Angular Momentum "Generation": Theory and Recent Experiments

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Abstract. The accretion theory of the spontaneous rotation phenomenon discovered in magnetically confined axisymmetric plasmas is shown to account for the most recent experimental observations. The interpretation, based on the quasilinear theory of the relevant modes that is consistent with earlier experiments on this phenomenon as well as with the most recent ones, is presented.

1. Introduction

The accretion theory [1] of the spontaneous rotation phenomenon [2] in axisymmetric toroidal plasmas can account for all the main recent experimental observations concerning the nature of this phenomenon. The principal elements of this theory are:

- The toroidal rotation of the plasma column, in the absence of externally injected angular momentum, is connected to its transport properties and to the excitation of the modes that determine these properties.
- The "source" of angular momentum is at the edge of the plasma column.
- The propagation of the angular momentum toward the center is "anomalous", that is not explained by a collisional transport theory, and is associated with the prevalent electrostatic modes in the main body of the plasma column.
- Angular momentum in the opposite direction is transferred to the material wall that surrounds the plasma column.
- The expectation, confirmed by the experiments, that the direction of the plasma rotation should change in the transition from regimes where the ion thermal energy is well confined such as the H-mode regimes to regimes where the ion thermal energy confinement is poor such as the L-mode regimes.
- The rotation velocity direction being the same as that of the ion diamagnetic velocity in the good ion confinement regimes. (e.g. H-mode regimes).
- The derivation of the quasilinear theory demonstrating the inward transport of angular momentum by the prevalent ion temperature gradient driven modes.
- The assumption that the direction of the angular momentum transferred to the surrounding material wall is correlated with the modes that are excited in the outermost region (edge) of the plasma column.

We note that recent experiments [3] have confirmed that the source of the rotation is at the edge of the plasma column, and shown that the rotation propagates toward the center of the plasma column at a faster rate than predicted from collisional transport theory. This is consistent with the assumption made within the accretion theory that Ion Temperature Gradient (ITG) driven modes, whose growth rates are enhanced by the radial gradient of the toroidal velocity in the outer region of the plasma column, are responsible for the inward momentum transport. The experimental observation that the central rotation disappears when a steep density gradient is produced well inside the plasma column [4], as a result of an induced transport barrier, is consistent with the suppression of the ITG modes within the main body of the plasma column.

Toroidal rotation in the direction of the ion diamagnetic velocity has been observed with similar characteristics in the H regimes with only ohmic as well as ICRH heating by the Alcator C-Mod machine and with ICRH heating in the enhanced ion-confinement regime produced by the Tore Supra machine [5]. In Alcator C-Mod, the toroidal velocity at the center of the plasma column is found to increase linearly with the total plasma thermal energy. In Tore Supra, the central velocity is considered to increase linearly with the central ion pressure. Since the plasma volume remained fixed in the Alcator C-Mod experiments, we may argue that both observations are consistent with a theory that relates the toroidal velocity to the ion pressure gradient.

The argument can be made that the amplitude of the prevailing ITG mode, which is responsible for the inward transport, can be sufficiently enhanced that, at the highest plasma pressures the inward and outward flux of angular momentum can compensate each other in the inner region of the plasma column while the velocity profile is peaked at the center. We note that the inward gradient of the velocity penalizes the growth rate of the mode producing inward angular momentum transport. At lower pressures and consequently lower mode amplitudes, we argue that the two fluxes can compensate each other provided the velocity profile does not develop an inward gradient, in the main body of the plasma column, according to the analysis given in the following sections. In experiments carried out by Alcator C-Mod, peaked velocity profiles have been observed in H-mode regimes [2] for the highest levels of injected ICRH power corresponding to the highest ion thermal energy contents. Recently, flat velocity profiles over a considerable part of the plasma radius have been observed [6], for considerably lower values of the thermal energy in H-mode regimes with ohmic heating only as shown in Fig. 1. These observations are consistent with the indications of the theory that is given.

2. Supporting Theory

We consider propagating modes with phase velocities along the magnetic field that are considerably smaller than the electron thermal velocity. For simplicity, we refer to a plane, one dimensional plasma where the magnetic field **B** is in the positive z direction and where a plasma flow velocity $\mathrm{V}_{\parallel}(x)$ is in the same direction as that of **B**. The plasma velocity $\mathrm{V}_{\parallel}(x)$ is smaller than the ion thermal velocity v_{thi}. We consider electrostatic modes with $\hat{\boldsymbol{E}} \simeq -\nabla\hat{\Phi}$ being the fluctuating field. Thus the relevant nominal modes are of the form $\hat{\Phi} = \tilde{\phi}\left(x\right)\exp\left(-i\omega t + ik_y y + ik_{\parallel} z\right)$. The modes of interest are in the regime where $\left|\omega - k_{\parallel}\mathrm{V}_{\parallel}\right| > \left|k_{\parallel}\right|\mathrm{v}_{thi}$. Thus the Doppler shifted frequency, $\omega - k_{\parallel}\mathrm{V}_{\parallel}$, can be replaced by ω in the rest of the analysis that follows. Then the fluctuating density is

$$\frac{\hat{n}^k}{n} \simeq \frac{e\tilde{\phi}^k}{T_e} \tag{1}$$

and the ion mass conservation equation, in the guiding center approximation gives

$$-i\omega_k \hat{n}^k + \hat{\mathrm{v}}^k_{Ex}\frac{dn}{dx} + ik_{\parallel} n \hat{u}^k_{i\parallel} = 0\,, \tag{2}$$

where $\hat{\mathrm{v}}^k_{Ex} = -ik_y\hat{\phi}^k c/B.$

Then we note that the quasilinear flux of momentum is represented by

$$\Gamma_J = m_i n\left\langle \hat{\mathrm{v}}_{Ex}\hat{u}_{i\parallel}\right\rangle \tag{3}$$

We use Eq.(1) and (2) to derive $\hat{u}^k_{i\parallel}$ and, for $\Gamma_J = \Sigma_k \Gamma^k_J$, we obtain

$$\Gamma_J^k \simeq -m_i n 2 D_B \frac{k_\perp}{k_\parallel} \gamma^k \frac{e^2 \left\langle \left|\tilde{\phi}\right|^2 \right\rangle}{T_e^2}, \tag{4}$$

where $\gamma^k = \operatorname{Im} \omega^k$ and $D_B \equiv cT_e / (eB)$.

Now in the absence of a source of angular momentum at the edge of the plasma column, we may argue that the ITG modes which can be excited have equal probability to have $k_\parallel / k_y > 0$ as $k_\parallel / k_y < 0$. Therefore the total induced flux of angular momentum induced by the combination of them would be insignificant. On the other hand, when a positive velocity gradient, $d\mathrm{V}_\parallel / dx$, is produced in the outer region of the plasma column, we argue that the mode with positive $k_\parallel / k_y$ acquires a larger growth rate, as we shall show later, and becomes predominant. We assume that this mode remains predominant well into the main body of the plasma column. In this region if we consider a set of modes with the same absolute values of k_y and $k_\parallel$ but opposite values of $k_y / k_\parallel$ the resulting total momentum flux can be written as

$$\Gamma_J^k \simeq 2 m_i n \left|\frac{k_y}{k_\parallel}\right| D_B \left(-\gamma_+^k \left\langle \left|\frac{\hat{n}^k}{n}\right|_+^2 \right\rangle + \gamma_-^k \left\langle \left|\frac{\hat{n}^k}{n}\right|_-^2 \right\rangle \right), \tag{5}$$

where the + sign refers to the modes with $k_y / k_\parallel > 0$.

Thus when a negative velocity gradient is formed, the growth of the mode with $k_\parallel / k_y > 0$ decreases and the total flow of angular momentum tends to vanish. We note that the momentum flux is inward for $k_\parallel / k_y > 0$. Consequently, the transport associated with the corresponding modes is in the direction of the flow velocity gradient if $d\mathrm{V}_\parallel / dx$ is negative and this would correspond to a "momentum pinch" effect. As indicated earlier, it is reasonable to consider that in the regimes with the highest levels of injected power, the value of $\left|\hat{n}^k / n\right|_+$ exceeds that of $\left|\hat{n}^k / n\right|_-$ by an amount that compensates the fact that $\gamma_+^k < \gamma_-^k$ when $d\mathrm{V}_\parallel / dx < 0$ and peaked velocity profiles are produced.

In order to assess the magnitude of Γ_J^k, we may assume that the mode saturation level is related to $k_\perp r_T$, for $r_T \equiv -\left(dT_i / dx\right)^{-1} T_i$, we may take
$\left|\hat{n} / n\right| \sim \alpha_s / \left(k_\perp r_T\right)$. Then

$$\Gamma_J^k \sim m_i n D_B \frac{\gamma_k}{k_\parallel k_\perp r_T^2} \alpha_s \tag{6}$$

and we can verify from this that the rate of momentum transport can be relatively fast.

The estimate of the corresponding flux of ion thermal energy

$$\Gamma_T = n \left\langle \hat{\mathrm{v}}_{Ex} \hat{T}_i \right\rangle$$

is well-known [7] and gives $\Gamma_T^k = -D_{QL} dT_i / dx$ where

$$D_{QL} = \frac{2\gamma^k}{\left|\omega^k\right|^2} \left\langle \left|\hat{\mathrm{v}}_{\mathrm{Ex}}^{\mathrm{k}}\right|^2 \right\rangle.$$

The transport rate for thermal energy and momentum can then be compared easily by referring to the transport velocities $v^k_{T,D} = D_{QL}/r_T$ and $v^k_{J,D} = \Gamma^k_J/\left(m_i n V_\parallel\right)$. It is evident that is $v^k_{T,D}$ outward as long as $d\ln T_i/dx$ is negative.

3. Linearized Approximation

The simplest form of the dispersion relation for Ion Temperature Gradient driven modes in the presence of a velocity gradient can be obtained by adding the longitudinal ion momentum conservation equation to Eqs.(1) and (2). This equation is

$$-i\omega_k m_i n\hat{u}^k_\parallel + \hat{v}^k_{Ex}\frac{dV_\parallel}{dx} \simeq ik_\parallel n\hat{T}_i \tag{7}$$

for $T_e \lesssim T_i$. The relevant equation of state, in the limit where $\eta_i \equiv \left(d\ln T_i/dr\right)/\left(d\ln n/dr\right)$ is sufficiently larger than unity, is simply

$$-i\omega_k\hat{T}_i + \hat{v}_{Ex}\frac{dT_i}{dx} \simeq 0. \tag{8}$$

We note that H-mode regimes are characterized by rather flat density profiles. Thus the assumption made on η_i is realistic. Then, by combining Eqs.(1), (2), (7), and (8) we obtain

$$1 = \frac{\omega_{*e}}{\omega} - k_\parallel k_y D_B\frac{1}{\omega^2}\left(\frac{dV_\parallel}{dx} + \frac{k_\parallel}{\omega m_i}\frac{dT_i}{dx}\right) \tag{9}$$

where $\omega_{*e} = -k_y\left(cT_e dn/dx\right)/\left(eB\right)$. When ω_{*e}/ω is negligible and $dV_\parallel/dx = 0$, the dispersion relation reduces to the well-known form $\omega^3 \cong k^2_\parallel D_B k_y\left(dT_i/dx\right)/m_i$ and the unstable root is characterized by $\mathrm{sgn}\left(\mathrm{Re}\,\omega/k_y\right) = \mathrm{sgn}\left(dT_i/dx\right)$. That is the unstable mode has a phase velocity in the direction of the ion diamagnetic velocity. In this case, the instability is not affected by the sign of $k_\parallel/k_y$. When $dV_\parallel/dx$ is introduced, the sign of $k_\parallel/k_y$ relative to that of $dV_\parallel/dx$ becomes important. Therefore if $dV_\parallel/dx$ is positive as is the case for the outer region of the plasma, the destabilizing sign of $k_\parallel/k_y$ is positive. Thus the mode with $k_\parallel/k_y$ should prevail. In fact, the mode with $k_\parallel/k_y$ negative can be brought to marginal stability. If we consider for simplicity, the case where $\left|\omega_{*e}/\omega_T\right| \ll 1$ where $\omega^3_T \equiv k^2_\parallel k_y D_B\left(dT_i/dx\right)/m_i$ the marginal stability condition is $C_v = \omega^2_v/\omega^2_T = 3/2^{2/3}$ where $\omega^2_v \equiv \left(k_\parallel k_y dV_\parallel/dx\right)D_B$. For $C_v < 3/2^{2/3}$, the mode remains weakly unstable if a small rate of momentum dissipation introduced into the ion momentum balance equation can be considered as physically meaningful. If we take $\omega_T = M_T\left|k_\parallel\right|v_{thi}$ where $M_T > 1$, the condition $C_v = 3/2^{2/3}$ corresponds to $\left|dV_\parallel/dx\right|r_T/v_{thi} = 3/4M_T 2^{1/3} < 1$, consistent with the assumptions made in our derivation. We note also that $\left|k_\parallel r_T\right| = \left(T_e/T_i\right)\left|k_y\rho_i\right|/\left(4M^3_T\right) \ll 1$, where $\rho_i \equiv v_{thi}/\Omega_{ci}$ is the ion gyroradius. Furthermore, if we take $\left|\omega^k\right| \simeq \varepsilon_T k_y cT_i/\left(eBr_T\right)$ with $\varepsilon_T < 1$, we can verify easily that the transport velocities for the angular momentum and the ion thermal energy can be of the same magnitude.

When the inflow of momentum is sufficiently strong that $dV_\parallel/dx$ begins to become negative in the center of the plasma column, a similar condition for marginal stability can be found for the mode with $k_{\|}/k_y$ positive, and we may argue again that in the case where this may be approached a remnant growth rate associated with a small rate of dissipation in the ion momentum balance equation can be found.

The experiments carried out by the Alcator C-Mod machine on the L to H-regime transition indicate that the plasma begins to rotate first, in the ion diamagnetic velocity direction, near the outer edge of the plasma column. Then the motion propagates toward the center [3] as angular momentum is transported inward. The estimated outward transport time for the ion thermal energy and that for the inward transport of angular momentum are comparable [3]. This is clearly consistent with the interpretation of the analysis given in Section 2.

In the same device, steep density profiles are produced when an internal transport barrier is established in plasmas already in the H-mode regime with either ohmic or ICRH heating. In this case, we expect the prevalent ITG to be suppressed and the inward flow of angular momentum also to cease within the region inside the barrier. In particular, the experiments show that the rotation in the region where the density profile is steep as well as in the central core slows down, while the rotation outside the barrier region drops with a significant delay [3].

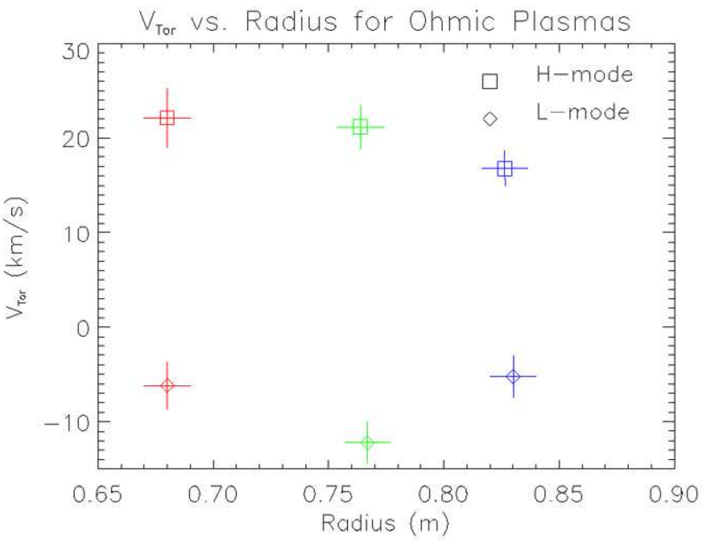

Figure 1 Toroidal velocity profile measured in the Ohmic H-mode regime with the Alcator C-Mod machine by J.E. Rice and W.D. Lee (August 2002, unpublished, courtesy of the authors).

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