Feedback Control of Resistive Wall Modes in Toroidal Devices

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Abstract. Feedback of nonaxisymmetric resistive wall modes (RWM) is studied analytically for cylindrical plasmas and computationally for high beta tokamaks. Internal poloidal sensors give superior performance to radial sensors, and this is explained by the distribution of poles and residues for the transfer functions. A single poloidal array of feedback coils allows robust control with respect to variations in plasma pressure, current and rotation velocity. The control analysis is applied to advanced scenarios for ITER. Studies are also shown of configurations with multiple poloidal coils and of feedback systems for nonresonant MHD instabilities in reversed field pinches.

1. Introduction

We study feedback control of RWM's, starting from open loop transfer functions for the plasma response in the frequency domain. The open-loop system is shown in Fig. 1. Here, s is the Laplace transform variable, I_f the current in the active coils, V_f the voltage applied to the active coils, and L_f is the self-inductance, and R the resistance of the active coils. V_s is the voltage measured by the sensor loop, M_{sf} a nominal mutual inductance between the active coil and the sensor loop, and Ψ_s the magnetic flux through the sensor loop. The non-dimensional transfer function $P_1(s)$ gives the response at the sensor loop of the plasma-wall system to currents in the active coils, and $P_2(s)$ is the normalized loaded self-inductance of the feedback coils.

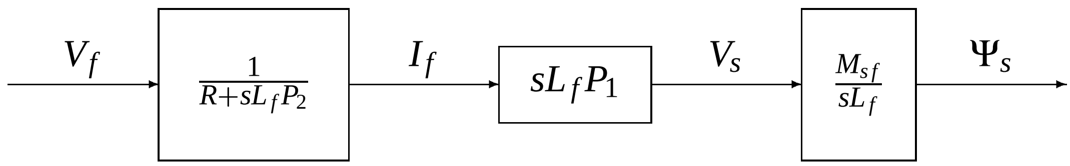

FIG.1. Open loop of the resistive wall mode control scheme.

For *current control*, the loop is closed between I_f and Ψ_s. The normalized flux in the sensor loops Ψ_s/M_{sf} is amplified by the controller $K(s)$ and fed back with the opposing polarity, so that $I_f = -K\Psi_s/M_{sf}$, where K is the feedback gain. The growth-rates of the closed-loop current control system satisfy the characteristic equation $1+K(s)P_1(s) = 0$.

For *voltage control*, the loop is closed between V_f and V_s. The voltage applied to the active coil is $V_f = s\Psi_f + RI_f = (sL_fP_2 + R)I_f$, and the characteristic equation for the closed loop system is $1+K(s)P_1(s)/[P_2(s)+1/s\tau_f] = 0$, with $\tau_f = L_f/R$. The transfer functions $P_1(s)$

and $P_2(s)$ can be constructed analytically for cylindrical equilibria [1], and computationally with the MARS-F code for two-dimensional toroidal equilibria [2].

2. Pole-residue Expansion of Transfer Functions and Robust Control

Both toroidal computations [2,3] and experiments [4,5] have shown that better control can be achieved with internal poloidal sensors than with radial sensors. This can be understood from the distribution of poles and residues of the transfer function $P_1(s)$. Cylindrical theory [1] shows that P_1 can be represented as a pole-residue expansion $P_1(s) = \sum_{i=0}^{\infty} R_i/(s-\gamma_i)$. The poles $\{\gamma_i\}_{i=0}^{\infty}$ are the RWM growth rates in the absence of feedback (generally, only *one* is positive). The residues $\{R_i\}_{i=0}^{\infty}$ reflect the reaction of each eigenmode to the feedback current. For toroidal equilibria, the residues can be computed by studying the closed loop eigenvalues for current control with small gains [6].

Figure 2 shows the poles and residues for a toroidal case with poloidal and radial sensors. Similar patterns are found in cylindrical theory [1]. The exact response function involves infinitely many terms, but a three-pole approximation, indicated by crosses in Fig. 2, generally gives an excellent approximation for real frequencies. Poloidal sensors give two advantages over radial ones: first, the residue connected with the unstable RWM is dominant for poloidal sensors. Secondly, there is considerable cancellation between the contributions from the stable modes. Both these properties reduce the undesired coupling of the feedback to the stable modes, and this facilitates control.

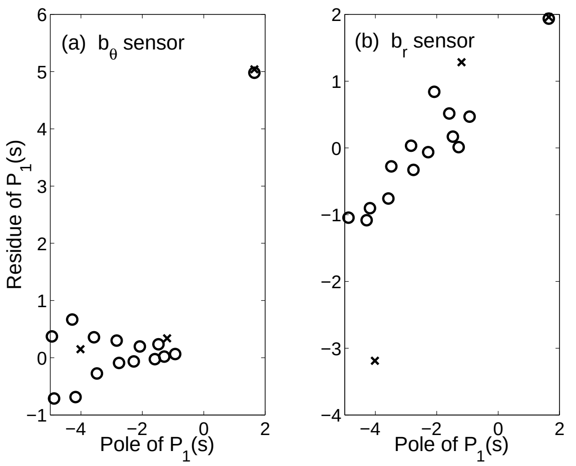

FIG.2. Poles and residues of complete transfer function $P_1(s)$ ("o") and three-pole approximation ("x").

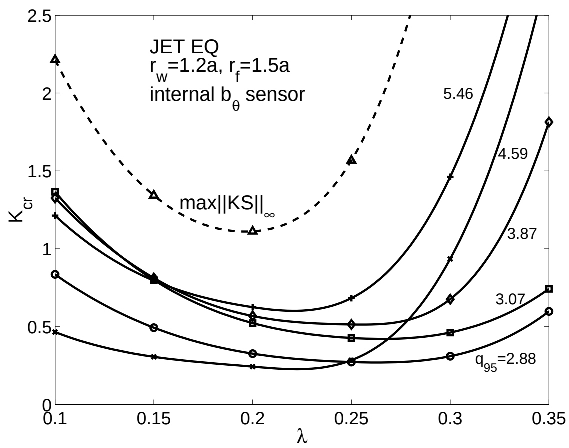

FIG.3. The minimum gain for stability with proportional current control (solid) versus poloidal width λ of the active coils for different q_{95}. Dashed line: maximum control activity for a single controller optimized for all equilibria.

Robust control of the RWM with respect to variations in the plasma parameters is rather easily achieved with internal poloidal sensors [7,6,3]. In [7], we showed that a single controller can control plasmas with β_N above no-wall limit by 63%. A controller designed for a high pressure works also for lower pressure, so that dynamic tuning of the control is not necessary [7]. We also studied robustness to variations in plasma current and toroidal rotation. We examined five advanced equilibria of JET shape, that are unstable to the $n=1$ RWM. These equilibria all have the same profile for the parallel current density, but different total current and $q_{\min}$. In each equilibrium, the pressure was raised until the $n = 2$ or $n = 3$ external mode reached the no-wall beta limit. The normalized plasma current varies by almost a factor of two. The critical gain for proportional current control needed to stabilize these equilibria is plotted in Fig. 3. The solid curves show the critical gain versus the fraction λ of the poloidal circumference subtended by the feedback coil array. The radii of the wall and sensors are $r_w = r_s = 1.2a$, and for the feedback coils the radial position is $r_f = 1.5a$. The optimum coil width decreases with increasing safety factor q. For a broad range of λ all equilibria can be well controlled. To achieve robust control, a single PD controller has been optimized for all the equilibria, by minimizing the control activity $J_u = \max_{\omega\in R}|K(j\omega)S(j\omega)|$ with a constraint on the stability margin $J_S \equiv \max_{\omega\in R}|S(j\omega)| < 2$, where $S(s) \equiv 1/[1+K(s)P(s)]$ is the sensitivity. The dashed line shows the maximum control activity J_u over *all* the equilibria. The robustness against variations in q can be understood by noting that for all the different q_{95}, the unstable RWM has a similar mode structure, with strong ballooning on the low field side [6,3].

With poloidal sensors, the feedback system can also be made robust with respect to toroidal plasma rotation. We studied a JET equilibrium with uniform toroidal plasma rotation, and different rotation frequencies $\omega_0/\omega_A = 0$, 0.02, 0.04, and 0.05. For all these rotation frequencies, a single proportional current controller $K = K_p = 0.63$ gives stability and good performance $J_S < 1.5$, $J_u < 0.9$. The feedback system works best, if a toroidal phase shift is introduced such that the feedback system pushes the mode rotation in the same direction as the plasma flow.

3. RWM Control for ITER

In [1], an extensive study of RWM feedback for ITER-like double wall structure and superconducting coils was made using cylindrical theory. Here we report new results from toroidal computations for ITER parameters: an advanced 9MA steady state (Scenario 4). We have used a best up-down symmetric fit of the plasma shape, with two conformal resistive walls at radii $r_1 = 1.375a$ and $r_2 = 1.725a$, respectively. The wall time for each wall is 0.15 s. The active coil is placed at $r_f = 3.0a$, covering about 12.5% of the total poloidal circumference ($\lambda = 0.125$), and β_N is 15% over the no-wall limit, half-way between the ideal-wall and no-wall limits.

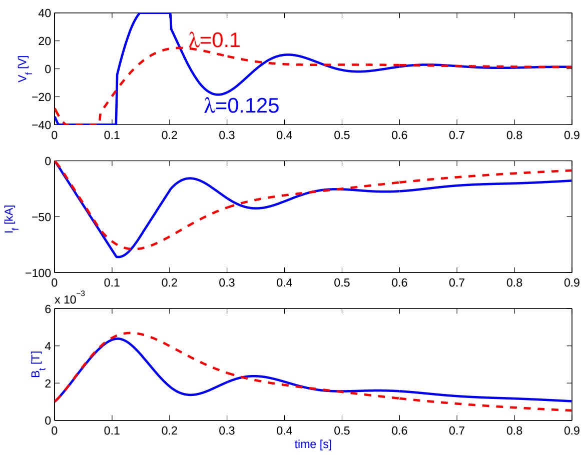

FIG.4. The time responses of the amplifier voltage V_f, feedback current I_f, and the detected poloidal field perturbation B_t for ITER design.

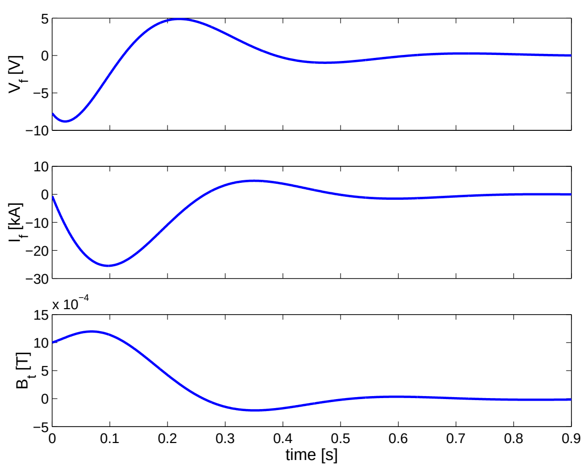

FIG. 5. Active coil voltage and magentic field when the coil is close to the outer wall $r_f = 1.75a$.

We have optimized proportional controllers to stabilize the RWM for the ITER plasma, with the designed active coil width $\lambda = 0.125$ and a slightly smaller value $\lambda = 0.1$. We consider voltage control and minimize the maximum voltage of the time response. (For this coil configuration, $P_2 \simeq 1$, and the inductance of the feedback coil is only weakly affected by the distant wall and plasma.) The time responses of the amplifier voltage V_f, feedback current I_f, and the detected poloidal field perturbation B_t, are plotted in Fig. 4. The feedback system is turned on when the detected signal exceeds 1 mT, and a voltage saturation level of 40 V is assumed. The comparisons show that feedback coil of width $\lambda = 0.1$ gives a better time response than the ITER original design $\lambda = 0.125$.

We note that the maximum voltage depends strongly on the radial position of the active coils. The maximum voltage can be significantly decreased, if the active coils are placed closer to the plasma. Figure 5 shows time responses for the wall positions $r_1 = 1.3a$, $r_2 = 1.55a$ and feedback coil radius $r_f = 1.75a$.

4. Feedback with Multiple Poloidal Coils

We have also investigated feedback with several sets of active and sensor coils along the poloidal angle, which is a multiple-input-multiple-output (MIMO) control system. We have developed cylindrical theory, similar to the single coil case [1], to generate transfer function *matrices* and carried out controller optimization by minimizing the controller activity, subject to constraints on control performance. The results for *poloidal* sensors show that good control can be achieved when the neighboring coils overlap only slightly, and also when they are well separated. However, with a simple control system, the results are not improved from the case of a single poloidal array. By contrast, the MIMO system makes it possible to stabilize RWM with *radial* sensors, which is generally not possible with a single coil array.

We have also constructed transfer function matrices for the MIMO control of toroidal equilibria from MARS-F computations, and optimized a system of three identical PD controllers applied *independently* to the three corresponding sets of feedback coils. The results for poloidal sensors are summarized in Table 1, together with results for a single array (SISO). The single coil system achieves better performance, with less control activity than the diagonal MIMO system. We find similar results for multiple-input-single-output (MISO) control systems.

Table 1: *Controller optimization results for a JET-shaped advanced equilibrium. The MIMO system with three coil arrays is compared with the SISO system for a single coil array.*

	K_p	J_S	J_u
MIMO	0.62	2.11	1.32
SISO	1.35	1.00	0.98

5. RWM control for Reversed Field Pinches

Feedback control of nonresonant RWM in RFP's is more difficult, because there are many instabilities with different toroidal numbers n, that need to be stabilized simultaneously. These multiple instabilities can be decoupled by using a sufficient number of evenly spaced feedback coils in both the poloidal and toroidal directions [8,9]. Therefore, an easy way is to stabilize each unstable mode (m,n) independently, using single-input-single-output control system.

We have studied model equilibria with zero pressure and $\nabla \times \mathbf{B} = \sigma(r)\mathbf{B}$ with $\sigma(r) = 2\Theta_0/a \times (1-(r/a)^{\alpha})$ and computed the RWM growth rates γ_{mn} for two equilibria with $\alpha = 3,\ a/R = 0.25$ and $\Theta_0 = 1.71,\ \Theta = 1.58,\ F = -0.2$, or $\Theta_0 = 1.85,\ \Theta = 2.0,\ F = -0.80$, respectively. The unstable modes are $-6 \le n \le +3$ for the low-Θ, and $n = -7, +1 \le n \le +4$ for the high-Θ equilibrium. In order to stabilize all the unstable modes, it is necessary to have at least $M = 3$ coils in the poloidal and $N = 12$ coils in the toroidal direction. However in order to avoid coupling to resonant, resistive "dynamo" modes, we choose $M = 4$ and $N = 24$. The wall is placed at $r_w = 1.1a$, and the active coils at $r_f = 1.25a$. The plasma response model for an RFP is obtained in a similar way as for the cylindrical tokamak. We applied proportional current control, with different sensor types, to stabilize the modes. Figure 6 shows the limiting curves for the critical gain as function of the toroidal angle $\Delta\phi_f$ spanned by each active coil for the two equilibria. The lower and upper limits are constructed by considering the most stringent condition imposed by all the unstable modes. For non-overlapping coils ($\Delta\phi_f \le 15^o$) the poloidal and toroidal sensors can operate within a larger range of coil widths than the radial sensors. Moreover, poloidal sensors have the advantage of not presenting an upper limit to the gain. But in practice the toroidal sensors may be preferred because they give a better signal/noise ratio in an RFP. In general there exists a window in coil width where each of the sensors can stabilize all the modes if the gain is chosen appropriately.

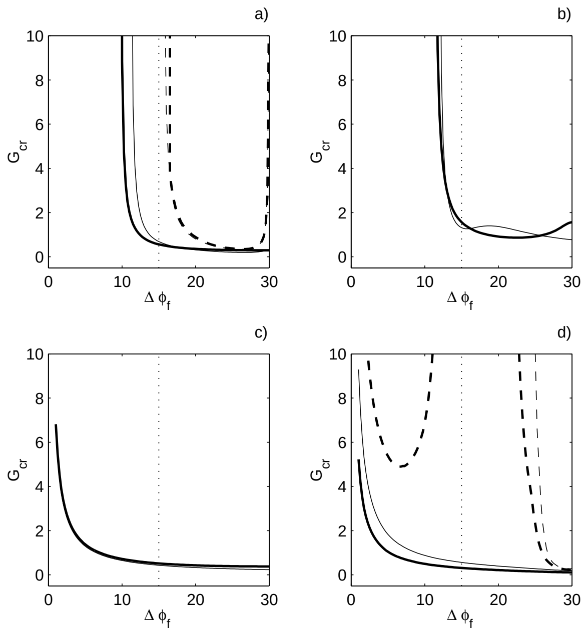

FIG.6. Minimum (solid) and maximum (dashed) gain G_{cr} necessary to stabilize all the modes, as function of the toroidal width of the active coils, for (a) radial small, (b) radial wide, (c) poloidal internal, (d) toroidal internal sensors. Bold lines refer to the equilibrium $\Theta_0 = 1.71$ and normal lines to $\Theta_0 = 1.85$. Dotted lines corresponds to complete coverage without overlapping. $M = 4, N = 24$ and $\Delta\theta_f = 90^o$.

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