Fusion Reaction Burn in Cylindrical Magnetized Target

A. Sid 1), A. Ghezal 2)

1) Laboratoire de Physique des Rayonnements et de leurs interactions avec la Matière (PRIMALAB), Département de Physique, Faculté des Sciences, Université de Batna, Batna, 05000 DZ, Algeria.
2) Commissariat à l'Energie Atomique, Centre de Recherche Nucléaire de Draria Division de Sûreté Nucléaire et Radioprotection, Draria, Alger, Algeria.
E-mail contact of mail author: a_sid@univ-batna.dz, sid_abdelaziz@hotmail.com

Abstract. A model has been proposed in order to describe the D-T fusion reaction burn in a magnetized cylindrical target. The reaction is supposed initiated in a central hot spot in the end of the implosion phase. In this model the thermonuclear fuel is composed by an interfering of three fluids which are the electrons, the deuterium and the tritium fluids.

Ions density decreasing due to the fusion reaction is taken into account. Importance is given for the alpha energy deposition in the thermonuclear fuel and for the effect of the applied magnetic field on the ignition conditions. The energy losses due to bremsstrahlung emission and by heat conduction are computed. Screening effect due to dense plasma is discussed and considered in this work.

1D space and 2D velocity space Eulerian simulation has been carried out in this work.
Keywords: Inertial Fusion, Magnetized Cylindrical Target, Magnetic field, Burn.

1. Introduction

In the way of inertial thermonuclear fusion, the fusion reaction begins initially in a central hot spot compressed and heated by the shock wave. This hot spot is formed by an external source or by converting of the shock wave energy into heat energy.

It is important to well know how the burn wave propagates outward leading to the ignition of the surrounding cod fuel. This permits to calculate the fraction of burned fuel and then to optimize the implosion conditions.

The pertinent parameter of the fusion reaction burn in cylindrical magnetized target is the surface density ρR, that the Lawson criterion can be presented in this situation as a threshold condition on ρR. Then the self burning of the fusion reaction, for a sufficient confinement time, needs a threshold value of ρR which depends on the nature of the used fuel. For example in a mixture of deuterium and tritium, $\rho R > 0.2g\ /\ cm^2$ and this value is significantly reduced by the applied magnetic field.

If we look at transport processes on a global scale - for example, the total heat transport in the initial interaction phase, it is not necessary or practical to study the plasma in such detail as it includes different collisions between electrons and ions. For describing the evolution of the macroscopic quantities such as density, temperature and pressure, a simplified picture of the fluid plasma is often enough. In the burning phase, a charge separation occurs over a length scale which it can be comparable to the Debye length. In this case the plasma cannot be considered as a single component fluid.

In this paper, we look to fuel plasma as an interfering of several fluids via collisions. Each king of particles is considered as a fluid characterized by hydro-dynamical equations: continuity equation, conservation of momentum and conservation of interne energy. This work is organized as follow: in section 2, we present the simple fluid model intensively reported in the literature under some approximations. In section 3, we present our multi fluids model and the physical effects taken into account in this model. Finally, in section 4, a conclusion is given.

2. Fluid model

In this model somewhat simplified and widely reported in the literature, the plasma is treated as a single conducting fluid. For a homogeneous fluid, the equation of continuity of matter in the fuel plasma, supposed a mixture of deuterium and tritium, is presented as:

$$\frac{dn}{dt} = -n_D n_T \langle\sigma v\rangle, \quad (1)$$

where n is the number of thermonuclear fusion reaction per volume unit, n_D is the deuterium density and n_T is the tritium density $n_D n_T\langle\sigma v\rangle$ means the fusion reaction rate, where σ is the cross section of the fusion reaction and v is the relative velocity of two fusing nucleus.

In order to estimate the fraction of burned fuel, we regard to thermonuclear fuel as homogeneous plasma. In this situation, the fraction of burned fuel can be deduced from the above equation. If the instantaneous densities of tritium and deuterium are the same, they are given by: $n_D = n_T = \frac{n_0}{2} - n$, where n_0 is the initial ion density (the sum of the initial densities of deuterium and tritium).

Introducing the fraction of burned fuel, f_b, given by: $f_b = \frac{n}{n_0/2}$, the continuity equation can be rewritten in terms of f_b as:

$$\frac{df_b}{dt} = \frac{n_0}{2}(1-f_b)\langle\sigma v\rangle, \quad (2)$$

The solution of this equation is:

$$f_b = \frac{n_0\tau_b\langle\sigma v\rangle/2}{1+(n_0\tau_b\langle\sigma v\rangle/2)} \quad (3)$$

where τ_b corresponds to the fusion reaction detonation time.

The detonation time is related to the sound speed in the plasma approximately by : $\tau_b = r/3C_s$ in the case of a spherical target and by: $\tau_b = r/2C_s$ in the case of a cylindrical target which corresponds to our case, so :

$$f_b = \frac{n_0 r\langle\sigma v\rangle/4C_s}{1+(n_0 r\langle\sigma v\rangle/4C_s)}. , \quad (4)$$

By introducing the mass density, $\rho \cong m_D n_D + m_T n_T$, the above equation rewrite as:

$$f_b = \frac{\rho R}{\rho R+\psi(T)}, \quad (5)$$

where $\psi(T) \sim C_s/\langle\sigma v\rangle$.

The reaction rate depends strongly on the temperature. An expression of this was introduced by Hiverly (1979), so:

$$\langle\sigma v\rangle = exp(\frac{a_1}{T^b} + a_2 + a_3 T + a_4 T^2 + a_5 T^3 + a_6 T^4), \quad (6)$$

where $a_1 = -21.377692$, $a_2 = -25.204054$, $a_3 = -7.1013427 \times 10^{-2}$, $a_4 = 1.9375451 \times 10^{-4}$, $a_5 = 4.9246592 \times 10^{-6}$, $a_6 = -3.9836572 \times 10^{-8}$ and $b = 0.2935$. Here the temperature is in KeV.

For a Deuterium-Tritium fusion reaction in the temperature range of T = 20-40 keV, the fraction of burned fuel can be approximated by the practice following expression:

$$f_b = \frac{\rho R}{\rho R + 6(\frac{g}{cm^2})} \quad (7)$$

In the way of inertial thermonuclear fusion, we always try to maximize the surface density of the hot spot in order to obtain a more effective reaction detonation and consequently to get a maximum of burned fuel.

In practice, the thermonuclear energy deposed in the thermonuclear fuel is that of the alpha particles. Because the mean free path of neutrons in plasma at a temperature of about 10 keV is very large compared to that of the alpha particle (~ 40 times). For a well self-heating of the fuel, it is necessary that the radius of the cylinder containing the fuel is much higher than the mean free path of the alpha particle l_α: $R \gg l_\alpha$. The thermonuclear power produced in a central region can be approximated by:

$$\frac{dE_{fusion}}{dt} = \pi r^2 \langle \sigma v \rangle \frac{n_0^2}{4} W_\alpha, \quad (8)$$

whereW_α is the energy of the alpha particle deposited in the plasma. This corresponds to the change of thermal energy:

$$\frac{dE_{int}}{dt} = \frac{d}{dt}(\pi r^2 n_0 T) = \pi r^2 n_0 \frac{dT}{dt} + 2\pi r n_0 T \frac{dr}{dt}. \quad (9)$$

By considering that the thermonuclear energy absorbed is transformed into internal energy: $\frac{dE_{int}}{dt} = \frac{dE_{fusion}}{dt}$, we obtain the equation of change of the radius of the detonation region, so:

$$\frac{dr}{dt} = n_0 \frac{\langle \sigma v \rangle W_\alpha r}{12T} - \frac{r}{3T}\frac{dT}{dt} \quad (10)$$

One can deduce from this equation that the burning velocity $v_b = \frac{dr}{dt}$ is greater than the sound speed in the plasma C_s. For example, if $T > 15\ KeV$T, $v_b > 2C_s$.

3. **Multi-fluids model**

The single fluid model is not enough to describe the spatiotemporal evolution of the inertial thermonuclear fusion reaction. Because, a space charge occurs on a scale length of the burning evolution which can be comparable to the Debye length and the quasi neutrality is not established. In order to describe, more rigorously; the evolution of the fusion reaction to the cold fuel, we developed a multi fluids model: the electrons, the deuterium ions and the tritium ions. This model takes into account the spatiotemporal dependence of the reaction rate which strongly depends on the plasma temperature. In this model, we are taking into account the energy losses by bremsstrahlung and by thermal conduction, the energy deposited by produced alpha particles in the fuel and the effect of the applied magnetic field on the plasma and the alpha particles.

3.1 Physical Effects

- **Alpha particles energy Deposit in the fuel**

The produced alpha particle deposits its energy in the fuel predominantly by the dynamical friction mechanism. The solution of the classical motion equation of an alpha particle allows to determine the fraction of the energy deposited in the thermonuclear fuel after a path, so: $f_{\alpha s} = \frac{E_\alpha(s)}{E_{\alpha 0}} = 2\left(\frac{s}{l_\alpha}\right) - \left(\frac{s}{l_\alpha}\right).$ (11)

The average value of $f_{\alpha s}$:

$f_\alpha = \frac{2}{2\pi R^2}\int_0^R rdr \int sin(\theta)\, d\theta \int f_{\alpha s}(r,\theta,\varphi)d\varphi$, can be exactly evaluated for a uniform sphere. But in the case of a cylindrical geometry, an approach form of this integral is calculated as: $f_\alpha = \frac{x_\alpha + x_\alpha^2}{1+\frac{13x_\alpha}{9}+x_\alpha^2}$, (12)

where $x_\alpha = \frac{8}{9}\left(\overline{R} + c^2/\sqrt{2c^2+1000}\right)$, $\overline{R} = \frac{R}{l_\alpha}$, $c = R\omega_\alpha/v_{\alpha 0}$ and $v_{\alpha 0}$ is the initial velocity of the alpha particle (at birth).

- **Energy loss by bremsstrahlung**

The specific power of radiation emitted by bremsstrahlung mechanism is calculated by AI Akhiezer and VB Berestetskii (1965) using a relativistic Maxwellian distribution. The following formula is obtained:

$$P_{br} = c_0 n_e^2 \sqrt{\frac{T_e}{m_e c^2}}\left(K^{ee}(T_e) + K^{ei}(T_e)\right), \tag{13}$$

where $c_0 = \frac{16}{3}\sqrt{2\pi/3}\,\alpha r_e^2 m_e c^3$, α is the fine structure constant and r_e is the classical radius of the electron. The functions $K^{ee}(T_e)$ and $K^{ei}(T_e)$ were calculated using a numerical fit, so:

$$K^{ee}(y) = 1.78y - 0.15y^2 + 0.58y^3$$

$$K^{ei}(y) = 1.1 + 0.59y + 3.06y^2 - 2.56y^3 + 0.85y^4$$

- **The fusion reaction rate**

The most used formula for the Deuterium-Tritium fusion reaction rate is established by Hively (1983) by considering a relativistic Maxwellian distribution:

$$\langle\sigma v\rangle = 9.1\times 10^{-16}\, exp\left(-0.572\left(ln\left(\frac{T}{64.2}\right)\right)^{2.13}\right) cm^3/s. \tag{14}$$

In this expression the temperature is keV.

The previous expression must be corrected by taking into account the Debye screen on the potential of Coulomb repulsion between the nucleuses. Because in a dense plasma, such the inertial fusion burning fuel, the potential is given by the Debye potential: $V_D(r) = \frac{q}{4\pi\varepsilon_0 r}\exp\left(-\frac{r}{\lambda_D}\right) \approx V_c(r) - E_s$, where $V_c(r)$ is the Coulomb potential, $E_s = \frac{q}{4\pi\varepsilon_0\lambda_D}$ is the energy of Debye screen produced by the dense plasma and λ_D is the Debye length.

Taking into account the Debye screen, the rate of fusion reaction for a mixture of deuterium and tritium at temperature T, became:

$$R_{fs} = R_{fns}\left[1 + \sqrt{3}\Gamma_e^{\frac{3}{2}}\right] =$$

$$9.1 \times 10^{-16}\, n_D n_T\left[1 + \sqrt{3}\Gamma_e^{3/2}\right]_T exp\left(-0.572\left(ln\left(\frac{T}{64.2}\right)\right)^{2.13}\right) cm^{-3}/s, \qquad (15)$$

where $\Gamma_e = \frac{e^2}{4\pi\varepsilon_0 a_B T}$ and a_B is the Bohr radius.

3.2. Model. Equations

The equations of this multi-fluids model are the hydrodynamic equations of each species of particles in the thermonuclear fuel. These equations are obtained by calculating the moments of the transport equation.

- **Continuity equation**

This equation is presented for electrons in the form:

$$\frac{\partial n_e}{\partial t} + \vec{\nabla}.(n_e\vec{V}) = 0 \qquad (16)$$

And for deuterium and tritium in the form:

$$\frac{\partial n_D}{\partial t} + \vec{\nabla}.(n_e\vec{V}) = -n_D n_T\langle\sigma v\rangle. \qquad (17)$$

$$\frac{\partial n_T}{\partial t} + \vec{\nabla}.(n_T\vec{V}) = -n_D n_T\langle\sigma v\rangle. \qquad (18)$$

The second member of these equations describes the reduction of ion density by the fusion reaction.

- **Momentum conservation equations**

The momentum conservation equations for electrons, deuterium and tritium are presented as:

$$m_e\frac{d(n_e\vec{V}_e)}{dt} = -\vec{\nabla}(n_e T) - e n_e\vec{V}_e \times \vec{B} - n_e(\nu_{eD}\vec{V}_{eD} + \nu_{eT}\vec{V}_{eT}), \qquad (18)$$

$$m_D \frac{d(n_D \vec{V}_D)}{dt} = -\vec{\nabla}(n_D T) + e n_D \vec{V}_D \times \vec{B} - n_D(\nu_{De} \vec{V}_{De} + \nu_{DT} \vec{V}_{DT}), \quad (19)$$

and $$m_T \frac{d(n_T \vec{V}_T)}{dt} = -\vec{\nabla}(n_T T) + e n_T \vec{V}_T \times \vec{B} - n_T(\nu_{TD} \vec{V}_{TD} + \nu_{Te} \vec{V}_{Te}) \quad (20)$$

Here $\frac{\mathrm{d}}{\mathrm{dt}} = \frac{\partial}{\partial \mathrm{t}} + \vec{\mathrm{V}}.\vec{\nabla}$ is the time convective derivative. The first term in the right-hand side in these equations corresponds to the force due to kinetic pressure gradient, the second term is the force due to applied magnetic field and the third term characterizes the interaction via collisions between the three fluids at their interference in the fuel plasma.

- **Energy conservation equation**

The equation of energy conservation that takes into account the energy loss by bremsstrahlung, the heating of the fuel by the alpha particles energy and the energy loss by heat conduction is presented as:

$$\frac{3}{2}\frac{d(n_e T)}{dt} + \frac{3}{2}\frac{d(n_D T)}{dt} + \frac{3}{2}\frac{d(n_T T)}{dt} + n_e T \vec{\nabla}.\vec{V}_e + n_D T \vec{\nabla}.\vec{V}_D + n_T T \vec{\nabla}.\vec{V}_T =$$

$$-\vec{\nabla}.(\vec{q}_e + \vec{q}_D + \vec{q}_T) - P_b + E_{\alpha 0} f_\alpha R_f, \quad (21)$$

where $\vec{q}_i$ is the heat flux of species i, P_b is the radiated power per unit volume by bremsstrahlung mechanism,$\mathrm{E}_{\alpha 0}\mathrm{f}_\alpha \mathrm{R}_\mathrm{f}$ is the energy deposited per unit time and per unit volume in the fuel, where $\mathrm{E}_{\alpha 0}, \mathrm{f}_\alpha$ and R_f are respectively the initial energy of the alpha particle (at birth), the fraction of the energy of the particle Alpha deposited in the fuel and the rate of thermonuclear fusion reaction.

3.3 Numerical simulation

In the case of a magnetized cylindrical target, the simplest numerical simulation is 1D space and 2D velocity space:

$\mathrm{n_e} = \mathrm{n_e}(\mathrm{t,r}), \mathrm{n_D} = \mathrm{n_D}(\mathrm{t,r}), \mathrm{n_T} = \mathrm{n_T}(\mathrm{t,r}); \vec{\mathrm{V}}_\mathrm{e}(\mathrm{t,r}) = \mathrm{V_{er}}(\mathrm{t,r})\vec{\mathrm{u}}_\mathrm{r} + \mathrm{V_{e\theta}}(\mathrm{t,r})\vec{\mathrm{u}}_\theta$; $\vec{\mathrm{V}}_\mathrm{D}(\mathrm{t,r}) = \mathrm{V_{Dr}}(\mathrm{t,r})\vec{\mathrm{u}}_\mathrm{r} + \mathrm{V_{D\theta}}(\mathrm{t,r})\vec{\mathrm{u}}_\theta$;$\vec{\mathrm{V}}_\mathrm{T}(\mathrm{t,r}) = \mathrm{V_{Tr}}(\mathrm{t,r})\vec{\mathrm{u}}_\mathrm{r} + \mathrm{V_{T\theta}}(\mathrm{t,r})\vec{\mathrm{u}}_\theta$ and $\mathrm{T} = \mathrm{T}(\mathrm{t,r})$

In this case we can represent the model equations in a dimensionless form, where

, $n_i \mapsto \frac{n_i}{n_0}, V_i \longmapsto \frac{V_i}{V_0}, T \longmapsto \frac{T}{T_0}, t \longmapsto \frac{t}{t_0}, r \longmapsto \frac{r}{r_0},$

where n_0 is the initial density of deuterium (tritium), $V_0 = \sqrt{T_{ig}/m_e}$ is thermal velocity of electrons at the ignition temperature, $t_0 = 1ns$ and r_0 is the initial radius of the cylinder containing the thermonuclear fuel.

-Continuity equations:

$$\frac{\partial n_e}{\partial t} = -a_1 \frac{\partial V_{er}}{\partial r} \quad (22)$$

$$\frac{\partial n_D}{\partial t} = -a_1 \frac{\partial V_{Dr}}{\partial r} - R_f \quad (23)$$

$$\frac{\partial n_T}{\partial t} = -a_1 \frac{\partial V_{Tr}}{\partial r} - R_f \quad (24)$$

Où ,$a_1 = \frac{t_0 V_0}{n_0 r_0}$ et $\tilde{R}_f = \frac{n_0 R_f}{t_0}$

-Momentum equations:

$$\frac{\partial(n_e V_{er})}{\partial t} = -b_1 V_{er} \frac{\partial(n_e V_{er})}{\partial r} - b_2 \frac{\partial(n_e T)}{\partial r} + \Omega_{ce} V_{e\theta} - n_e(\nu_{eD} V_{eDr} + \nu_{eT} V_{eTr}), \quad (25)$$

$$\frac{\partial(n_e V_{e\theta})}{\partial t} = -\Omega_{ce} V_{er} - n_e(\nu_{eD} V_{eD\theta} + \nu_{eT} V_{eT\theta}), \quad (26)$$

$$\frac{\partial(n_D V_{Dr})}{\partial t} = -b_1 V_{er} \frac{\partial(n_D V_{Dr})}{\partial r} - b_2 \frac{\partial(n_D T)}{\partial r} + \Omega_{cD} V_{D\theta} - n_e(\nu_{De} V_{eDr} + \nu_{eT} V_{DTr}), \quad (27)$$

$$\frac{\partial(n_D V_{D\theta})}{\partial t} = -\Omega_{cD} V_{Dr} - n_e(\nu_{De} V_{De\theta} + \nu_{eT} V_{DT\theta}), \quad (28)$$

$$\frac{\partial(n_T V_{Tr})}{\partial t} = -b_1 V_{Tr} \frac{\partial(n_T V_{Tr})}{\partial r} - b_2 \frac{\partial(n_T T)}{\partial r} + \Omega_{cT} V_{T\theta} - n_T(\nu_{Te} V_{Ter} + \nu_{TD} V_{TDr}), \quad (29)$$

$$\frac{\partial(n_T V_{T\theta})}{\partial t} = -\Omega_{cT} V_{Tr} - n_T(\nu_{Te} V_{Te\theta} + \nu_{TD} V_{TD\theta}), \quad (30)$$

where $b_{1=} \frac{V_0 t_0}{r_0}$, $b_2 = \frac{t_0 T_0}{m_e V_0 r_0}$, $\Omega_{ci} = t_0 \frac{eB}{m_e}$ is the normalized cyclotron frequency for particle of species i and ν_{ij} is a normalized collision frequency ij.

-Energy conservation equation:

$$\frac{\partial(n_e T + n_D T + n_T T)}{\partial t} = b_1 V_{er} \frac{\partial(n_e T)}{\partial r} - b_1 V_{Dr} \frac{\partial(n_D T)}{\partial r} - b_1 \frac{\partial(n_T T)}{\partial r} - \frac{2}{3} b_1 n_e T \frac{\partial V_{er}}{\partial r} - \frac{2}{3} b_1 n_D T \frac{\partial V_{Dr}}{\partial r} - \frac{2}{3} n_T T \frac{\partial V_{Tr}}{\partial r} - \frac{2}{3}\frac{\partial q_e}{\partial r} - \frac{2}{3}\frac{\partial q_e}{\partial r} - \frac{2}{3}\frac{\partial q_e}{\partial r} - \frac{2}{3} P_b + \frac{2}{3} c f_\alpha \tilde{R}_f, \quad (31),$$

where $c = E_{\alpha 0}/T_0$ et $E_{\alpha 0} = 3.5\ MeV$ is the energy of the alpha particle at birth.

The model equations are composed by a set ten dimensionless coupled partial differential equations. The numerical solution of this system allows us to study more rigorously the detonation of the fusion reaction in the case of a magnetized cylindrical target. We have shown in Figure 1, the reaction rate vs. the temperature of the plasma. We presented in figure 2, the fraction of alpha particle energy deposited as a function of the applied magnetic field.

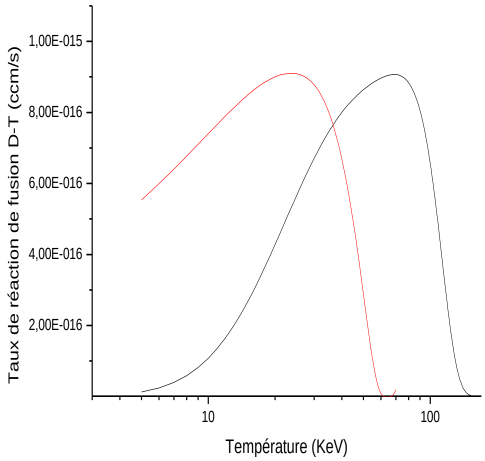

Figure 1
Fusion reaction rate vs. the fuel temperature
Red curve corresponds to the shielding potential
Black curve corresponds to the non shielding potential

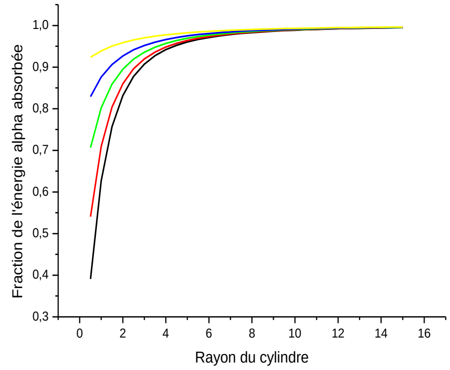

Figure 2
Alpha particle energy fraction deposited in the fuel vs. the cylinder radius reported to the alpha main free path; $\overline{R} = R/l_\alpha$ *, for several values of normalized magnetic field* $\overline{B} = \frac{2eR}{m_\alpha V_{\alpha 0}} B$.
Black curve: $\overline{B} = 0$; *Red curve:* $\overline{B} = 3$; *Green curve:* $\overline{B} = 5$; *Blue curve:* $\overline{B} = 7$ *and Yellow curve:* $\overline{B} = 1$

4. Conclusion

In order to well describe the thermonuclear fusion burn in a magnetized cylindrical target, a multi fluids model is proposed for the fuel plasma. The following physical effects are considered in this model: - the energy losses by bremsstrahlung emission and by thermal conduction, -the confinement of fuel plasma and the produced alpha particles by the applied magnetic field, -the self heating of the thermonuclear fuel and -the Debye shielding effect on the fusion reaction rate. The numerical resolution of the model equations permits to determine the spatio-temporel evolution of the fuel burning. It has been shown that the fraction of burned fuel undergoes an increasing due to the augment of the fusion reaction rate by the screening effect and due to the magnetic field.

References

[1] E. N. Avrorin et al., Sov. J. Plasma Phys. 10 (3), May-June 1984.
[2] A. I. Akhiezer and V.B. Berestetskii: Quantum Electrodynamics. New York, Interscience Publishers. (1965).
[3] MM Basko et al., Nuclear Fusion, Vol. 40, No. 1 (2000).
[4] S. I. Braginskii, in Review of Plasma Physics, Vol. 1 Consult. Bureau, N. Y., 205 (1965).
[5] L. M. Hively, Nuclear Techn. Fusion Vol. 3 (1983) pp. 199-200.