇T_i driven	∇B driven
fluid regime in torus	$-1/L_n$ Inward	0	$-4/R$, for $\tau = 1$ Inward
kinetic regime near marginality in a torus	$1/L_n$ Outward	$-\left(\frac{5}{2} - \alpha_c(\omega_k)\right)/L_{T_i}$ Inward	$-\frac{8}{5}\alpha_c(\omega_k)/R$ Inward

3. Summary of Poloidal Turbulent Momentum Flux

Within this section we will be interested in developing a framework for describing poloidal rotation which incorporates both neoclassical as well as turbulent stresses. For a small inverse aspect ratio plasma, an expression for the poloidal flow may be written as

$$\frac{\partial}{\partial t}\left(n_i m_i r \overline{u}_\theta\right) + \left\langle \nabla \cdot \left(\overline{r\hat{e}_\theta \cdot \mathbf{\Pi}_{turb}}\right)\right\rangle = -n_i m_i r \mu_{ii}\left(\overline{u}_\theta - u_\theta^{neo}\right), \qquad (12)$$

where $\mu_{ii} \sim \tau_{ii}^{-1}$ and u_θ^{neo} is the neoclassical rotation off-set. The perpendicular Reynolds stress (likely the dominant contribution for $B_\theta \ll B_\varphi$), has been directly evaluated in Ref. [29]. It was found that the relevant spectral moment is given by $\langle v_{gr}\left(k_y/\omega_k\right)\rangle \sim \langle v_{gr}\rangle \neq 0$, where v_{gr} is the radial group velocity, and thus some form of symmetry breaking (i.e. anisotropy) is required in order to render the stress nonvanishing. Later work demonstrated that either parallel flow shear or $E \times B$ shear are capable of inducing the necessary anisotropy in the turbulence spectrum to drive a finite perpendicular stress [31]. Recent experimental work on the basic plasma experiment CSDX has observed the formation of an azimuthal flow driven by a nondiffusive, residual contribution to the turbulent stress, further reinforcing the critical role of off-diagonal transport contributions to mean flow formation [32].

The above observations, while providing useful criteria for when turbulent flow generation is possible, are not sufficient for determining regimes in which turbulent stresses are likely strong enough to drive experimentally relevant rates of poloidal rotation. Within this section, we will utilize an additional constraint on the perpendicular Reynolds stress with the motivation of providing an alternate perspective on poloidal flow generation. In particular, we will be interested in exploiting a Taylor identity, which allows the perpendicular stress to be expressed in terms of the flux of potential vorticity. In perhaps its simplest form, a Taylor identity may be written as [33]

$$\frac{\partial}{\partial x}\left\langle \delta u_y^E \delta u_x^E \right\rangle = -\left\langle \delta u_x^E \delta q \right\rangle, \qquad (13)$$

where $\delta \mathbf{u}^E = (\hat{b} \times \nabla_\perp \delta\phi)$, $\delta q \equiv (1 - \nabla_\perp^2)\delta\phi$ can be identified as the potential vorticity for idealized plasma models such as the Hasegawa-Mima equation, we have normalized length and time scales by ρ_s and ω_{ci}, respectively, and the electrostatic potential by e/T_e. Note that while Eq. (13) requires translational invariance in the zonal direction, no small amplitude assumption of the fluctuation intensity was necessary.

Equation (13), while useful for the treatment of highly idealized fluid models, is not sufficient for the present analysis. An analogous form to Eq. (13), appropriate to an electrostatic gyrokinetic formulation, can be shown to have the form [30]

$$\overline{\sum_s q_s \int d^3\overline{v}\,\delta F_s \left(\hat{e}_\varphi \times \nabla_\perp J_0 \delta\phi\right)_r} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 m_i \frac{c}{B}\overline{\delta E_\theta \delta\left(n_i u_y\right)}\right], \qquad (14)$$

where δF_s is the gyrocenter distribution function. Thus, for the limit $B_\theta \ll B_\varphi$, it is clear that the perpendicular stress divergence can be linked to the flux of gyrocenter charge. This latter quantity can be easily estimated via the gyrokinetic equation. A systematic quasilinear calculation utilizing a phase space conserving gyrokinetic equation was performed in Ref. [30]. It was found that at stationarity, deviations from neoclassical predictions of poloidal rotation can be explicitly linked to the spatial growth/dissipation profile of the underlying fluctuations, consistent with the small amplitude limit of a Charney-Drazin theorem [34, 35]. Here, however, we will be interested in discussing a different perspective on this result. Namely, it will be useful to consider a simplified fluid description, where the ion gyrocenter center evolves according to

$$\frac{d}{dt}\left(\frac{N_i}{B^2}\right) = \hat{b}\cdot\left(\frac{\nabla P_i \times \nabla B}{m_i \omega_{ci} B^2}\right) + \frac{\nu}{B^2}\nabla_\perp^2 N_i. \tag{15}$$

Here we have we have assumed $\hat{b}\cdot\nabla \rightarrow 0$, taken the long wavelength limit, and approximated $\nabla \times \hat{b} \approx \hat{b} \times \nabla_\perp \ln B$ for simplicity. From Eqs. (15), (14) and (12), assuming adiabatic electrons, a straightforward calculation in the small amplitude limit yields at stationarity

$$\overline{u}_\theta = u_\theta^{neo} - \frac{\omega_{ci}}{\mu_{ii}}\left[D_{PV}\frac{B^2}{n_0}\frac{\partial}{\partial r}\left(\frac{\overline{N}_i}{B^2}\right) + V_{PV}^{Th}\frac{\overline{N}_i}{n_0}\right], \tag{16a}$$

with

$$D_{PV} \equiv c_s^2\rho_s^2 \mathrm{Re}\sum_k \frac{i\left(\hat{b}\times\mathbf{k}_\perp\right)_r^2}{\Omega_k + i\nu_k}\left|\frac{e\delta\phi_k}{T_e}\right|^2, \quad V_{PV}^{Th} \equiv -2\mathrm{Re}\sum_k \frac{i\omega_{d\nabla B}}{\Omega_k + i\nu_k}\delta u_{r,-k}^{EB}\frac{\delta T_{i,k}}{T_i}, \tag{16b}$$

and we note that in physical coordinates the mean gyrocenter density can be written

$$\overline{N}_i \approx n_i + \nabla_\perp\cdot\left(n_i m_i \frac{c^2}{eB^2}\overline{\mathbf{E}}_\perp\right) - \frac{1}{2}\nabla_\perp\cdot\left(n_i\rho_i^2\nabla_\perp \ln \overline{P}_i\right) + \ldots. \tag{17}$$

Here $\Omega_k \equiv \omega_k - \omega_{d\kappa} - \omega_{d\nabla B}$, $\omega_{d\kappa}$ and $\omega_{d\nabla B}$ are curvature and grad-B drifts, $\nu_k \equiv \nu k_\perp^2$, and we have utilized the inequality $B_\theta \ll B_\varphi$. From Eq. (16), turbulence driven deviations from neoclassical rotation levels can be seen to be linked to the flux of the ion gyrocenter density N_i, which can be seen to be composed of diffusive, TEP and thermoelectric contributions. From Eq. (16) it is clear that turbulence induced deviations are likely most significant for regimes of low collisionality, as would be naively anticipated. Similarly, the thermoelectric contribution can be seen to be closely associated with the turbulent heat flux. Indeed, a simple closure of this term performed in Ref. [30] reveals strong temperature gradient dependence of this term.

4. Conclusion

In this analysis, we have presented a unified formulation of toroidal and poloidal flow generation. The primary results of this analysis are as follows:

a) The relevant spectral moments and associated symmetry breaking mechanisms for toroidal flow generation have been derived, and are summarized in Table 1. It is noted that aside from the now extensively studied $E \times B$ shear driven residual stress, numerous other residual stress contributions are likely present within the toroidal momentum flux.

b) A quasilinear expression for a pinch of toroidal momentum has been derived for ITG turbulence near marginality. Turbulent equipartition as well as thermoelectric contributions to the toroidal pinch velocity are derived, and their dependencies are summarized in Table 2.

c) Equation (14), which possesses an analogous form as the classic Taylor identity, has been utilized in order to link the perpendicular stress divergence to the flux of gyrocenter charge. This flux has been shown to be composed of diffusive, TEP and thermoelectric contributions, suggesting multiple means of driving deviations from neoclassical levels of rotation.

References

[1] RICE, J. E. et al., Nucl. Fusion **47** (2007) 1618.
[2] SOLOMON, W. M. et al., Nucl. Fusion **49** (2009) 085005.
[3] YOSHIDA, M. et al., Plasma Phys. Control. Fusion **48** (2006) 1673.
[4] TALA, T. et al., Phys. Rev. Lett. **102** (2009) 075001.
[5] DEGRASSIE, J. S., Plasma Phys. Control. Fusion **51** (2009) 124047.
[6] BORTOLON, A. et al., Phys. Rev. Lett. **97** (2006) 235003.
[7] IDA, K. et al., Phys. Rev. Lett. **74** (1995) 1990.
[8] WANG, W. X. et al., Phys. Rev. Lett. **102** (2009) 035005.
[9] HOLOD, I. et al., Phys. Plasmas **15** (2008) 092302.
[10] KOIDE, Y. et al., Phys. Rev. Lett. **72** (1994) 3662.
[11] BELL, R. E. et al., Phys. Rev. Lett. **81** (1998) 1429.
[12] CROMBÉ, K. et al., Phys. Rev. Lett. **95** (2005) 155003.
[13] SOLOMON, W. M. et al., Phys. Plasmas **13** (2006) 056116.
[14] DIF-PRADALIER, G. et al., Phys. Rev. Lett. **103** (2009) 065002.
[15] DIAMOND, P. H. et al., Nucl. Fusion **49** (2009) 45002.
[16] PEETERS, A. G. et al., Phys. Rev. Lett. **98** (2007) 265003.
[17] HAHM, T. S. et al., Phys. Plasmas **14** (2007) 072302.
[18] HAHM, T. S. et al., Phys. Plasmas **15** (2008) 055902.
[19] GÜRCAN, Ö. D. et al., Phys. Rev. Lett. **100** (2008) 135001.
[20] YOON, E. S. et al., Nucl. Fusion **50** (2010) 064006.
[21] DIAMOND, P. H. et al., Phys. Plasmas **15** (2008) 012303.
[22] GÜRCAN, Ö. D. et al., Phys. Plasmas **14** (2007) 042306.
[23] GÜRCAN, Ö. D. et al., Residual parallel reynolds stress due to turbulence intensity gradient in tokamak plasmas, submitted to Physics of Plasmas, 2010.
[24] LEE, C. J. et al., APS-DPP Conference, Atlanta, 2009.
[25] CAMENEN, Y. et al., Phys. Rev. Lett. **102** (2009) 125001.
[26] MCDEVITT, C. J. et al., Phys. Rev. Lett. **103** (2009) 205003.
[27] PEARLSTEIN, L. D. et al., Phys. Rev. Lett **23** (1969) 220.
[28] HORTON, W., Rev. Mod. Phys. **71** (1999) 735.
[29] DIAMOND, P. H. et al., Phys. Fluids B **3** (1991) 1626.
[30] MCDEVITT, C. J. et al., Poloidal rotation and its relation to the potential vorticity flux, in press, Physics of Plasmas, 2010.
[31] DOMINGUEZ, R. R. et al., Phys. Fluids B **5** (1993) 3876.
[32] YAN, Z. et al., Phys. Rev. Lett. **104** (2010) 065002.
[33] TAYLOR, G. I., Phil. Trans. Roy. Lond. **215** (1915) 1.
[34] CHARNEY, J. G. et al., J. Geophys. Research **66** (1961) 83.
[35] DIAMOND, P. H. et al., Plasma Phys. Control. Fusion **50** (2008) 1.