Peculiarities of Destabilization of Alfvén Modes by Energetic Ions in Stellarators

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Abstract. Alfvén eigenmodes (AE) associated with the breaking of the axial symmetry in stellarators are considered. Specific calculations are carried out for the Helias reactor HSR4/18. An explanation of the temporal evolution of Alfvénic activity observed in experiments on W7-AS is suggested.

1. Introduction

Analysis of energetic-ion-driven Alfvén instabilities observed experimentally in stellarators is often based on a theory developed for tokamaks. This is explained by the fact that most works dealing with the theory of the Alfvén instabilities are relevant to tokamaks. However, in general, the physics of the Alfvén instabilities in stellarators differs from that in tokamaks in that the absence of axial symmetry strongly changes the structure of the Alfvén continuum, leading to additional gaps and new discrete eigenmodes inside these gaps [1–3]. Furthermore, specific wave-particle resonances do exist in stellarators, which may strongly enhance the instabilities and change the conditions of the wave destabilization [4]. The latter is associated with the fact that non-axisymmetric harmonics (e.g., the helical one) in the Fourier spectrum of the magnetic field strength often have a dominant influence on the particle motion in stellarators.

This work further develops the theory of Alfvén eigenmodes in helical systems, the specific examples being relevant to the Helias reactor HSR4/18 [5], and suggests an interpretation of some experimental observations of Alfvén instabilities in Wendelstein 7-AS.

2. Alfvén Eigenmodes in Ideal and Resistive MHD

The Wendelstein-line stellarators are characterized by weak magnetic shear, $\hat{s} \sim 0.1$. On the other hand, the dominant Fourier harmonics of the magnetic field strength ($\epsilon_B^{(\mu\nu)}$, with μ, ν the poloidal and toroidal coupling numbers) and the metric tensor ($\epsilon_g^{(\mu\nu)}$), which couple the Fourier components of the electromagnetic perturbation (E_{mn} and $E_{m+\mu,n+\nu N}$, with N the number of the magnetic field periods, m and n are the poloidal and toroidal mode numbers, respectively), are not small. The largest is the $\epsilon_g^{(21)}$ harmonic ($\epsilon_g^{(21)} \gtrsim 0.5$) associated with the rotation of the strongly elongated plasma cross section along the large azimuth of the torus [2]. Therefore, we have extended the analysis of Ref. [2] by keeping all the terms with $\epsilon^{(\mu\nu)}$ rather than only those terms with $\epsilon^{(\mu\nu)}$ which contain the second radial derivative of the perturbation. We have derived the equations for the Helicity-induced Alfvén Eigenmodes with $(\mu, \nu) = (2, 1)$, i.e., the HAE$_{21}$ modes (we follow the notations for Alfvén Eigenmodes introduced in Ref. [2]), and the Mirror-induced Alfvén

Eigenmodes, MAE, for which $(\mu, \nu) = (0, 1)$. In our calculations we took into account only two coupled harmonics.

To solve the derived equations, the code BOA-E (Branches of Alfvén modes - Extended), which extends the BOA code [2], has been developed. Below we present the results of calculations with the BOA-E code for the parameters of HSR4/18 [5].

At the beginning, we assume that the plasma is homogeneous, in which case the gaps in the Alfvén continuum are open. We have found that there are multiple discrete HAE_{21} modes below the upper continuum curve. This picture considerably differs from the spectrum obtained with the use of truncated equations [2]. The main difference is that the eigenfrequencies in the lower half of the gap disappear. In order to compare the results of ideal MHD with the results allowing for non-ideal effects, the spectrum of the HAE_{21} modes considered was calculated with the use of the resistive MHD equations derived in Ref. [6]. We found that the resistivity did not affect the real part of the frequency (ω) and led only to $\mathrm{Im}(\omega) \neq 0$.

The results obtained for MAE modes were surprising. Both calculations with the BOA-E code and analytical consideration showed that the MAE gap was "empty", i.e., there were no discrete modes inside it in the approximation of two coupled harmonics and the cold plasma. But later we took into account plasma pressure and found that finite pressure led to the appearance of discrete MAE modes, see Fig. 1.

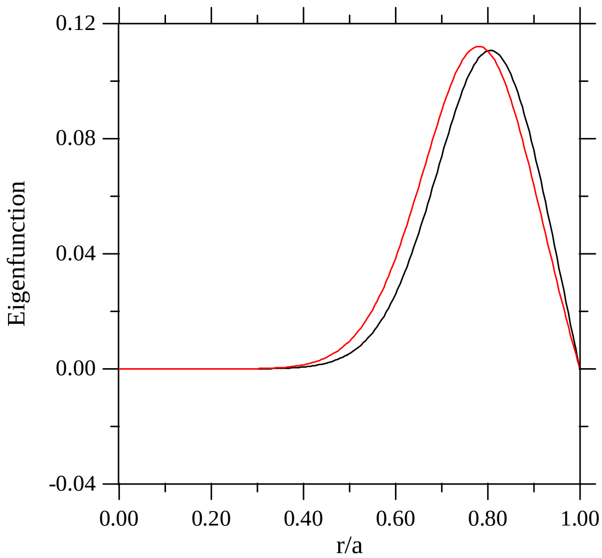

FIG. 1. MAE mode associated with the finite plasma pressure in HSR4/18 with a homogeneous plasma for the coupled $m = 9$, $n = 6$ and $m = 9$, $n = 10$ harmonics.

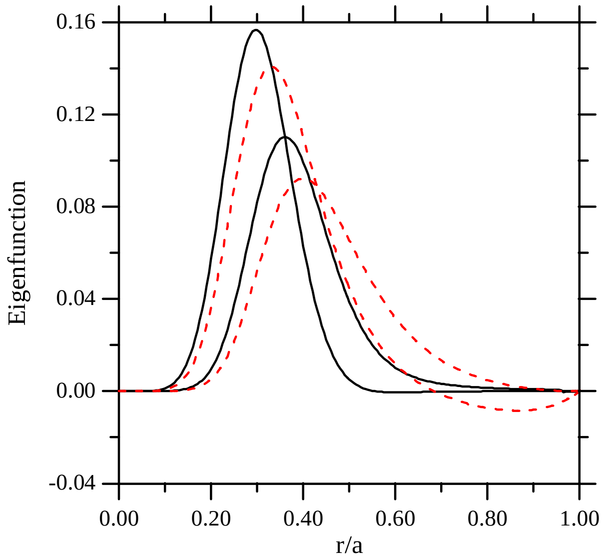

FIG. 2. The radial structure of the HAE_{21} mode with the normalized eigenfrequency $\lambda \equiv \omega R/v_A(0) = 1.6957$ (R is the major radius of the torus) for the coupled $m = 7$, $n = 5$ and $m = 9$, $n = 9$ harmonics in an inhomogeneous plasma (solid lines) and in a homogeneous plasma (dashed lines) of HSR4/18. The radial profile of plasma density was taken in the form $\rho(r) = \rho(0)[1+(rx_n^{-1}a^{-1})^{10}]^{-1}$, with $x_n = 0.7$.

Previously [2], we showed that the plasma inhomogeneity strongly affects AEs in low-shear systems, tending to kill the modes. New calculations with the BOA-E code confirmed this. But on the other hand, we found that there are certain mode numbers for which the gap in the Alfvén continuum in the two-coupled harmonic approximation is open, which helps the modes to survive even in realistically inhomogeneous plasmas. An example of such modes is given in Fig. 2. The presence of other harmonics coupled with the pair of harmonics considered closes the gap, leading to continuum damping. However, the continuum damping resulting from the satellite harmonics is much less than the damping existing when the gaps are closed in the two-harmonic approximation.

3. Estafette of Resonances During NBI Experiments on W7-AS

In NBI experiments on W7-AS the Alfvénic activity was strongly dependent on whether v_b/v_A (v_b is the beam particle velocity, v_A is the Alfvén velocity) is less or more than unity, which was not surprising (and followed from the tokamak theory). But till now it was not clear why in the regime with $v_b > v_A$ a small decrease of the Alfvén velocity strongly changed the character of MHD activity. This was the case in, for example, shot #43348 [7]. In this shot the ratio v_b/v_A varied approximately from 0.5 to 1.8. When v_b/v_A slightly exceeded unity, an Alfvén instability was observed at $\omega \sim 50\,\mathrm{kHz}$. Later, for a larger magnitude of v_b/v_A, the unstable waves were characterized by multiple frequencies in the range of $50-250\,\mathrm{kHz}$. After that all the instabilities disappeared, and, finally, when v_b/v_A reached the maximum magnitude (about 1.8), an instability with $\omega \sim 230\,\mathrm{kHz}$ appeared.

We will show that this picture can be understood if we take into account the structure of the Alfvén continuum and the fact that the fast ion drive strongly grows with the particle energy. First of all, we analyse the Alfvén continuum calculated with the code COBRA for shot #43348 (see Ref. [6]). We observe several gaps in the continuum in the frequency range of $50-500\,\mathrm{kHz}$. The discrete eigenmodes that may reside in them or the corresponding Energetic Particle Modes (EPM) may be destabilized by injected ions. A necessary condition for the destabilization is that these ions should be in resonance with the Alfvén waves. We use the resonance condition derived in Ref. [4], which we extended to include the so-called "compound resonance" introduced in Ref. [8]. A sufficient condition is that the instability drive should exceed the wave damping. Unfortunately, experimental data provide no information on the radial localization and the mode structure of the instabilities. Therefore, a detailed comparison of the theoretical and experimental results and, in particular, a calculation of the damping is not possible. Therefore, we introduced an adjustable parameter v_{min}, which is the minimum magnitude of the beam velocity required for the fulfillment of the condition $\gamma_\alpha > \gamma_{damp}$, where γ_α is the beam drive rate, γ_{damp} is the damping rate. In other words, we assumed that the instability is possible only when $v_{min} \leq v_\parallel^r \leq v_0$, where v_0 is the maximum velocity of the injected particles, $v_\parallel^r$ is the longitudinal velocity of the resonance particles.

The picture we obtained is shown in Fig. 3. It agrees with the experimentally observed evolution of the MHD activity described above. In the calculations we took $r_*/a = 0.3$ (r_* is the radius around which the mode is localized) and $v_{min} = 0.9v_0$. The latter assumption seems reasonable because in W7-AS the drive of particles with $v_\parallel = 0.9v_0$ is less than the drive produced by the particles with $v_\parallel = v_0$ by a factor of 2. (This estimate follows from $\gamma_\alpha \propto (v_\parallel^r)^7 f_b(v_\parallel^r)$ and the fact that in W7-AS the dependence of f_b on v is rather weak in the range of $26-50\,\mathrm{keV}$.) The main resonance velocities were used for all the modes except for the EAE modes, for which the compound resonance [8] with $\mu_s = 1$, $\nu_s = 0$ was used. The resonance numbers [4,8] $\mu_r = 1$ and $\nu_r = 0$ were chosen because the harmonic $\epsilon_B^{(10)}$ produced by toroidicity is the largest in W7-AS.

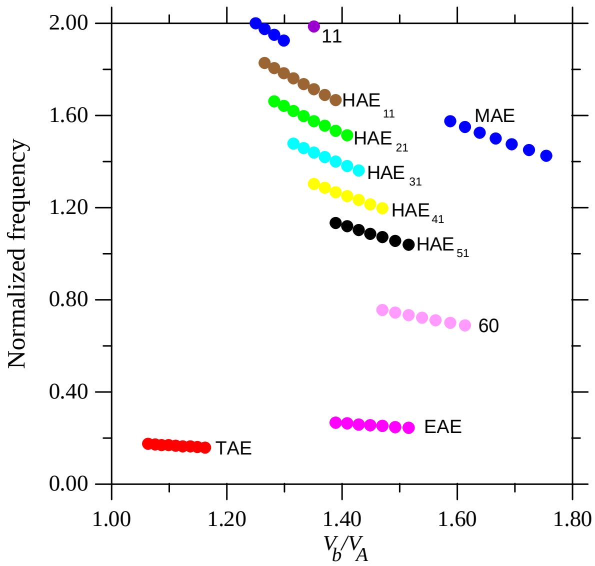

FIG. 3. Theoretically calculated dependence of the destabilized Alfvén spectrum on v_b/v_A in shot #43348 of W7-AS. This picture agrees with the temporal evolution of Alfvénic activity observed experimentally.

4. Summary

A new code, BOA-E (BOA extended), has been developed with the aim of calculating the AEs on the basis of the amended equations. Using this code, it is shown that when coupling parameters are relatively large, keeping all the terms with these parameters in the equation for AEs is of importance, which was demonstrated for the HAE_{21} and MAE eigenmodes. A general conclusion of Refs. [2,6] that plasma inhomogeneity tends to kill Alfvén eigenmodes with a global structure is confirmed. At the same time, modes that survive in realistically inhomogeneous plasmas are found.

An explanation of varying MHD (Alfvénic) activity observed in the Neutral Beam Injection (NBI) experiments on W7-AS [7] has been suggested. Namely, the temporal evolution of the Alfvén velocity leads to an “estafette” of resonances: Depending on the Alfvén velocity, various gap modes (TAE, EAE, HAE_{21}, MAE) or EPM with frequencies in the vicinity of these gaps are destabilized. At certain magnitudes of v_A, no modes can be in resonance with ions having an energy that is about the energy of the injected particles (50 keV), in which case the MHD activity disappears.

Recently (in July 2002) new experiments on Alfvén instabilities driven by energetic ions were carried out on W7-AS, where both low-frequency and high-frequency oscillations were observed. Their interpretation is in progress.

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