Nonlinear MHD and Energetic Particle Modes in Stellarators [1]

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Abstract. The M3D (Multi-level 3D) project carries out simulation studies of plasmas using multiple levels of physics, geometry, and grid models. The M3D code has been applied to ideal, resistive, two fluid, and hybrid simulations of compact quasi axisymmetric stellarators. When β exceeds a threshold, moderate toroidal mode number ($n \sim 10$) modes grow exponentially, clearly distinguishable from the equilibrium evolution. The β limits are significantly higher than the infinite mode number ballooning limits. In the presence of resistivity, these modes occur well below the ideal limit. Their growth rate scaling with resistivity is similar to tearing modes. At low resistivity, the modes couple to resistive interchanges, which are unstable in most stellarators. Two fluid simulations with M3D show that resistive modes can be stabilized by diagmagnetic drift. The two fluid computations are done with a realistic value of the Hall parameter, the ratio of ion skin depth to major radius. Hybrid gyrokinetic simulations with energetic particles indicate that global shear Alfvén TAE - like modes can be destabilized in stellarators. Computations in a two - period compact stellarator obtained a predominantly $n = 1$ toroidal mode with the expected TAE frequency. It is found that TAE modes are more stable in the two - period compact stellarator than in a tokamak with the same q and pressure profiles. M3D combines a two dimensional unstructured mesh with finite element discretization in poloidal planes, and fourth order finite differencing in the toroidal direction.

I. Introduction

The M3D (Multi-level 3D) project carries out simulation studies of plasmas using multiple levels of physics, geometry, and grid models. The M3D code has been applied to ideal, resistive, two fluid, and hybrid simulations of compact quasi axisymmetric stellarators. For β above a threshold, ideal MHD low mode number ballooning - like modes appear. In the presence of resistivity, these modes occur well below the ideal limit. Their growth rate scaling with resistivity is similar to tearing modes. Stellarators tend to be resistive interchange unstable, so the resistive modes are not stabilized at small resistivity, as in tokamaks.

Two fluid simulations indicate that the resistive modes are stabilized by diamagnetic drifts. This might account for the lack of experimental evidence of these modes. The two fluid simulations were done with a realistic value of the Hall parameter, which determines the speed of the diamagnetic drift relative to the Alfvén speed.

Hybrid simulations with energetic particles indicate that global shear Alfvén TAE - like modes can be destabilized in stellarators. A sequence of equilibria was studied, with the amount of three dimensional distortion varying smoothly from no distortion (tokamak) to full three dimensional distortion (stellarator). The rotational transform and pressure was the same for all equilibria in the sequence. It was found that the TAE growth rate was reduced by increasing the three dimensional distortion.

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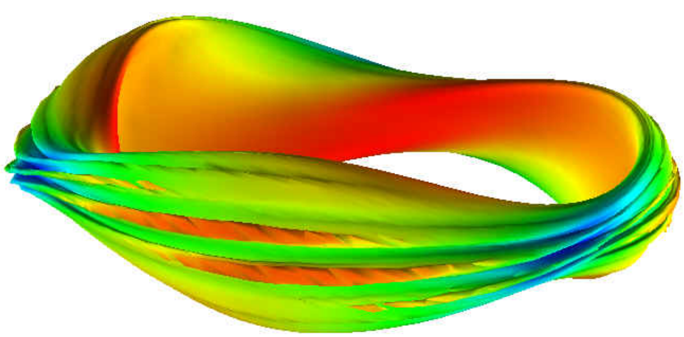
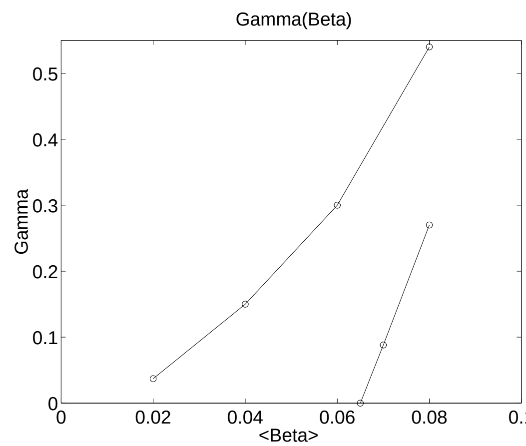
M3D combines a two dimensional unstructured mesh with finite element discretization in poloidal planes [2], and fourth order finite differencing in the toroidal direction. The code is parallelized and runs on both shared and distributed memory computers. Equilibria from the VMEC code [3] are used to construct the M3D grid, and initialize the magnetic field and pressure.

II. Stellarator Equilibrium and Ideal MHD Stability

To obtain equilibria, the initial state is relaxed under the influence of viscous and (possibly) resistive dissipation, cross field thermal conduction, and parallel thermal conduction (simulated with the "artificial sound" method [4]), using source terms to maintain the toroidal current and pressure. The kinetic energy is removed by viscous damping. Equilibria have been obtained for the proposed NCSX compact quasi-axisymmetric (QAS) stellarator, li383. Simulations are done with a fixed conducting wall boundary condition. Simulations can be done of a single period or the whole torus. M3D equilibria agree well with VMEC, except that M3D magnetic fields can contain islands.

To study β limits, we multiply the pressure in a VMEC equilibrium by a constant, and allow the equilibrium to relax. When β exceeds a threshold, intermediate toroidal mode number ($n \sim 10$) modes grow exponentially, clearly distinguishable from the equilibrium evolution. The β limits are significantly higher than the infinite mode number ballooning limits. An example of a nonlinear pressure driven mode in the proposed NCSX design is shown in Fig. 1(a). The modes follow field lines, modulated by magnetic curvature, typical of ballooning modes.


(a)



(b)

Fig.1 (a) Pressure isosurface in a ballooning unstable NCSX stellarator design (b) Growth rate γ of Ideal and Resistive Mode vs. β

III. Resistive Simulations

With nonzero resistivity, the long wavelength modes become resistive ballooning modes, which are unstable for almost all values of β. Fig. 1 (b) shows growth rate as a function of β. The leftmost curve is with dimensionless resistivity $\eta = 1/S = 1.25\ 10^4$, and the rightmost curve is with zero resistivity. The resistive ballooning modes are similar in structure to ideal MHD ballooning modes, with a dispersion relation like tearing modes [5]. For kinematic viscosity $\mu \gtrsim \eta$, the growth rate scales as $\sim \eta^{5/6}$, which is confirmed by simulations. For small $\eta \lesssim 10^{-5}$, the modes couple to interchange modes. In NSTX, as in most stellarators, resistive interchange modes are unstable.

Although most stellarators are resistive ballooning and resistive interchange unstable, it seems to have little effect experimentally. This suggests that some other physical effect is stabilizing the resistive modes.

IV. Two Fluid Simulations

Two fluid drifts can stabilize resistive modes. The two fluid terms are proportional to the Hall parameter,

$$H = \frac{c}{\omega_{pi} R}$$

where c/ω_{pi} is the ion skin depth and R is the major radius. In the simulations, $H = 0.01$ which is comparable to the expected experimental value. We use a nonlinear drift formulation of the two fluid equations similar to the Braginski equations [6]. The following calculations are done with the gyroviscous term only. We neglect the electron pressure and temperature in Ohm's Law. We also take a constant density. An example computation in Fig. 2, compares the electrostatic potential in two resistive calculations, (a) with $H = 0$, and one with $H = 0.01$, both with $S = 8000$. The purely resistive case is unstable, with typical ballooning perturbations, while the two fluid resistive case is stable, with a large scale diamagnetic flow. The value of S is unrealistically large, so the growth rate is also too large, but the mode is still stabilized with a realistic value of H. This suggests that finite H should be very effective in suppressing resistive ballooning and interchange modes.

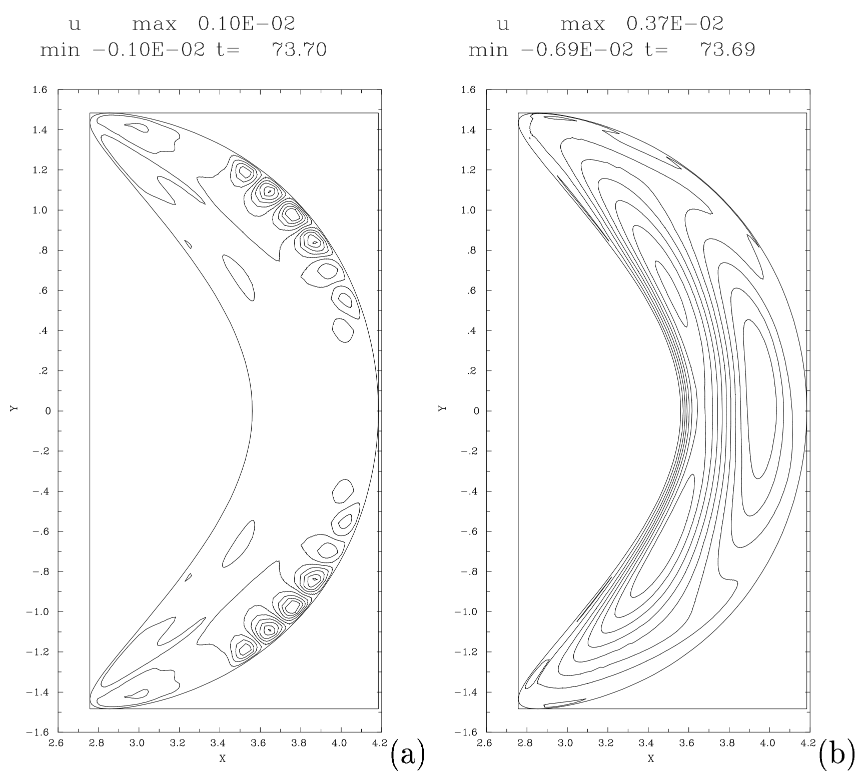


Fig. 2 Electrostatic Potential for (a) MHD, (b) 2 Fluid

When the resistive modes are stabilized by diamagnetic drifts, it is possible to consider resistive evolution of the equilibrium. In Fig. 3 are field line traces at the same time in their evolution, for the same NCSX case as previously. In (a), the field lines are stochastic in the outer region because of the resistive instability. In (b), the field lines are closed. In this case a large island, with poloidal / toroidal mode numbers 5/3, is present. This is consistent with PIES [7] results.

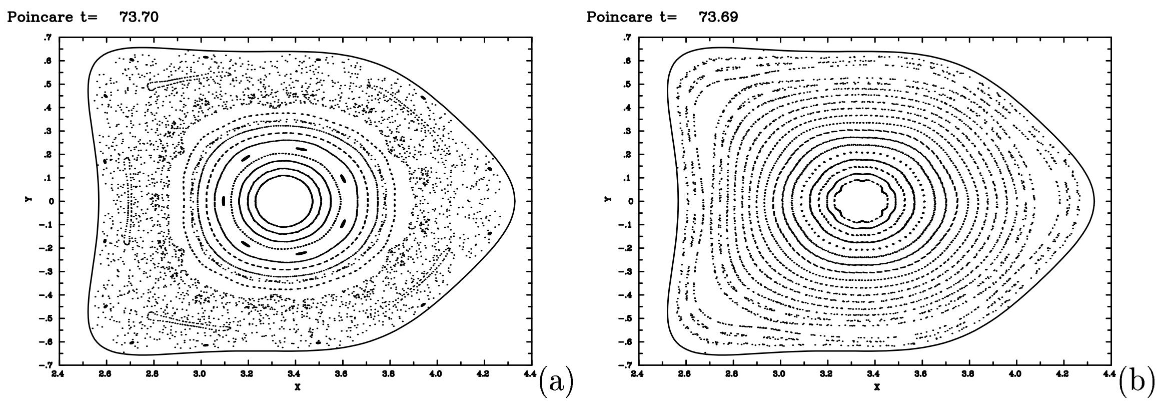


Fig. 3 *Poincaré plots for (a) MHD, (b) 2 Fluid*

V. Hot Particle Effects on Global Shear Alfvén Modes

The M3D code has a hybrid option, which has been generalized to use a 3D mesh for stellarator simulations. Energetic ions are treated as gyrokinectic particles and are coupled to the bulk plasma fluid through their contribution to the pressure tensor in the momentum equation. A δf method is used for noise reduction.

The particles are pushed on same mesh as the MHD fluid. Computations in a two - period compact stellarator indicate the destabilization of shear Alfvén eigenmodes by hot particles, as in tokamaks [8]. The following example has parameters $R/a = 4, \beta = 0.014$ and a slowing - down distribution of fast ions with $\rho_{fast}/a = 0.12, v_{fast}/v_A = 1.6$. The TAE mode is a predominantly $n = 1$ toroidal mode. The frequency is about $0.2V_A/R$, which is the expected TAE frequency $V_A/(2qR)$ at the q=2.5 continuum gap. The growth rate is linear in hot particle β_h, above a threshold, as shown in Fig. 4(a). It is possible to produce a sequence of equilibria varying smoothly between a stellarator and a tokamak, by diminishing the amplitude of the 3D perturbations of the outer boundary, while keeping the q and p profiles fixed. We find that 3 D geometry is stabilizing for this case. The growth decreases as we interpolate between tokamak and stellarator geometry, as shown in Fig. 4(b).

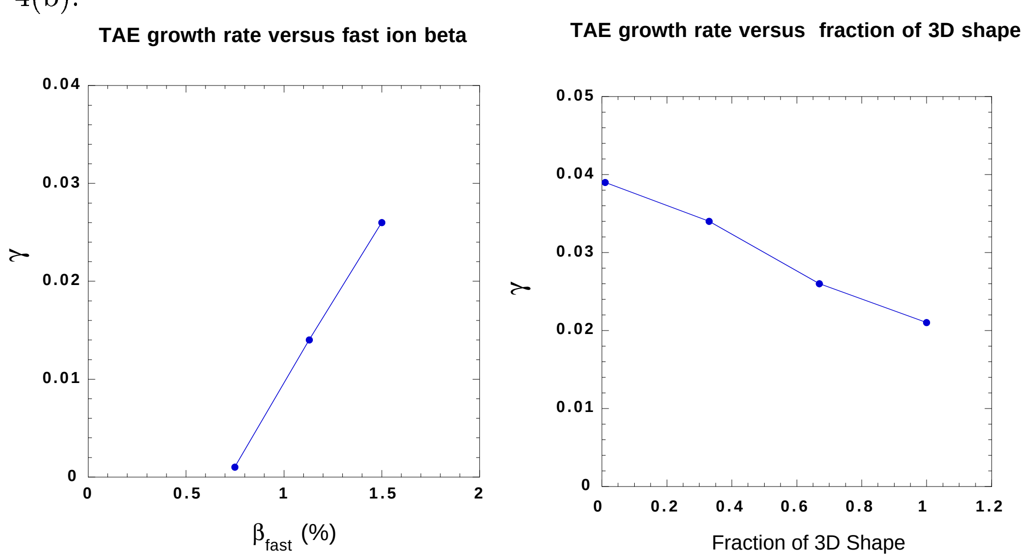


Fig. 4 (a) Growth rate γ vs. hot particle β (b) Growth rate γ vs. 3D shape for equilibria varying from tokamak to stellarator.

Fig. 5(a) shows the electrostatic potential in a poloidal plane, for the tokamak limit, while 5(b) is the electrostatic potential in the stellarator.

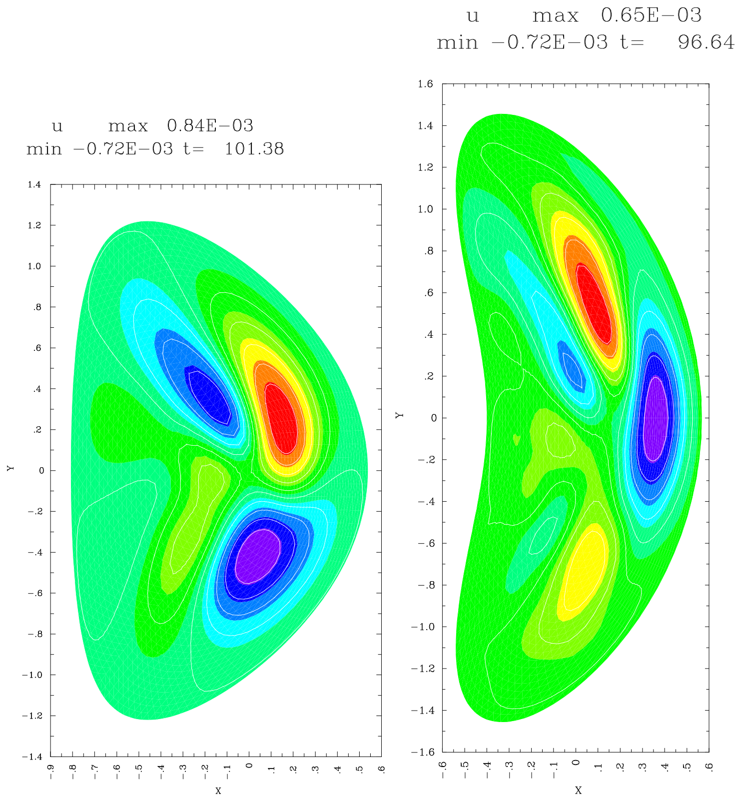


Fig. 5 (a) Electrostatic potential of a TAE mode in tokamak geometry. (b) Electrostatic potential of a TAE mode in a two field period stellarator.

VI. Conclusion

The M3D code has several levels of physics, which are applied to stellarators. Ideal MHD is used to calculate β limits. Resistive MHD computations find moderate toroidal mode number unstable resistive ballooning modes, well below the ideal MHD β limit. Including two fluid diamagnetic drifts has a stabilizing effect on the resistive ballooning modes. Hot particle kinetic effects are also included, to study TAE modes in stellarators. The TAE modes are confirmed by their frequency and linear scaling of growth rate with hot particle β. The growth rate in a two period stellarator is lower than in a tokamak with the same profiles and average boundary shaping.

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