IAEA-CN-94/TH/7-1Ra

Theory and Observations of High Frequency Alfvén Eigenmodes in Low Aspect Ratio Plasma*

N. N. Gorelenkov 1), E. Fredrickson 1), E. Belova 1), C. Z. Cheng 1), D. Gates 1), S. Kaye 1), R. White 1)

1) Princeton Plasma Physics Laboratory, P.O. Box 451, Princeton, NJ, USA 08543-0451

*This work supported by DoE contract No. DE-AC02-76CH03073

Theory and Observations of High Frequency Alfvén Eigenmodes in Low Aspect Ratio Plasma.[1]

N. N. Gorelenkov 1), E. Fredrickson 1), E. Belova 1), C. Z. Cheng 1), D. Gates 1) S. Kaye 1), R. White 1)

1) Princeton Plasma Physics Laboratory, P.O. Box 451, Princeton, NJ, USA 08543-0451

e-mail contact of main author: ngorelen@pppl.gov

Abstract. New observations of sub-cyclotron frequency instabilities in low aspect ratio plasmas in the National Spherical Torus experiment (NSTX) are reported. The frequencies of observed instabilities scale with the characteristic Alfvén velocity of the plasma. A theory of localized Compressional Alfvén Eigenmodes (CAE) and Global shear Alfvén Eigenmodes (GAE) in low aspect ratio plasmas is presented to explain the observed high frequency instabilities. CAE's/GAE's are driven by the velocity space gradient of energetic super-Alfvénic beam ions via Doppler shifted cyclotron resonances. Properties of such instabilities are investigated.

1. Introduction

Magnetic field activity measured by edge Mirnov coils during NBI injection in NSTX shows a broad and complicated frequency spectrum of coherent modes between $400kHz$ and up to $2.5MHz$, with the fundamental cyclotron frequency of background deuterium ions $f_{cD} = \omega_{cD}/2\pi = 2.3MHz$, calculated at the vacuum magnetic field at the geometrical axis of the plasma $B_{g0} = 0.3T$ [1–3]. The instability frequency spectrum has discrete peaks as shown in FIG. 1. Frequency spectrum peaks scale with the characteristic Alfvén velocity as magnetic field and plasma density are varied. NSTX is a low aspect ratio toroidal device with major and minor radii $R_{g0} = 0.85m$ and $a = 0.65m$ respectively. In NSTX the Alfvén speed is low compared to the injection velocity of $E_{b0} \simeq 80keV$ NBI deuterium ions, with $v_A/v_{b0} \simeq 1/4$. The power available to sustain these modes may be a fraction of the total auxiliary heating power larger than in conventional tokamaks and thus stronger Alfvén type instabilities are expected in NSTX. The excitation is sensitive to the NBI injection angle. Particle losses were not seen during these instabilities.

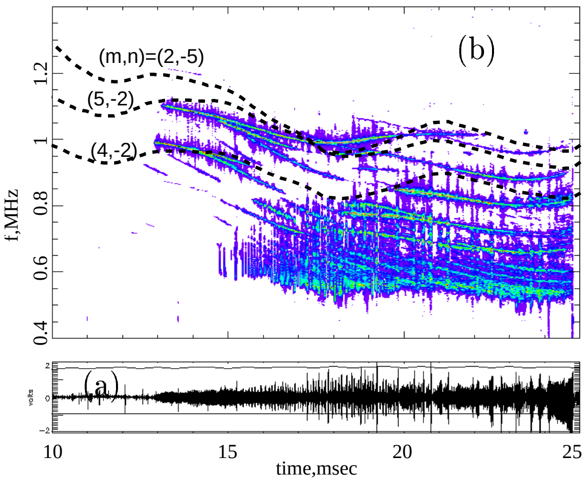

FIG. 1: *Time evolution of the Mirnov signal (a), and its frequency spectrum (b) in NSTX shot #108236.*

For the sub-cyclotron frequency instabilities under consideration, in the plasma there are shear Alfvén and compressional Alfvén (or magnetosonic) types of oscillations. For the purpose of application to the low aspect ratio plasma of Spherical Tokamaks (ST) we developed a new theory of radially and poloidaly localized Compressional Alfvén Eigenmodes (CAE)[3, 4]. CAEs were studied earlier in connection to the problem of ion cyclotron emission in tokamaks [5–11]. Initially observed instabilities were identified as CAEs driven

[1] This work supported by DoE contract No. DE-AC02-76CH03073

by energetic beam ions, since the predicted CAE frequency spectra were in agreement with the experiments, in which the magnetic signal peaks evolve parallel to each other. CAEs are localized radially at the plasma edge and poloidaly on the low magnetic field side of the torus. CAE frequencies are determined primarily by the Alfvén frequency at the mode location and the poloidal wave vector: $\omega_{CAE} = v_A m/r$, where m is the poloidal mode number, and r is the minor radius.

In new observations such as shown in FIG. 1 the spectrum peak lines intersect, indicating a more complicated dispersion than reported earlier for CAEs. We suggest that such modes are GAE's [12]. GAE's were found to be unstable in the nonlinear Hybrid MHD (HYM) code [13] modified for the ST geometry (to be reported elsewhere). In conventional tokamaks GAE's were considered stable against the pressure driven instability given by fast particles as a result of strong continuum damping. This is due to the toroidal coupling to the kinetic mode at the edge [15]. In this report we summarize the theory and observations of CAEs and GAE's in NSTX.

2. Compressional Alfvén Eigenmodes

For a typical low aspect ratio plasma of NSTX the dispersion of CAEs can be presented in the form [4]

$$\begin{aligned}\omega^2_{msn} \simeq\ & \frac{v^2_{A00}}{r_0^2}\left\{\frac{4(m+1/2)^2}{\kappa^2}\left(\epsilon_0-\alpha_0\right)+\frac{\kappa^4-1}{\kappa^4}\frac{m^2+m+3/2}{2}\right.\\ +\ & \left.\frac{2(2s+1)(2m+1)}{\kappa}\sqrt{\frac{(\epsilon_0-\alpha_0)(1+\sigma)}{2\sigma}}+n^2\left[\frac{q^2(r_0)}{\kappa^2}+\frac{R_0^2+4r_0^2}{4R_{00}^2}\right]\right\}.\end{aligned} \quad (1)$$

where m, n, and s are the poloidal, toroidal and radial wave numbers, $\alpha_0 \simeq B_\theta^2/2B_\varphi^2$ measures the weakening of the magnetic field well due to the poloidal field, $r_0 = 1/\sqrt{1+\sigma}$, κ is the ellipticity, where the plasma density profile was chosen as $n_e = n_{e0}(1-r^2/a^2)^\sigma$. The double subscript refers to the low field side point at $r = r_0$. To the lowest order in $m \gg 1$ the mode is localized at the low field side with a poloidal width of $\Theta = 1/\sqrt{\epsilon_0 - \alpha_0}$. Radially the CAE is localized within the domain $\Delta^2/r_0^2 = \kappa\sqrt{2\sigma/(1+\sigma)\left(\epsilon_0-\alpha_0\right)}/(2m+1)$. To avoid damping on the Alfvén continuum at the edge, CAEs must have low $k_{||}$ at the edge and thus low n's, so that k and the dispersion are primarily determined by the poloidal mode numbers. Note that m is a quantum number and becomes the cylindrical poloidal mode number in the limit of high m's and high aspect ratio[3, 4]. Spacing between the discrete peaks of the observed CAE frequency spectra is in agreement with the theoretically obtained dispersion expression. In discharges with pure CAEs the radial mode number corresponds to a frequency spacing on the order of $\Delta f_s \simeq 1MHz$ and is responsible for two bands of CAE peaks at around $f \simeq 0.9MHz$ and $f \simeq 1.8MHz$. Within each band peaks are typically separated by $\Delta f_m \simeq 100-150kHz$ corresponding to neighboring poloidal mode numbers m and $m+1$. Toroidal mode numbers produce the finest splitting of each frequency peak by $\Delta f_n \simeq 10-20kHz$. These features were observed in experiments[1].

3. Global shear Alfvén Eigenmodes

GAE's are formed just below the minimum of the Alfvén continuum $\omega \simeq \pm\omega_{Amin} = \left(k_{||}(r)v_A(r)\right)_{min}$[12]. The GAE eigenfrequency is slightly shifted downward from ω_{Amin}, and the shift depends on the q and density profiles. GAE is localized radially at the minimum of $\omega_{Amin}(r_0)$ and is dominated by one poloidal harmonic m. With the typically flat q profile the Alfvén continuum has a minimum at the plasma center, so that $\omega \simeq \pm v_{A0}\left(m/q_0 - n\right)/R_{ax}$, where R_{ax} is the major radius of the magnetic axes. One can see that if q is evolving in time, the eigenfrequencies of GAE's with different combinations of (m, n) will result in different evolutions. This can

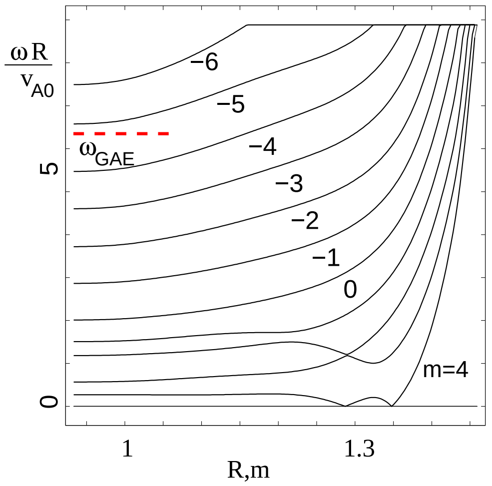

FIG. 2: *NOVA calculated Alfvén continuum and GAE eigenfrequency for NSTX shot #108236 at $t = 15msec$ and for $n = 2$. Included are poloidal harmonics from $m = -6$ to $m = 4$. Shown as a dashed line is the radial extent and the frequency of GAE with $(m, n) = (-5, 2)$.*

be seen from FIG. 1, where the instability peaks intersect. In FIG. 1 we plotted several simplified GAE eigenfrequencies $\omega \simeq -\omega_{Amin}$. The negative sign of the mode frequency is chosen to match the calculated frequency spectrum with the observed with toroidal mode numbers $|n| = 4 - 5$ and to satisfy the resonance condition of co-injected beam ions with GAE's (see next section). Some discrepancy with the measured frequencies are due to uncertainties in the measurements of the plasma parameters and in the reconstruction of the equilibrium with the EFIT code. We have calculated the AE continuum and one of the GAE's using the ideal MHD code NOVA [14] as shown in FIG. 2. The frequency of this mode in the laboratory frame is $f = (1.18 + nf_{rot})\,MHz$, where $f_{rot} \simeq 11kHz$ accounts for the plasma rotation. This frequency closely (within 5%) matches the observed frequency. The frequency separation between the different modes is primarily due to changes in n or m, so that as NOVA predicts $\Delta f = f_{m,n-1} - f_{m,n} = v_{A0}/R - f_{rot} = 175kHz$ or $\Delta f = f_{m+1,n} - f_{m,n} = v_{A0}/q_0 R = 150kHz$.

Finding the eigenmode structure requires numerical solution and was studied in Ref. [12]. Main damping contribution of GAE's is due to the interaction with the continuum on the tail of the eigenmode structure which propagates to the $m+1$ branch of the Alfvén continuum [15].

4. CAE/GAE Cyclotron Excitation by Beam Ions

For two branches of plasma oscillations one can find in WKB approximation that the electric field is polarized according to $E_1 = E_2\left(\epsilon_{11} - c^2k_{||}^2/\omega^2\right)/\epsilon_{12}$, where $\epsilon_{11} = \omega_{pi}^2/\left(\omega_{ci}^2 - \omega^2\right)$, $\epsilon_{12} = i\omega\omega_{pi}^2/\omega_{ci}\left(\omega_{ci}^2 - \omega^2\right)$, which in case of $|\omega| < \omega_{ci}$ yields $E_1 = E_2\epsilon_{12}\omega^2/k_\perp^2c^2$ for GAE's and in the limit of $k_{||}^2 \ll k^2$ $E_1 = -iE_2\omega_{ci}/\omega$ for CAEs. Here we have chosen direction 1 to be perpendicular to the vector $\mathbf{k}_\perp$, and to the equilibrium magnetic field.

The perturbation theory expression for the growth rate of the cyclotron instability driven by fast beam ions was derived in Ref. [16] and includes particle drift motion which strongly modifies the resonant interaction with the eigenmode:

$$\gamma_b = -\omega\int d\mathbf{r}\mathbf{E}^*\Im\epsilon_b^A\mathbf{E}/2\int d\mathbf{r}\mathbf{E}^*\epsilon\mathbf{E}, \tag{2}$$

where subscript b refers to the beam ions, and ϵ_b^A is the anti-hermitian part of beam ion dielectric tensor. For CAEs we obtain $\mathbf{E}^*\epsilon\mathbf{E} = |E_1|^2\,\omega_{pi}^2/\omega_{ci}^2$ and for GAE's $\mathbf{E}^*\epsilon\mathbf{E} = |E_2|^2\,\epsilon_{11}$. One can show that the strongest instability is the one with $l \neq 0$ in the resonance condition $\mathcal{R} \equiv \omega - l\omega_{cb} - k_{||}v_{||} = 0$, for which we restrict our analysis in this paper. In such a case the variation of the phase in the wave particle interaction comes from the $l\omega_{cb}$ term in the resonance condition. We introduce a function g according to the following. For CAE $g \equiv (J_0 - J_2)^2$ and for GAE $g \equiv (J_0 + J_2)^2$. In the case of co-injection we have to chose $l = -1$ and $\omega < 0$ to match the frequency of the measured and predicted GAE spectra. Then in the limit $\omega^2/\omega_{cb}^2 \ll 1$ the expression for the beam ion anti-hermitian part of the dielectric tensor is

$$\int d\mathbf{r}\mathbf{E}^* \Im\epsilon_b^A \mathbf{E} = -\frac{\pi\omega_{pb}^2\omega_{cb}}{n_b\omega^2}\int d\mathbf{r} dv_\| dv_\perp E_j^2 \left[\frac{\partial\left(v_\perp^2 g\right)}{\partial v_\perp} + \frac{v_\perp^3 g\omega}{v^2\omega_{cb}}\left(\frac{v\partial}{\partial v} - \frac{\lambda\partial}{\partial\lambda}\right)\right] f_b\delta\left(\mathcal{R}\right), \quad (3)$$

where $\lambda \equiv v_\perp/v$, ω_{pb} is the beam ion "plasma" frequency, j is 1 for CAE and 2 for GAE. NBI injection into NSTX is tangential. Typically the distribution function can be separated into two parts: almost tangentially confined passing with narrow width in pitch angle $f_{bp} = 3B^2\beta_b(1-\eta)e^{-\lambda/\delta\lambda_p}/v^3\delta\lambda_p^2 2^3\pi E_{b0}$ and trapped bump-on-tail in λ direction, which we assume as Gaussian with narrow width $f_{bt} = 3B^2\beta_b\eta\sqrt{1-\lambda_0^2}e^{-(\lambda-\lambda_0)^2/\delta\lambda_t^2}/v^3\delta\lambda_t\lambda_0 2^3\pi^{3/2}E_{b0}$, where η gives a fraction of trapped ions. For dense plasma η can be close to one, while for low density plasma it is much smaller than 1. The integrand determines conditions for the instability. We show the dependence of the driving term in the integrand of Eq.(3) in FIG. 3. From this figure one can conclude that at the low end for the instability one should have $1 < (-\omega/\omega_{cb})(v_{\perp b0}/v_A) < 2$ for CAEs and $2 < (-\omega/\omega_{cb})(v_{\perp b0}/v_A)\left(k_\perp/k_\|\right) < 4$ for GAE's. Both conditions can be met in NSTX. Note that the finite width of the distribution function in $v_\perp$(denote it $\delta\lambda_t$) is stabilizing. The requirement for such stabilization can be obtained from FIG. 3 so that for CAEs we need $\delta\lambda_t v > -v_A\omega_{cb}/\omega$, and for GAE's $\delta\lambda_t > -2v_A\omega_{cb}/\omega$. If the distribution function is narrower, than the assumption of a narrow pitch angle width of the trapped particle distribution function is reasonable. After some algebra we arrive at the expression for GAE growth rate:

$$\frac{\gamma_{bGAE}}{\omega} = \frac{T_i\beta_b}{E_{b0}\beta_i}\frac{3\pi\omega_{cb}\left(\omega_{cb}-\omega\right)}{2\omega^2}\left\{(1-\eta)\left[2+6\delta\lambda_p^2\frac{\omega}{\omega_{cb}}\right]+\eta\left[\frac{\partial\left(v_\perp^2 g_t\right)}{v_\perp\partial v_\perp}+\frac{\omega\lambda_0^2 g_t}{\omega_{cb}}\right]\right\},$$

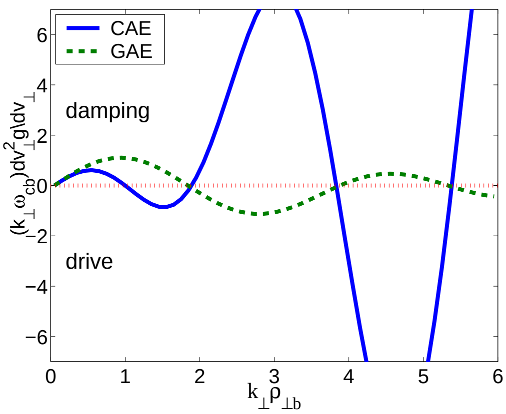

FIG. 3: *Driving factor from Eq.(3) which includes function g vs. FLR parameter* $k_\perp\rho_{\perp b}$.

where $g_t = g(\lambda = \lambda_0)$ and we assumed that beams ions and the plasma ions are the same species. For CAEs we obtain $\gamma_{bCAE} = \omega_{cb}^2\gamma_{bGAE}/\left(\omega_{cb}^2-\omega^2\right)$. These expressions give growth rates on the order of $\gamma_b/|\omega| \simeq n_{bt}/n_i \leq 1\%$, where n_{bt} is the characteristic density of trapped beam ions. The term due to passing particles in the growth rate expression may be driving if the width of the passing particle distribution function is $\delta\lambda_p^2 > -\omega_{cb}/3\omega$. The instability favors modes with low frequencies provided the resonance condition is satisfied. Typically the distribution function of beam ions becomes isotropic below the critical energy so that the drive is possible within the energy range $E_{b*} < E < E_{b0}$ or in NSTX $20keV < E < 80keV$, i.e. $v_{b0}/2 < v < v_{b0}$. Trapped particles can be in the resonance condition if their parallel velocity satisfies $v_\|/v_A > (1+\omega_{cb}/\omega)$ for GAE's and $v_\|/v_A > k_\perp(1+\omega_{cb}/\omega)/k_\|$ for CAEs.

5. Thermal Ion Stochastic Heating Due to High Frequency Alfvén Eigenmodes

A very important aspect of CAE instability in STs is their nonlinear evolution and saturation. An intriguing possibility has been suggested that large amplitude CAEs could stochastically heat the thermal ions [17]. Experimentally there is no enough evidence at the moment that CAE/GAE instabilities increase the background ion temperature [2]. We simulated this effect in more complex toroidal geometry (see FIG. 4) than in previous work [17]. The stochastic domain at low energy, corresponding to the plasma background ions, occurs at lower amplitude than in the slab plasma. The initial temperature profile of thermal ions is shown as a black curve in FIG.4. We used 21 CAEs to simulate measured CAE spectrum by edge Mirnov probes in NSTX with rms amplitude to be $\delta B/B = 0.5 \times 10^{-3}$ and with the collisional frequency $\nu = 0.01\omega_c$. Mode frequencies were in the range $0.2 < \omega/\omega_{cb}(R = R_{ax} + a) < 0.8$. The heating is proportional to the mode numbers if the amplitude is fixed. The stochastic heating could open a way of direct energy channeling from beam ions (or α-particles in a reactor) to the thermal plasma ions.

6. Conclusions

High frequency modes with frequencies below the fundamental cyclotron frequency of thermal ions were observed in NSTX. Based on the measured spectrum of high frequency modes we identify them as CAEs and GAE's. CAEs have similar time evolution as plasma parameters change, while GAE's may intersect due to q-profile relaxation. A theory has been developed to study the properties of these mode. Both types of instabilities are driven by the tangential NBI in NSTX. Beam ions excite CAE/GAE's through the Doppler shifted cyclotron resonance. The main source for the drive is the velocity space anisotropy of the beam ion distribution function. Our simulation of the effect the CAE/GAE's may have on plasma ions indicate that these modes may provide a channel for efficient energy transfer from fast ions directly to thermal ions.

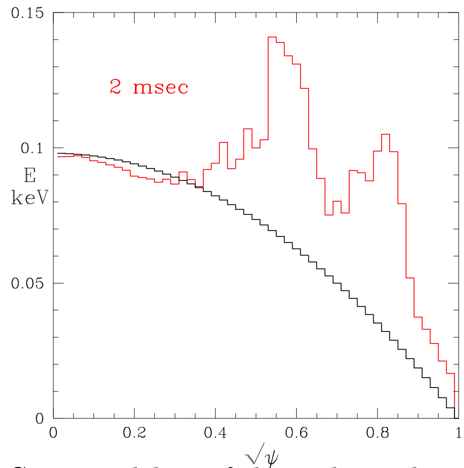

FIG. 4: *Modeling of thermal ions heating due to multiple CAEs during* 2*msec.*

[1] FREDRICKSON, E., *et. al.,* Phys. Rev. Lett. **87** (2001) 145001.
[2] FREDRICKSON, E., *et. al.,* Phys. Plasmas **9** (2002) 2069.
[3] GORELENKOV, N. N.,*et. al.,* Nucl.Fusion **42** (2002) 977.
[4] GORELENKOV, N. N., *et. al.,* Phys. Plasmas **9** (2002) 3483.
[5] GREENE, G. J., and the TFTR Group, in *Proceedings of 17th European Conference on Controlled Fusion and Plasma Heating*, Amsterdam, 1990, Part IV, Vol. 14B, p.1540.
[6] COTTRELL, G. A., DENDY, R. O. , Phys. Rev. Lett. **60** 33 (1988).
[7] MAHAJAN, S. M., and Ross, D. W., Phys. Fluids **26** 2561 (1983).
[8] COPPI, B., *et.al.*, Phys. Fluids **29** 4060 (1986).
[9] GORELENKOV, N. N., and CHENG, C. Z., Nucl. Fusion **35** (1995) 1743.
[10] FŰLŐP, T., *et.al.,* Nucl. Fusion **37** 1281 (1997).
[11] KOLESNICHENKO, Ya., *et.al.*, Nucl. Fusion **38** 1871 (1998).
[12] APPERT, K., *et. al.,* Plasma Physics **24** (1982) 1147.
[13] BELOVA E. V. et al., Phys. Plasmas **7** (2000) 4996. BELOVA E. V., *et.al.,* "Numerical simulations of CAE modes in NSTX", Bull. Am. Phys. Soc. v. 46, 334 (2001)
[14] CHENG, C. Z. , Phys. Reports **211** (1992) 1.
[15] VAN DAM, J.W., *et. al.,* Fusion Technology **18** (1990) 461.
[16] GORELENKOV, N. N., and CHENG, C. Z., Phys. Plasmas **2** (1995) 1961.
[17] GATES, D. A., *et. al.,* Phys. Rev. Lett. **87** (2001) 205003.