Theory of Cross Phase-Induced Transport Suppression in Strongly Sheared Flow

P. W. Terry 1), A.S. Ware 2), and D.E. Newman 3)

1) University of Wisconsin-Madison, Madison, Wisconsin 53706.
2) University of Montana-Missoula, Missoula, Montana 59812.
3) University of Alaska at Fairbanks, Fairbanks, Alaska 99775.

e-mail contact of main author: pwterry@facstaff.wisc.edu

Abstract. The critical role played in transport barriers by the cross phase between transport-causing fluctuations is elucidated theoretically. This work details how the cross phase contributes to flux reduction in the presence of flow shear, and accounts for a number of experimental observations that cannot be understood solely in terms of the response of fluctuation magnitudes. It is found that 1) in strong shear, the cross phase for an advected scalar responds more strongly to flow shear than do the amplitudes. Consequently, the flux reduction produced by flow shear is dominated by cross-phase suppression. The scaling of the cross phase with shear strength is such that the fluctuation level of either the scalar or the potential can increase, yet the flux decreases, as observed in some experiments. 2) The interplay of magnetic shear and flow shear inhomogeneities can lead to localized regions of negative cross phase, and therefore inward flux, provided the fluctuation spectrum is dominated by a single, or limited range of helicities. 3) In collisionless trapped electron mode turbulence the cross phase, particle flux, growth rate, and nonlinear coupling all depend on the density-potential correlation, and hence all decrease with flow shear. The decrease of growth rate and nonlinear coupling offset in the saturation balance, leaving fluctuation levels largely unchanged.

1. Introduction

The suppression of turbulence by **E**×**B** flow shear has been widely invoked to describe the physics of transport barriers and enhanced confinement regimes [1]. However, under closer scrutiny, it is not difficult to find measurements whose details appear to be at odds with the simple suppression paradigm. For example, probe measurements of the shear layers of a variety of devices show a marked decrease of the particle flux in the region of flow shear, while fluctuations of either the density or the potential actually increase [2-4]. In these cases, and other experiments where detailed measurements have been made [5], a significant portion of the particle flux reduction comes from a reduction in the sine of the density-potential cross phase. In probe-induced shear layers the flux is also observed to become inward locally if the shear is strong [6]. The flux reversal occurs toward the inside edge of the shear layer, in a region of positive shear, suggesting reproducible spatial structure. The paradigm of turbulence suppression by **E**×**B** flow shear is based on the simple notion that in stable sheared flow, the amplitude of passive scalar fluctuations is reduced [7], or that key instabilities are quenched altogether. Neither of these notions fit the experimental observations just cited, which do, however, suggest the importance of investigating the response of the cross phase to flow shear. The theoretical investigation of the cross phase that follows shows that flow shear generally creates an unfavorable cross phase for transport. In strong shear, the effect is more robust than amplitude suppression, and likely to be the dominant suppression mechanism in transport barriers.

2. Cross Phase of Advected Scalar in Strong Shear

The transport flux of a scalar χ due to advection by a turbulent **E**×**B** flow is $\Gamma = -cB_0^{-1}\sum_k k_y |\tilde{\chi}_{\mathbf{k},\omega}|\,|\phi_{-\mathbf{k},-\omega}| \sin\delta_{\mathbf{k},\omega}$, where $\tilde{\chi}_{\mathbf{k},\omega}$ is the Fourier amplitude of the scalar fluctuation, $\phi_{-\mathbf{k},-\omega}$ is the electrostatic potential, and $\sin\delta_{\mathbf{k},\omega}$ is the sine of the cross phase. The scalar magnitude and cross-phase are determined from the scalar evolution equation as a nonlinear response to the potential. We calculate the cross phase of a generic advected scalar, inverting the renormalized advection operator in uniform flow shear [8]. The scalar evolution equation is $[-i\omega + ik_y x v_0' - (\partial/\partial x)D_{\mathbf{k},\omega}(\partial/\partial x) - k_y^2 d_{\mathbf{k},\omega}]\ \tilde{\chi}_{\mathbf{k},\omega} = cB_0^{-1} ik_y \phi_{\mathbf{k},\omega} d\chi_0/dx$, where v_0' is the constant shearing rate of the mean flow, χ_0 is the mean scalar, and $D_{\mathbf{k},\omega}$ and $d_{\mathbf{k},\omega}$ are the renormalized turbulent diffusivities. The diffusivities arise from the turbulent advection of the scalar fluctuation, and are given by $D = c^2 B_0^{-2} \sum_k (k_y' - k_y) f \phi_{\mathbf{k},\omega}\, R_{\mathbf{k}\text{-}\mathbf{k}',\omega\text{-}\omega} f \phi_{\mathbf{k}',\omega'}$, where $f = (k_y')^{1/2}$ for $D = D_{\mathbf{k},\omega}$ and $f = (k_y)^{-1/2}\partial/\partial x$ for $D = d_{\mathbf{k},\omega}$. The function $R_{\mathbf{k}\text{-}\mathbf{k}',\omega\text{-}\omega'} = [-i(\omega-\omega') + i(k_y - k_y')x v_0' - (\partial/\partial x)D_{\mathbf{k}\text{-}\mathbf{k}',\omega\text{-}\omega'}(\partial/\partial x) - (k_y - k_y')^2 d_{\mathbf{k}\text{-}\mathbf{k}',\omega\text{-}\omega'}]^{-1}$ is the operator of the advective response time at wavenumber $k-k'$. Note that the same differential operator governs both $\tilde{\chi}_{\mathbf{k},\omega}$ and $R_{\mathbf{k},\omega}$. Therefore, in strong shear, when scalar fluctuations are suppressed, the advective response is also suppressed. This is a crucial feature of cross phase response suppression. In the limit that the ratio of shearing rate to dissipation rate becomes infinite, the scalar fluctuation is in phase with the potential, and the flux is zero. For a large but finite shearing rate (small but finite dissipation), the flux is nonzero, and the degree of suppression is proportional to the magnitude of the effective dissipation rate relative to the shearing rate. If the system is turbulent, the effective dissipation rate is in turn proportional to the turbulent diffusivities, which are themselves suppressed by the flow shear.

This effect is quantified by inverting the scalar evolution equation using a Green function and asymptotic analysis for the strong shear limit. The Green function localizes the advective response to $\Delta x S^{-1/3}$, the reduced radial correlation of scaling theory [7], where Δx is the nominal mode width, and $S = k_y v_0' \Delta x^3 / D_{\mathbf{k},\omega}$ is the shear strength parameter. The flux, which integrates over the Green function, is given by $\Gamma = \mathrm{Re}\sum_k ic^2 k_y^2\ |\psi_{\mathbf{k}}|^2 (d\chi_0/dx)[B_0^2 k_y v_0' \Delta x^3 A(x)]^{-1}$, where $\psi_{\mathbf{k}}$ is the frequency-integrated potential eigenmode, and $A(x) = [x/\Delta x - (\omega + ik_y^2 d_{\mathbf{k},\omega})/k_y v_0' \Delta x]$ is the structure function of the turbulent mixing. From previous calculations, the eigenmode is shifted off the rational surface by an amount proportional to v_0'. Changes in the mode width are weaker and are often ignored. The mixing structure function $A(x)$ exhibits a Kelvin neutral layer. Within this layer the mean flow vanishes, and the scalar is optimally mixed; outside, the flux is strongly suppressed. The width of the neutral layer is proportional to $d_{\mathbf{k},\omega}$, which in turn is proportional to S^{-1}, making it much smaller than the reduced radial correlation $\Delta x S^{-1/3}$ in the limit of large S, and much smaller than the eigenmode width. Moreover, as noted above, the width becomes narrower in strong shear because $d_{\mathbf{k},\omega}$, is itself suppressed by flow shear. To estimate the magnitude of $d_{\mathbf{k},\omega}$, we solve for $d_{\mathbf{k},\omega}$, using the Green function derived for the scalar evolution and the flux. Outside the neutral layer, and assuming a spectrum in which strongly turbulent scales (nondissipated) are coupled by a direct cascade to dissipated scales at somewhat higher wavenumber, an upper bound for the flux is obtained:

$$\Gamma = -\pi c^2 B_0^{-2} \sum_k k_y^2\, |\psi_{\mathbf{k}}|^2 (d\chi_0/dx)(k_y v_0' x)^{-1} \cdot [\pi c^2 B_0^{-2} \sum_{k'} |d\psi_{\mathbf{k}'}/dx|^2 (k-k')\mu (v_0' x)^{-3}], \qquad (1)$$

where μ is a viscous dissipation. The first part of the expression (before the [ ] brackets) represents the response of the scalar amplitude $\tilde{\chi}_{\mathbf{k},\omega}$ to the flow shear. The expression in brackets is the cross phase response. Both depend on the eigenmode amplitude squared, a factor that may increase, decrease, or remain unchanged in sheared flow, depending on the circumstance. However, the phase factor has an additional dependence of $(v_0')^{-3}$, while the fluctuation magnitude factor has a dependence of $(v_0')^{-1}$. The sine of the cross phase can therefore be reduced to a considerably larger degree than the fluctuation magnitude factor.

3. Magnetic Shear and Flux Reversal

The flux calculated in the previous section is positive definite. To identify physics that can produce localized flux reversals like those observed in experiment [6], we examine the cross-phase response for electron density advected in a sheared magnetic field by a turbulent flow with linearly varying mean. The nonadiabatic electron response governs transport. We study a nonadiabatic response characteristic of the edge plasma of Ref. 6. To evaluate the eigenmode we treat the nonadiabatic electron density as the electron contribution to ion temperature gradient (ITG) turbulence. Magnetic shear enters electron density evolution through a dissipated parallel flow. The competition between the inhomogeneities of flow and magnetic field is known to produce an oscillatory eigenfunction envelope in trapped electron turbulence, and therefore may affect the electron particle flux. The nonadiabatic electron density $h_{\mathbf{k},\omega}$, satisfies $[-i\omega + ik_y x v_0' + (v_e^2 k_y^2 x^2/\upsilon_e L_s^2) - (\partial/\partial x) D_{\mathbf{k},\omega}(\partial/\partial x) - k_y^2 d_{\mathbf{k},\omega}] h_{\mathbf{k},\omega} = i(\omega - k_y v_0' x - \omega_*) e\phi_{\mathbf{k},\omega}/T_e$, where v_e is the electron the thermal velocity, υ_e is the electron ion collision rate, L_s is the magnetic shear scale length, and $D_{\mathbf{k},\omega}$ and $d_{\mathbf{k},\omega}$ are given by the same expressions used in the previous section, but with the right hand side of the advective response time $R_{\mathbf{k}-\mathbf{k}',\omega-\omega'}$ augmented by the magnetic shear term $+\, v_e^2(k_y - k_y')^2 x^2/\upsilon_e L_s^2$. Using the asymptotic Green function technique the leading order flux in a strong shear expansion is

$$\Gamma = -\mathrm{Re}\sum_{\mathbf{k}} \frac{cT_e}{eB_0}\frac{n_0 k_y}{2}\left|\frac{e\phi_{\mathbf{k},\hat{\omega}}(x)}{T_e}\right|^2 \left[\frac{\hat{\omega}-\omega_*}{D_{k,\hat{\omega}}/\Delta^2} - S\frac{x}{\Delta}\right]\left[\frac{x^2}{\Delta^2} - \frac{iSx}{\Delta} - \frac{S^2}{8} + \frac{iS}{2}\left(\frac{\hat{\omega}}{\omega_s} - \frac{ik_y^2 d_{\mathbf{k},\hat{\omega}}}{\omega_s}\right)\right]^2$$

$$\left[\frac{x^2}{\Delta^2} + \frac{iSx}{\Delta} - iS\left(\frac{\hat{\omega}}{\omega_s} + \frac{ik_y^2 d_{\mathbf{k},\hat{\omega}}}{\omega_s}\right)\right]\left|\left[\frac{x^2}{\Delta^2} + \frac{iSx}{\Delta} - \frac{S^2}{8} - \frac{iS}{2}\left(\frac{\hat{\omega}}{\omega_s} + \frac{ik_y^2 d_{\mathbf{k},\hat{\omega}}}{\omega_s}\right)\right]^2\right|^{-2} \quad (S \to \infty), \qquad (2)$$

where $\hat{\omega} = \omega_r(k)\ i\omega_i(k) + ik_y v_0(x_r)$ is the Doppler-shifted frequency, $\omega_r(k)$ is the peak of the power density spectrum of the electrostatic potential in frequency space evaluated at wavenumber k, $\omega_i(k)$ is the width of the power density spectrum, x_r is the location of the rational surface, $\Delta = (D_{\mathbf{k},\hat{\omega}}\, \upsilon_e L_s^2/v_e^2 k_y^2)^{1/4}$ is the electron response width, and $\omega_s = k_y v_0'\Delta$ is the shearing frequency. The eigenfunction $|\phi_{\mathbf{k},\hat{\omega}}(x)|^2$ and the structure function of the electron density response [everything to the right of $|e\phi_{\mathbf{k},\hat{\omega}}(x)/T_e|^2$ in Eq. (2)] are deconvolved because the ion gyroradius-scale variation of the ITG eigenfunction is slow compared to the electron gyroradius-scale variation of the structure function.

The behavior of the flux is determined by $|\phi_{\mathbf{k},\hat{\omega}}(x)|^2$ and the structure function. The eigenfunction is positive definite, but the structure function has both positive and negative values, allowing the flux to change sign. Analysis of the seventh order polynomial obtained by taking the real part of the numerator of the structure function indicates that it has three zeros for large S. One zero occurs near x=0 and a pair of zeros occur for $x \approx \pm\Delta S/2$. The structure function is positive asymptotically for $x \to \infty$, and becomes negative between x=0 and the zero at $x \approx \Delta S/2$. For negative values of x the structure function is first positive and then becomes negative. The structure function is plotted in Fig. 1. The magnitude of the

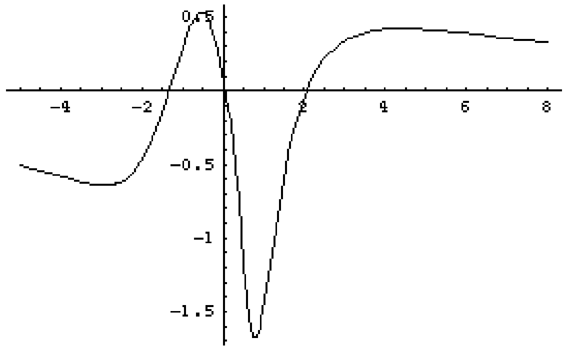

Fig. 1. Structure function for S=3, $(\omega_r(k)\text{-}k_y^2 Im\, d_{\mathbf{k},\hat{\omega}})/\omega_*=0.3$, $(\omega_i(k)+k_y^2 Re\, d_{\mathbf{k},\hat{\omega}})/\omega_*=-0.1$, *and* $\omega/\omega_*=0.3$

structure function at the extrema on either side of the origin decreases with increasing shear as S^{-4}. The magnitude of the structure function at the outer extrema around $x\approx\pm\Delta S$ is essentially independent of S. Besides having negative values, this structure function differs from that of the previous section in two ways. First, the width of the structure function is of order ΔS, whereas for the homogeneous magnetic field the width is of order ΔS^{-1}. Second, inside its width, the structure function has regions that both suppress the flux and yield optimal mixing. For the homogeneous field the flux is optimal, but only within the extremely narrow mixing layer. These differences arise from the inhomogeneity of the magnetic field. They indicate that the behavior of transport in the presence of flow shear does not have universal scaling properties if the turbulence is inhomogeneous. This means that if the only inhomogeneity of the system is the flow shear and its scale length is large, then the scaling properties of turbulence suppression can be extracted from a single coefficient of a Taylor expansion of the local flow profile. On the other hand, if there are other inhomogeneities present, such as that of a sheared magnetic field, the nature of transport is sensitive to the profile *of both the flow shear and the magnetic field shear*.

4. Eigenmode Contribution to Flux Reduction

We turn now to the eigenfunction, which has a significant effect on the flux. For ITG turbulence, the eigenfunction is determined by the ion dynamics and the adiabatic electron response. In the presence of linear flow shear, the eigenmode of the fluid ITG instability in a sheared slab is known from previous studies to shift off the rational surface by an amount proportional to S [9]. The spatial scale of the shift is governed by ion dynamics, i.e., by ρ_s, the ion gyroradius evaluated at the electron temperature. A simple estimate of the shift, obtained from the eigenmode potential, is $x_{ITG} = \rho_s(k_y v_0'\rho_s/\omega_*)(1+\eta_i)(T_e/T_i)(L_s/L_n)^2$. The eigenfunction is large at this value of x, and becomes exponentially small beyond a mode width. The mode width is also proportional to ρ_s, but with little dependence on S. Evaluating the flux from Eq. (2) at x_{ITG} yields $\Gamma = -\sum_k (cT_e/eB_0)\; n_0 k_y\; |e\phi_{\mathbf{k}}(x_{ITG})/T_e|^2\; k_y^2\rho_s^2$ $(\Delta/\Delta_{ITG})^4\; [(1+\eta_i)(T_e/T_i)(L_s/L_n)]^{-1}$, where $\Delta_{ITG} = \rho_s\; [(1+\eta_i)(T_e/T_i)(L_s/L_n)]^{1/2}$ is the nominal zero flow ITG linear mode width obtained using $\omega = (1+\eta_i)(T_e/T_i)$. Two features are evident. First, the electron structure function has a width Δ about the rational surface governed by the electron scale ρ_e. The eigenmode is shifted away from the rational surface by $x_{ITG} \sim \rho_s$. Hence, there is very little overlap between the eigenmode and the electron structure function, resulting in the very small factor $(\Delta/\Delta_{ITG})^4=\omega_* \upsilon_e L_s L_n/v_e^2$. Second, the shift of the eigenmode is proportional to S, making the parameter S drop out of the flux. Thus the flux is extremely small but does not decrease further if S is increased. Because the eigenfunction is exponentially small for $x<x_{ITG}$, the regions where the structure function is negative make a negligible contribution to the flux.

The situation is different if the eigenmode is a collisional electron drift wave. The electron gyroradius scale governs both the eigenmode and the structure function. Hence, the scale separation assumption in Eq. (2) is no longer valid. However, if the flux can still be approximated by some product of the eigenfunction and structure function, the eigenfunction, which shifts to $x=\Delta S$, will preferentially sample the positive (rightmost) lobe in Fig. 1. The negative lobe to the right of the origin will emerge, albeit with smaller amplitude. Thus, a flux reversal can be anticipated for an electron-scale eigenmode. The result is sensitive to the wavenumber sum in the flux expression. Because the sum samples different helicities, a smaller negative lobe associated with a given helicity will be cancelled by larger positive lobes of nearby helicities. However, if the spectrum is quasicoherent and dominated by a single helicity, the negative flux region can survive. In experiments the strong localization of the flow may effectively isolate the fluctuations occurring in the flow layer, allowing a localized flux reversal. In Eq. (2) the density gradient is maintained, otherwise the gradient will relax. A steady state flux with localized reversals requires additional transport, e.g., a collision-driven flux that is larger than the anomalous flux, or a flux that is not poloidally symmetric.

A different situation arises in collisionless trapped electron turbulence, where the electron density response can be iteratively inverted for strong shear and substituted into the ion equation. Both the growth rate and the advection of electron density depend directly on the correlation between the electron density and the potential, and therefore on the cross phase. Consequently, strong flow shear decreases both. Unstable long-wavelength fluctuations are saturated by the advection of electron density. Therefore, the decrease in growth rate is offset by the decrease in nonlinear coupling strength, *leaving the saturated potential amplitude largely unchanged*. The cross phase, which goes like the growth rate, is reduced by shear.

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