Two-Fluid Mechanism of Plasma Rotation in Field-Reversed Configuration*

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Abstract. End-shorting of the open magnetic field lines and particle loss are two mechanisms commonly considered to be sources of the ion toroidal spin-up in field-reversed configurations (FRCs). End-shorting mechanism requires conducting boundaries at the FRC ends, whereas the particle loss mechanism is related to loss of particles with preferential sign of toroidal velocity. An alternative spin-up mechanism, which does not require conducting boundaries or particle losses, is demonstrated. This mechanism relies on two-fluid description of plasma and finite parallel gradients of the plasma density. In particular, it is shown that the electron differential toroidal rotation results in generation of an anti-symmetric toroidal field, which in turn leads to the ion toroidal spin-up. Time scale of the toroidal field generation is comparable to the Alfven time scale for reasonable values of the parallel density gradient. In experiments, regions of parallel density gradient can be formed, for example, near the FRC ends, where the open-field-line plasma comes into contact with the wall, or during the FRC formation. Two-fluid simulations using the HYM code demonstrate that plasma toroidal velocity can be comparable to the ion diamagnetic velocity even in the simulations with periodic boundary conditions (i.e. no end-shorting) and without the particle losses.

1. Introduction

Toroidal spin-up in the ion diamagnetic direction, and subsequent excitation of the n=2 rotational mode are always observed in prolate FRC experiments. Average ion rotation rate observed in the experiments is comparable to the ion diamagnetic frequency, with typical spin-up time on the order of tens of Alfven times. Several spin-up mechanisms have been proposed and studied [1]. The two mechanisms commonly considered to be sources of the ion toroidal rotation in FRCs are end-shorting of the open magnetic field lines and particle loss mechanism [1]. End-shorting mechanism requires conducting boundaries at the FRC ends, where the tangential component of the electric field is zero, and force-balance condition, applied along the open magnetic field lines, requires finite ion toroidal flow velocity. This mechanism has been studied recently using 2D Hall-MHD simulations, which demonstrated good agreement with measured FRC rotation rates [2]. However, the end-shorting mechanism does not explain observed ion rotation in the experiments with the insulating end-walls.

The particle loss mechanism is related to loss of the ions with preferential sign of toroidal velocity, which has been studied using 2D hybrid simulations [3]. It was shown that particle loss from the open magnetic field-line region, as well as loss of weakly confined particles from the close-field-line region due to resistive decay of magnetic flux, leads to FRC rotation rates consistent with observations. However, a discrepancy between the experimentally observed spin-up times and experimentally measured particle loss times was found for larger FRC devices [1].

An alternative spin-up mechanism, which does not require conducting boundaries or particle losses, is demonstrated using 2D Hall-MHD simulations using the HYM code. This mechanism relies on two-fluid description of plasma and finite parallel gradients of the plasma density. In particular, it is shown that the electron differential toroidal rotation due to nonuniform electron density profiles results in a generation of toroidal magnetic field, which in turn leads to the ion toroidal spin-up.

2. Hall mechanism of toroidal field generation and ion spin-up

Assuming two-component plasma, quasineutrality condition, and zero initial ion flow velocity, it can be shown that equilibrium electric field can be written as $\mathbf{E} = \frac{\nabla p_i}{en}$, where $p_i=p_i(\psi)$ is the ion pressure, and ψ is poloidal flux. When the plasma density is also a function of ψ, the electric field is curl-free, and no magnetic field can be generated. In cases with finite parallel density gradients, we have

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \sim \nabla n \times \nabla \psi \cdot \frac{p'}{n^2}$$

$$\frac{\partial \mathbf{B}}{\partial t} \sim -\frac{p'}{n^2}(RB_0 \nabla_{\parallel} n)\hat{\phi},$$

where we have used $\mathbf{B} = \nabla \phi \times \nabla \psi,$ and $\nabla \psi = -R\hat{\phi} \times \mathbf{B}$, and φ is toroidal angle. It can be seen that finite parallel density gradient results in the generation of the toroidal component of the magnetic field. The time scale of the toroidal field generation is proportional to $\tau \sim (S^* L_n/R_c)t_A$, where L_n is the scale length of the parallel density gradient, S^* is the FRC kinetic parameter (ratio of the separatrix radius to the ion collisionless skin depth), and the Alfven time is defined as ratio of the flux conserver radius and the characteristic Alfven velocity, $t_A=R_C/V_A$. For L_n/R_c ~0.5 and $S^*=13$, the time scale τ is approximately 6-7 Alfven times.

Physical mechanism of toroidal field generation is related to the electron differential rotation. Since in the two-fluid model, the magnetic field is frozen into the electron fluid, and the electron angular rotation frequency $\omega_e \sim (dp/d\psi)/n_e$ is not a function of poloidal flux, the poloidal magnetic field lines will be twisted, producing finite toroidal field component near the FRC ends. The generated toroidal field is anti-symmetric relative to the FRC midplane, so that the net toroidal magnetic flux remains zero. Previous two-fluid simulations [2] have demonstrated how the generation of a toroidal magnetic field produces a torque on the plasma near the FRC separatrix, causing the ions to spin-up in the direction of the current (i.e. the ion diamagnetic direction).

In order to demonstrate that the proposed Hall mechanism of FRC rotation is not related to the applied boundary conditions, we have performed 2D Hall-MHD simulations using the HYM code with the periodic boundary conditions. In the case of periodic boundary conditions, the total angular momentum of the FRC plasma must be conserved (zero), so that the generation of positive (i.e. co-rotating) toroidal ion flows near the FRC midplane, must be accompanied by the corresponding negative rotation near the end-walls. This is an artifact of the periodic boundary condition model. In the general case, the angular momentum conservation can be written as:

$$\frac{dL}{dt} = 2\pi \int R^2 (B_z B_\phi - \rho V_z V_\phi) dR \Big|_{-Z_c}^{Z_c}$$

where the integration limits are from R=0 to $R=R_c$, and Z_c is the flux conserver half-length. It can be seen that any boundary conditions allowing angular momentum flux outflow from the simulation region will result in the net toroidal rotation. In particular, finite V_z at the end-walls (particle loss boundary condition) is sufficient for the generation of the net angular momentum.

3. Numerical model

Two-dimensional nonlinear numerical simulations have been performed using two-fluid version of the HYM code [3], which solves the MHD equations including the Hall term in the generalized Ohm's law. Conducting boundary conditions at the radial wall, and a choice of periodic or wall boundary conditions in axial direction have been used in the simulations. Grad-Shafranov solver was used to generate the initial conditions (Fig.1a), assuming zero initial ion velocity and the current carried by the electrons at t=0. Numerical simulations have been performed for following FRC parameters: S*=13, E=3.8, $S=10^3$, $Re=2\cdot10^3$, where E is the separatrix elongation, and S and Re are the Lundquist and Reynolds numbers respectively. Several set of simulations have been performed using different plasma density profiles for different values of the resistivity.

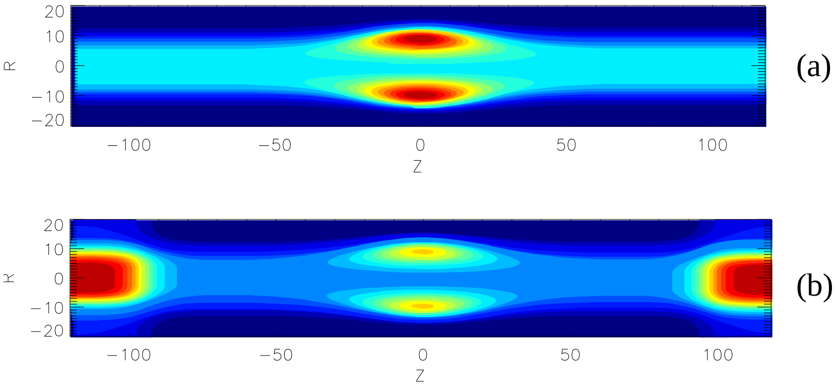

Figure 1. *Contour plots of (a) initial FRC pressure profile, (b) initial density profile from 2D Hall-MHD simulations of FRC spin-up.*

4. Hall-MHD simulations of generation of the toroidal filed and the ion rotation by parallel density gradients

A typical non-uniform density profile used in the simulations is shown in Fig. 1b. At the FRC ends, the plasma density is increased and temperature is decreased in order to model the cold-wall plasma boundary (Fig. 1b). The nonuniform initial density profile ($n_e \neq n_e(\psi)$) results in a significant shear in the electron angular rotation profile $\omega_e \sim dp/d\psi/n_e$ near the FRC ends. Oppositely-directed toroidal fields are initially generated near the end regions, but propagate eventually towards the midplane. Time evolution of the toroidal component of the magnetic field is shown in Fig 2. Peak amplitude of the generated toroidal field $B_\varphi/B_0=5.6\cdot10^{-2}$ occurs at $t\sim5t_A$. At later times, toroidal field profile broadens in both radial and axial directions, and its amplitude reduces to $B_\varphi/B_0=1.6\cdot10^{-2}$ as the configuration reaches a quasi-steady state at $t\geq30t_A$.

It is interesting that a relatively weak self-generated toroidal magnetic field can lead to a significant ion toroidal spin-up in the diamagnetic direction as shown in Fig.3. The spin-up mechanism is similar to the ‘sling-shot’ effect discovered in the counter-helicity spheromak merging studies [4], when a stretched and twisted poloidal magnetic field lines exert a toroidal torque on the ions through the **J**x**B** forces. If the periodic boundary conditions are applied, the total angular momentum of the system is still conserved, due to negative rotation near the FRC ends (Fig.3). In contrast, the free-outflow boundary conditions allowing a net flux of toroidal momentum out of the system result in a gain of angular momentum by the system and stronger ion spin-up.

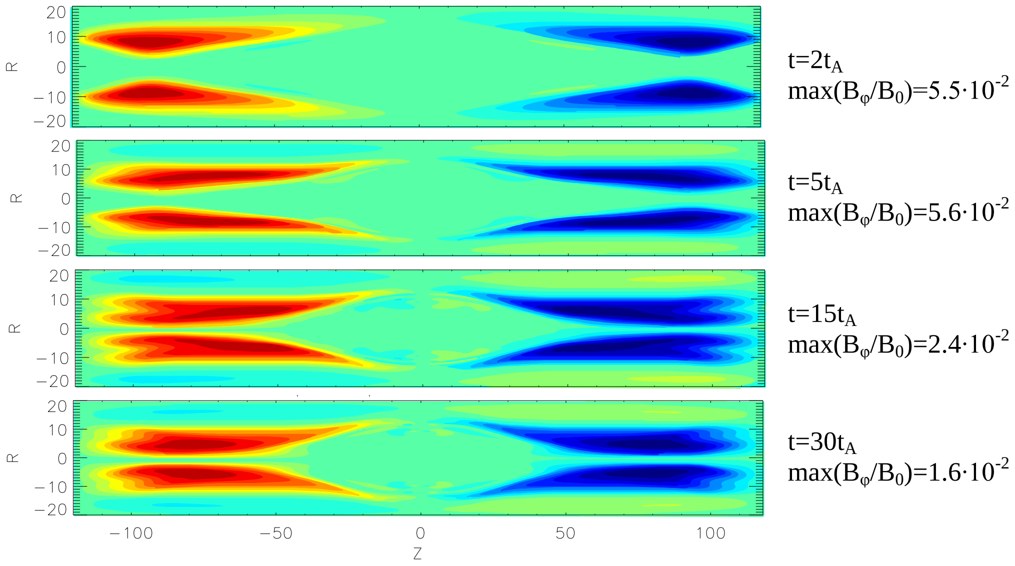

Figure 2. *Contour plots of self-generated toroidal magnetic field at $t=2t_A$-$30t_A$ from 2D Hall-MHD simulations of FRC spin-up including non-uniform density and periodic boundary conditions.*

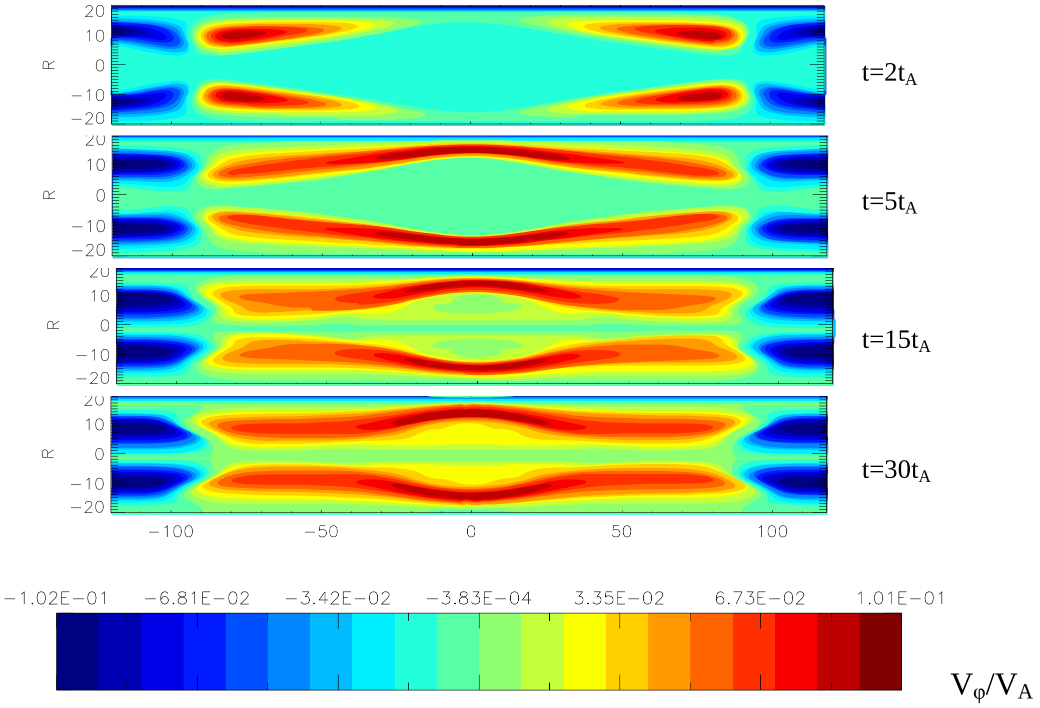

Figure 3. *Contour plots of ion toroidal flow velocity in poloidal plane from 2D Hall-MHD simulations of FRC spin-up including non-uniform density and periodic boundary conditions.*

Figure 4 shows radial profiles of plasma toroidal velocity at different times. The initial spin-up causes the ion flow in the open-field-line region near the separatrix ($t=5t_A$). However, the rotation extends to the plasma inside the separatrix region at later times ($t=30$-$50t_A$) due to finite plasma viscosity. The quasi-steady state is reached, when the rotation profile approaches a rigid-rotor profile (Fig. 4c). Peak value of the generated toroidal velocity is $V_\varphi \sim 0.1 V_A$, which is comparable to the ion diamagnetic drift velocity for FRC with $S^*=13$.

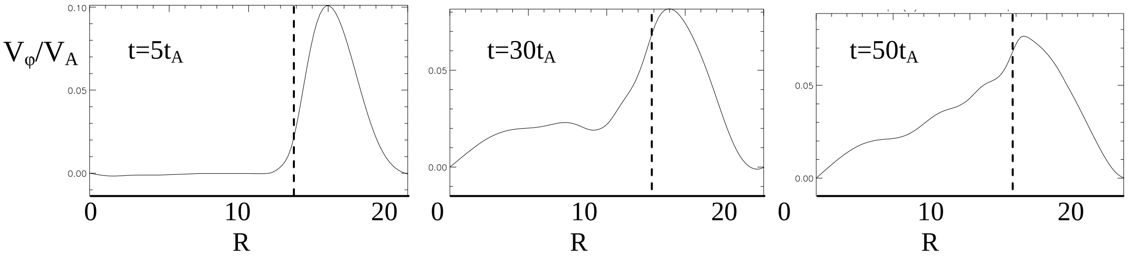

Figure 4. *Radial profiles of ion toroidal flow velocity for normalized viscosity value corresponding to the Reynolds number* $Re=2\cdot 10^3$. *The separatrix radius is shown by a dashed line.*

As shown in Section 2, the time scale of the toroidal field generation can be estimated as $\tau \sim S^* L_n/V_A \sim 6$-$7t_A$. Figure 5 shows that the ion spin-up time is approximately 5-10 Alfven times, consistent with the above estimate. Initially, only the open-field-line plasma rotates, whereas the spin-up time for the plasma inside the separatrix depends on the plasma viscosity. The rotation profiles inside the separatrix approach the rigid-rotor profile (Fig.4c) for normalized viscosity values corresponding to the Reynolds number $Re=10^3$. It is also found that the magnitudes of the generated toroidal field and the rotation profiles depend strongly on the Lundquist number S, and that the generation of significant torsional Alfven waves is possible for high values of S.

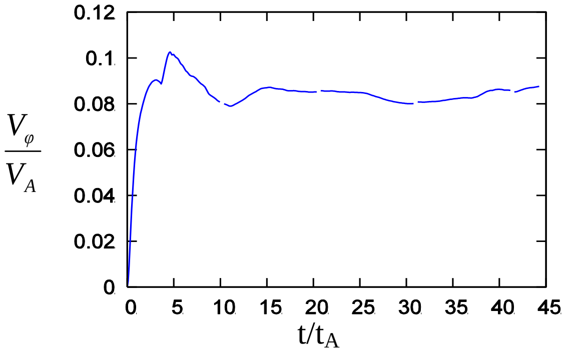

Figure 5. *Time evolution of peak toroidal velocity normalized by Alfven velocity from Hall-MHD simulations of FRC spin-up,* $t_A=R_c/V_A$.

Time evolution of the peak ion toroidal flow velocity is shown in Fig. 5. It is seen that the toroidal magnetic field and the ion flow generation is not a transient effect, and that the configuration reaches a quasi-steady state equilibrium with final toroidal fields and the ion flows, even though the initial non-uniform density profile was allowed to relax in these simulations. Maintaining a non-uniform density profile with higher plasma density (i.e. low temperature) near the FRC ends results in a higher spin-up rate. In addition, numerical simulations with non-uniform viscosity profile show that end-localized viscosity profiles have strong effect on long-time velocity evolution. Large plasma viscosity near the end region result in an increased plasma rotation, as well as a generation of the net toroidal angular momentum due to viscous decay of the negative rotation at the FRC ends.

5. Resistive relaxation of two-fluid plasma

Main difference in the resistive relaxation of the one-fluid resistive MHD and Hall-MHD plasmas is also related to the ion toroidal spin-up in the Hall-MHD model. In particular, a generation of toroidal rotation has been observed during 2D numerical simulations of the resistive relaxation of two-fluid FRC plasmas. Unlike in the previous section, the initial configuration in this case was chosen to be a true ideal MHD equilibrium with $n=n(\psi)$, and $\partial \mathbf{B}/\partial t=0$, and periodic boundary conditions. Evolution of the magnetic field configuration due to resistive decay results in the generation of the plasma poloidal flows mostly near the FRC separatrix and in the end regions. Related slow changes in the plasma density profile occur, so that the density is no longer function of the poloidal flux. The spin-up mechanism is then the same as described in previous section, however much smaller rotation rates up to $V_\varphi \sim 0.02 V_A$ have been obtained for relatively low plasma resistivity with $1/S=10^{-3}$.

6. Discussion

A generation of transient toroidal magnetic fields and the localized ion rotation during the tearing reconnection in early stages of the FRC formation had been studied before using the Hall-MHD model [5,6]. However, the importance of the electron density profile near the end-walls and the electron differential rotation for the FRC spin-up and the steady-state rotation had not been realized. The presented two-fluid simulations using the HYM code, demonstrate the FRC toroidal spin-up due to the plasma density profile effects and the Hall effects. It is shown that the conducting end-wall boundaries forcing $E_R=0$, are not necessary to explain the FRC toroidal rotation, which may explain the observed ion rotation in the experiments with the insulating end-walls.

Previous Hall-MHD simulation study of FRC spin-up due to end-shorting of the electric field has considered both perfectly conducting and the insulating boundary conditions at the FRC end walls [2]. It is interesting that in both cases similar spin-up rates have been obtained. In the case of perfectly conducting boundaries, the observed plasma spin-up was attributed to the end-shorting mechanism with $E_R=0$ at the ends. The insulating boundary was modeled by very large values of plasma resistivity and viscosity in the narrow layer near the end caps. The authors argued that in this case, both the radial electric field and rotation velocity must be small in the end regions, therefore the spin-up mechanism is the same as the end-shorting mechanism. It appears, however, that obtained plasma spin-up in Ref. [2] can be explained by the two-fluid effects and the density profile used in these simulations. Numerical simulation using the HYM code have been performed for similar electron density profiles and the FRC parameters (but with periodic boundary conditions) showing very similar rotation rates.

The two-fluid spin-up mechanism considered here is particularly important in experiments with small S* parameter (low plasma density), since in this regime the two-fluid effects will dominate. Recent RMF-FRC experiments have been operated with S*=0.1-2, i.e. very far from the MHD regime. Since any deviation of the plasma initial state from the MHD equilibrium will result in the shear in the electron angular rotation profile, it is expected that the two-fluid mechanism of generation of toroidal magnetic field and plasma rotation considered in this paper, will play a major role in dynamics of the low-S* devices.

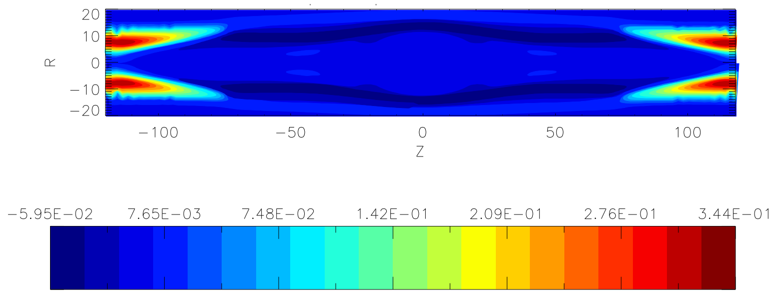

Figure 6. *Contour plots of ion toroidal flow velocity at* $t=15t_A$ *from 2D Hall-MHD simulations using the HYM code. Compared to the Fig. 3, the 'inversed' density profile is used, with smaller plasma density in the end regions.*

Reduction in the plasma rotation rate in the FRC can stabilize or at least delay the growth of the destructive n=2 rotational instability. Rotation control is also important in Magnetized target fusion (MTF) applications. Figure 6 shows that it is possible to reverse the direction of plasma rotation in FRC (counter-rotation) by the according changes in the initial density profiles. If the electron density profile can be controlled, it is possible to utilize the two-fluid rotation mechanism to counter-balance other ion spin-up mechanisms such as the particle loss mechanism.

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