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MODELLING OF ELM-LIKE PHENOMENOLOGY VIA MIXED SOC-DIFFUSIVE DYNAMICS

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ABSTRACT: A sandpile model describing some of the features of plasma turbulent transport dynamics in L-mode is extended to exhibit a transition to enhanced confinement modes. ELM-free and Type-I ELM-like H-modes can both be obtained. Each exhibits features reminiscent of what is observed in confined plasmas. The interplay between an avalanche and a diffusive transport mechanism is shown to be essential, in this context, for the system to display periodic edge ELMing.

1 Introduction

A few years ago, self-organized criticality [1] (SOC) was proposed as a possible paradigm for plasma turbulent transport dynamics in fusion devices [2,3]. SOC systems have the ability to organize themselves and to fluctuate around a state marginal to a major disruption, exhibiting scale-free transport. The key ingredient for the appearance of SOC is the existence of two disparate time scales, respectively associated with the drive and with the instability relaxation. Since this condition is usually fulfilled in systems with instability thresholds, it was proposed that SOC might apply to a plasma confined in L-mode [3]. The idea of the plasma staying in a SOC-like state is attractive, since it can help in understanding several experimental facts encountered in L-mode, such as the Bohm scaling of the confinement time, the existence of canonical profiles and the superdiffusive propagation of cold and heat pulses. Some experimental evidence consistent with this idea has already been reported. It claims self-similarity for electrostatic edge fluctuations [4] and the radial propagation of avalanche-like events in L-mode discharges [5].

The existence of a H-mode confinement state and its transition from the L-mode opens a new challenge to the SOC transport paradigm. Quasi-periodic oscillations have been found in SOC systems [6-8], but their relevance to the H-mode is still unclear. Therefore, in this paper, we explore whether the sandpile model for the L-mode can be extended, in a physically meaningful way (for fusion plasmas) so that it exhibit a transition into H-mode. This extended model can then provide us with a highly simplified system, in which the interaction of the physical mechanisms thought relevant can be isolated and studied, helping to shed some light on the relevance (if any) of SOC in this context.

2 The diffusive sandpile: a model for L-mode dynamics

We use a running diffusive sandpile [6] as the simplest model that captures the essential dynamical features of plasma transport in L-mode. The sandpile consists of L cells, labeled by the index $n \in [1, L]$. Each cell stores an amount of sand h_n. U_0 grains of sand are dropped randomly on every cell at each iteration with probability P_0. The external drive per cell is thus $S_0 = U_0 P_0$. SOC dynamics appear because of the existence of a critical slope Z_c that, when locally overcome, triggers the removal of N_F grains of sand to the next cell. To this avalanche transport channel, a second channel is added. A local diffusive flux Γ is computed at each cell as $\Gamma_n = D_0\,(Z_{n+1} - Z_n)$, where D_0 is the diffusion coefficient and $Z_n = h_{n+1} - h_n$ is the local slope. Finally, the sandpile has an open boundary at $L = N$, from which sand is removed.

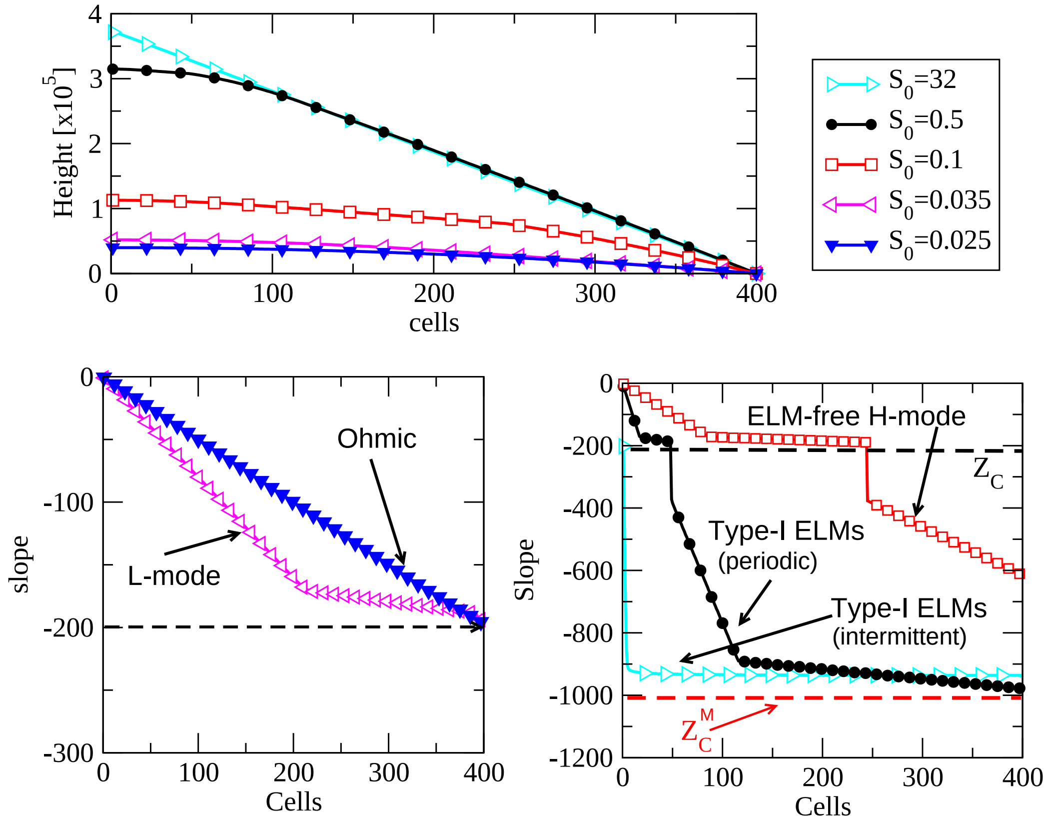

Fig. 1. Height and slope profiles of the diffusive sandpile as the external power is increased, showing the path of the system across the different regimes.
(Parameters: $D_0 = 0.05; L = 400; Z_c = 200; N_f = 30; Z_c^M = 1000; N_F^M = 120; k = 10$)

A typical slope profile of the diffusive sandpile is shown in the lower left part of Fig. 1. It consists of two regions: 1) a diffusive core (with a linear slope profile) where transport is dominanted by the diffusive channel, and 2) a SOC region (with a constant slope just below Z_c), where transport is mainly driven via avalanches. Interestingly, For $S_0 > D_0(Z_c - N_F/2)/L$, this sandpile has many of the dynamical characteristics of L-mode: transport in the SOC region is scale-free, with contributing scales only limited by the system size; the slope profile stays on average very close to Z_c; finally, any perturbation of the slope profile can propagate ballistically (if $\Delta Z >> N_f/2$) or diffusively (if $\Delta Z << N_f/2$). On the other hand, for $S_0 < D_0(Z_c - N_F/2)/L$, the system has an *ohmic* phase, where the slope profile is determined by the strength and distribution of the source.

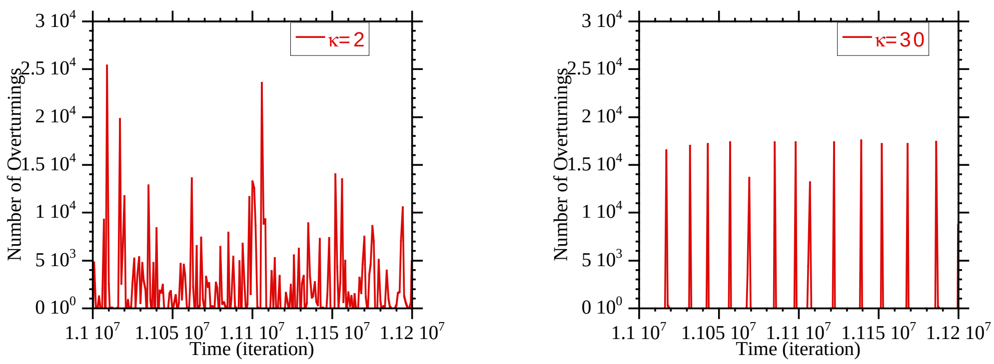

Fig. 2. Time traces of avalanche activity in the sandpile below (left; $D_0 N_F^2/S_0 = 2$) and above (right; $D_0 N_F^2/S_0 = 30$) the dynamical transition to quasi-periodical edge discharges.

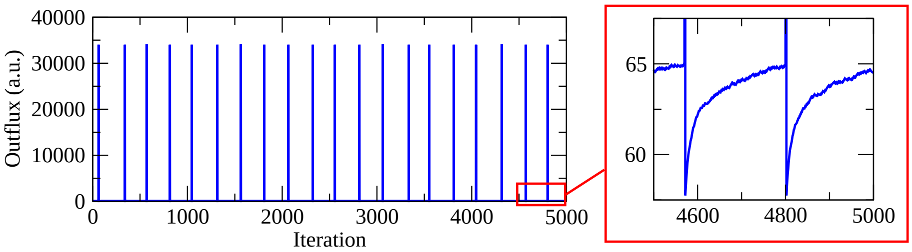

Fig. 3. Time traces of the flux leaving the sandpile for $S_0 = 0.5$ in the Type-I ELMy H-mode.

Since the relevant limit in fusion plasmas is $D_0/P_0 << 1$, it might seem that this sandpile does not intrinsically differ from the standard non-diffusive sandpile [3]. However, even in this limit it can be made that $\kappa \equiv D_0 N_F^2/P_0 >> 1$. In this case, the non-zero diffusion allows access to a very interesting new regime [9]. In it, diffusion is not yet important enough to dominate the total transport (still mainly driven through the avalanche channel), but it can erase all inhomogeneities in the slope profile so efficiently that SOC disappears. In its place, a quasi-periodic relaxation of the edge that empties the SOC region with frequency $\nu \sim S_0/N_F$ balances the incoming flux to maintain steady-state (see Fig. 2). As we will see, this transition may open a possible route to ELM-like relaxations once the system has entered into H-mode.

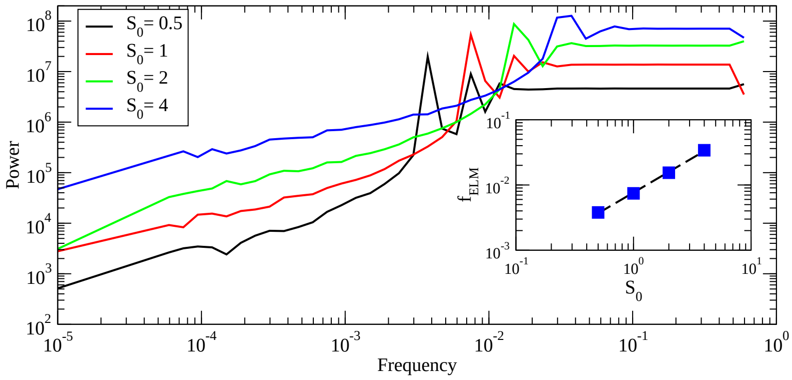

Fig. 4. Power spectra of outfluxes in ELMy H-mode. Inset: ELMing frequency scaling with S_0.

3 Physics-based extension of the SOC-diffusive sandpile

In tokamak plasmas, H-mode is achieved when the external power exceeds the threshold value for the L-H transition [10]. A transport barrier then forms at the edge that is thought to be caused by the sheared-flow-induced suppression of turbulence. After the transition, the confinement time seems to follow a gyro-Bohm scaling, at least in its ion channel. Once the system is in H-mode, several relaxation processes successively appear as power increases [11]. Type-III ELMs appear first. These are small periodic relaxations with frequency [amplitude] that decreases [increases] with the external power. As power continues to increase, type-III ELMs give way to an ELM-free phase. Finally, at even larger powers, large type-I ELMs appear, with a frequency that increases with power. The standard view of these phenomena relates type-III ELMs to resistive modes, while type-I ELMs are associated to the much faster ideal ballooning modes.

To allow our sandpile to have a transition into some kind of H-mode, we have simulated the suppresion of local transport by a turbulence-generated shear-flow by varying the relaxation rule for those cells in which the slope must stay above Z_c to balance the external power. These cells begin to exist as soon as $S_0 > S_0^{L-H}$, being $S_0^{L-H} \sim (N_F - D_0 Z_c)/L$. The new relaxation rule for any supercritical cell is to substitute the transfer of N_F grains to the next cell by a 'turbulent' diffusive flux, with diffusion coefficient given by $D_t = N_F/(kZ_c)$, that is added to the ambient diffusion given by D_0. The effective diffusivity is then $D_e = D_0 + D_t$. Note that with this prescription, transport is reduced by k times (relative to the normal overlapping rule) when the slope is close to Z_c. But at the same time, the diffusive turbulent flux brings into the dynamics the radial transport decorrelation associated with the shear-suppression mechanism, which is suspected to be responsible for the transition of the confinement time towards gyro-Bohm scaling.

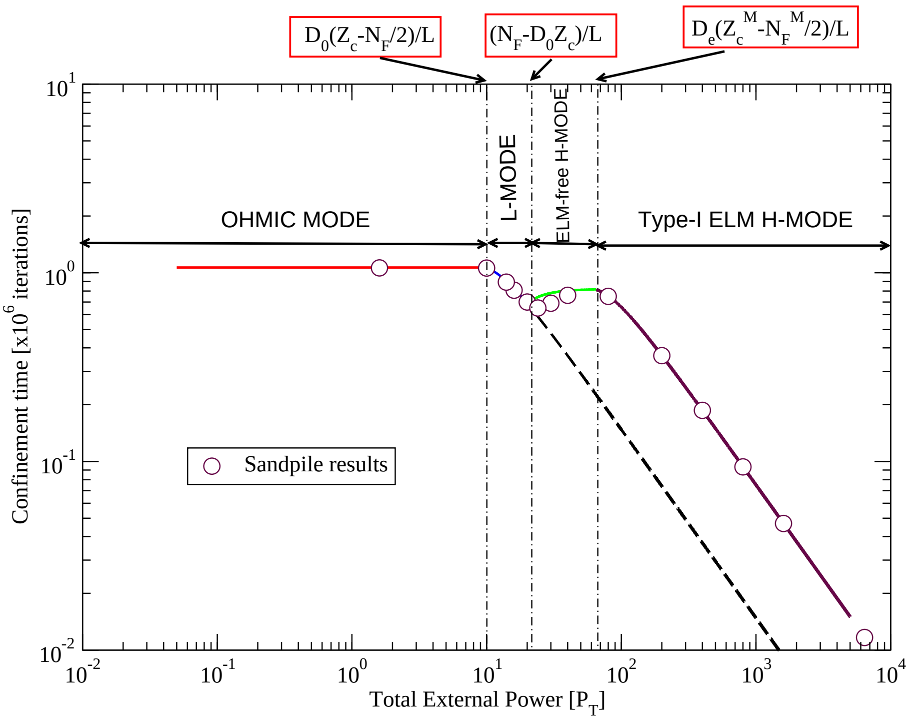

Fig. 5. Comparison of the analytic formulae for τ_E with numerical calculations. Dashed line extrapolates L-mode beyond the power threshold for the L-H transition for easier comparison.

With this modified rule, a 'diffusive pedestal' is first formed just after $S_0 > S_0^{L-H}$ (see open squares slope profile in Fig. 1). This new regime will be called the sandpile ELM-free H-mode, since confinement (as shown in Sec. 4) improves but no ELMs are yet present. But a sandpile type-I ELMy H-mode can also be obtained at even larger powers, if a second critical gradient, $Z_c^M >> Z_c$, is introduced (and its associated $N_F^M >> N_F$). Then, for $S_0 > D_e(Z_c^M - N_F^M/2)/L$, the sandpile enters into type-I ELMy H-mode. Since in a real plasma, type-I ELMs are associated with ideal MHD temporal scales [10], their much faster time scales are included by relaxing them differently: instead of constraining the relaxation to affect only one cell per iteration, the relaxations relative to Z_c^M will be continued *in the same iteration* until no more free energy is available. In this way, after an ELM-free H-mode phase, a second (and much steeper) pedestal is formed (see the profile in filled circles in Fig. 1), with type-I ELMS being triggered whenever Z_{edge} reaches Z_c^M. As an example, the temporal trace of the flux leaving the sandpile for $S_0 = 0.5$ is shown in Fig. 3. Clearly, a periodic relaxation takes place and between ELMs, an increasing diffusive flux (see zoom of the same figure) reveals the increase of Z_{edge} towards Z_c^M. In ELMy H-mode, the frequency of the ELMs increases linearly with S_0, as shown in Fig. 4.

The fact that such a strong ELM periodicity is observed is, however, unexpected, since the access to the second critical gradient should in principle yield scale-free SOC transport. The hidden cause for this strict periodicty is found in the large value of $D_e(N_F^M)^2/S_0$ (even when $D_e/S_0 << 1$). This comes from the large N_F^M associated to the ideal mode. Therefore, it turns out that the SOC pedestal is in a regime similar to that previously found in the diffusive sandpile, where quasi-periodic edge-triggered relaxations take over the transport dynamics of the system from the scale-free avalanches (see Fig. 1). This is confirmed by many diagnostics, in particular by the linear slope profile found all across the pedestal [9]. In fact, a different ELMy H-mode with intermittent scale-free relaxations is also possible at much higher powers (see right triangle slope profile in Fig. 1), when the average constant slope and the intermittent edge relaxations characteristic of SOC are recovered.

4 Scaling of the diffusive sandpile confinement time

Finally, we will estimate the scaling of the confinement time τ_E of the sand stored in the system with respect to the relevant parameters (D_0, $D_e \equiv D_0 + D_t$, L, $P_t \equiv S_0 L$, Z_c, Z_c^M, N_F and N_F^M) to ascertain to what extent the sandpile H-mode found here is an enhanced confinement regime. To help the interpretation of the results, we will formally use $\Delta x(=1)$ as the size of a cell, and define $\rho = \Delta x/L$. Finally, as a working definition we use here $\tau_E \equiv M/P_T$, where M is the total sand stored, and $P_T \equiv S_0 L$ the total external power. Since the profiles in all regimes can be estimated analytically in the relevant regimes, τ_E can be approximated as:

$$\begin{aligned}
\tau_E^{\mathrm{Ohmic}} &= \frac{L^2}{3D_0} \sim \rho^{-2}, \\
\tau_E^{\mathrm{L-mode}} &= \frac{(Z_c - N_F/2)L}{2S_0} - \frac{D_0^2(Z_c - N_F/2)^3}{6S_0^3 L} \sim \rho^{-1} P_T^{-1} L, \\
\tau_E^{\mathrm{ELM-free\ H-mode}} &= \frac{L^2}{3D_e} - \frac{(N_F - D_0 Z_c)^2}{2S_0^3 L}\left[\frac{2(N_F - D_0 Z_c)}{3D_e} - (Z_c - \frac{N_F}{2})\right] - \\
&\quad - \frac{D_0^2(Z_c - N_F/2)^3}{6S_0^3 L} \sim \rho^{-2} \qquad (1) \\
\tau_E^{\mathrm{ELMy\ H-mode}} &= \frac{(Z_c^M - N_F^M/2)L}{2S_0} - \frac{D_e^2(Z_c^M - N_F^M/2)^3}{6S_0^3 L} - \frac{D_0^2(Z_c - N_F/2)^3}{6S_0^3 L}, \\
&\quad - \frac{(N_F - D_0 Z_c)^2}{2S_0^3 L}\left[\frac{2(N_F - D_0 Z_c)}{3D_e} - (Z_c - \frac{N_F}{2})\right] \sim \rho^{-1} P_T^{-1} L.
\end{aligned}$$

These estimations agree very well with the numerical results for all regimes, as shown in Fig. 5. From them, it is clear that τ_E has a gyro-Bohm scaling for both the Ohmic phase and ELM-free H-mode, as reflected by $\tau_E \sim \rho^{-2}$, which implies that the transport process takes place through steps of size ρ, but has zero mean and finite variance (as any diffusive process). However, in L-mode and in ELMy H-mode, the scaling becomes Bohm-like, being proportional to ρ^{-1}. This is characteristic of a process with non-zero mean displacement. Finally, power degradation is observed in L-mode and ELMy H-mode. But confinement is (Z_c^M/Z_c) times better in ELMy H-mode than in L-mode for the same P_T.

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