Plasma and Momentum Transport Processes in the Vicinity of a Magnetic Island in a Tokamak

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Abstract. In the vicinity of a magnetic island in tokamaks, the toroidal symmetry in the magnitude of the magnetic field |***B***| is broken. This leads to enhanced radial transport fluxes and momentum dissipation. The radial electric field can be determined from the quasineutrality condition, or equivalently the momentum equation, on the island magnetic surface. The equation that governs the radial electric field is nonlinear and can have bifurcated solutions. This may suppress turbulence fluctuations and improve plasma confinement. The theory remains valid for a rotating island with an appropriate re-interpretation of the radial electric field.

I. Introduction

Magnetic islands are nearly ubiquitous in magnetically confined plasmas. They play an important role in fusion plasmas through their effect on plasma confinement. For example, low *m* neoclassical islands limit the plasma beta, which is the ratio of plasma pressure to magnetic field pressure, in fusion grade plasmas [1-3]. Here, *m* is the poloidal mode number. When magnetic islands are present, the equilibrium symmetry in the magnitude of the magnetic field $B = |\mathbf{B}|$ is broken. The broken symmetry in B is, however, usually ignored. This is because the perturbed magnetic field strength δB due to magnetic islands is small compared with the equilibrium value of B. The symmetry breaking effect on B is thought to be of order $(\delta B/B)^2 << 1$. However, if B is not spatially uniform, *e.g.*, B=B(x), with x the radial variable, the symmetry breaking effect in B is of the order of $B'(x_0)(\Delta x)$, Here, x_0 is the position of the singular layer, prime denotes d/dx, and Δx is the width of the island [4]. Because Δx is proportional to $(\delta B/B)^{1/2}$, the symmetry breaking effect becomes much more important than previously perceived. Here, we discuss the implications of this broken symmetry in B on plasma confinement, and momentum dissipation, and extend the theory to a rotating island.

II. Magnetic Field Model

The magnetic field strength B in a large aspect ratio tokamak is $B/B_0 = 1 - \varepsilon \cos\theta$, where B_0 is B at the magnetic axis, ε is the inverse aspect ratio, and θ is the poloidal angle. In the presence of a magnetic island, the magnetic flux surface is distorted. On this distorted surface, B is modified to [4]

$$\frac{B}{B_0} = 1 - \left[\frac{r_S}{R} \pm \frac{r_W}{R} \left(\overline{\Psi} + \cos\xi \right)^{1/2} \right] \cos\theta \ , \tag{1}$$

where r_S is the minor radius r at ψ_S, R is the major radius, $\overline{\Psi} = - \Psi/\tilde{\psi}$ a normalized radial island coordinates, and $r_W = [2q_S^2 \tilde{\psi}/(q_S' B_0 r_S)]$ is a measure of the island width, ψ_S is the poloidal flux function at the singular surface, Ψ is the helical flux function, q_S is the safety factor and $q_s^{\prime} = dq/dr$ at ψ_S, respectively. The helical angle $\xi = m\,(\theta - \zeta / q_S)$ in which ζ, is the toroidal angle. It is clear that toroidal symmetry in B is broken in tokamaks due to the presence of the islands.

The symmetry breaking is especially significant for low m islands. For an m=2 mode, r_w / r_s can be of the order of 10%. This is similar to a rippled tokamak with toroidal ripple strength of the order of a few percent in the core region which results from the finite number of toroidal field coils. With this magnitude of symmetry breaking perturbation in B, transport and dissipation processes in the vicinity of the island will be modified significantly from the standard neoclassical theory [5-7]. The transport processes in this region are similar to those in stellarators.

III. Plasma Confinement

Because the toroidal symmetry is broken, trajectories of the toroidally trapped particles, *i.e.*, bananas, are no longer closed on themselves in a poloidal plane. They drift off the perturbed helical flux surface Ψ. This leads to enhanced transport over the conventional neoclassical fluxes. It has been shown [4] by solving the drift kinetic equation that when the standard collisionality parameter $\nu_* < 1$, the flux-surface-averaged particle flux $\Gamma = \langle N\boldsymbol{V}\cdot\boldsymbol{\nabla}\Psi\rangle$ is

$$\Gamma = -\frac{C_1}{2}\frac{\left(I\mathbf{n}_0\bullet\nabla\theta\right)^2}{M^{7/2}\Omega^2}\left(\frac{q_s^{\prime}}{q_s}r_w\right)^2 m^2\delta_w^2\,\varepsilon^{3/2}\,H(\overline{\Psi})\int dW\,W^{5/2}\frac{1}{\nu}\frac{\partial f_M}{\partial\Psi}, \qquad (2)$$

where N is plasma density, M is mass, Ω is the gyro-frequency, $\mathbf{n}_0$ is a unit vector in the direction of the unperturbed magnetic field, $\boldsymbol{V}$ is the flow velocity, $\delta_W = r_w/R$, f_M is a Maxwellian distribution function, angular brackets denote a flux-surface-average on the island magnetic surface, ν is the collision frequency, C_1= 0.684, $W = M\mathrm{v}^2/2$, $H(\overline{\Psi})$ is a form factor that is a function of $\overline{\Psi}$ and can be inferred from results given in Ref.[4]. The flux in Eq.(2) is applicable in the region outside the island. In cylindrical coordinates, the radial flux $\Gamma_r \sim 0.5N(cT/eBr)(m^2\delta_w^2\varepsilon^{3/2}/\nu)H(\overline{\Psi})[(dP/dr)/P + e(d\Phi/dr)/T + 2.5(dT/dr)/T]$ if one uses the approximation $\nu \sim \mathrm{v}^{-3}$ in evaluating the energy integral. Here, T is the temperature, P is the pressure, e is the charge, Φ is the electrostatic potential, and c is the speed of light. Note, however, because the transport process is relative to the distorted helical flux surface, the cylindrical coordinate form is only for the reference purposes. The heat flux Q is similar to the particle flux given in Eq.(2) except there is an extra factor $(W - 5T/2)$ in the $\int dW$ integral. The ratio of the heat flux to that of the standard axisymmetric banana regime flux [5-7] is of the order of $(m\delta_W/\varepsilon\nu_*)^2$. Thus, when $\nu_* < 1$, the island induced transport fluxes can be larger than the banana flux (for both electrons and ions) and significant when compared with the ion anomalous transport fluxes. As can be seen from Eq. (2), the flux increases as the collision frequency decreases. However, this $1/\nu$ dependence cannot persist indefinitely. Eventually the finite drift orbit width will limit the transport. Neglecting super bananas and the effects of the collisionless detrapping/retrapping, we find, by solving the drift kinetic equation, that when the collision frequency $(\nu/\varepsilon) < \omega_E (RB_P)(q_S' r_W / q_s)$, the particle flux becomes [8]

$$\Gamma = -0.22\, N \nu (cT/eBr)^2 (\delta_W / \omega_E)^2 \varepsilon^{-1/2} G(\overline{\Psi})(P'/P + e\Phi'/T - 0.5T'/T), \qquad (3)$$

where primes denote $d/d\Psi$, $\omega_E = cE_\Psi/(Br)$ is the $E\times B$ angular speed, $E_\Psi = -d\Phi/d\Psi$ is the radial electric field, $G(\overline{\Psi})$ is a form factor defined in Ref.[8]. The singularity at $\omega_E = 0$ can be removed either by joining Eq.(3) to Eq.(2) or including the effects of the super bananas. The cylindrical form can be obtained by replacing $d/d\Psi$ with d/dr in Eq.(3). The flux depends nonlinearly on the radial electric field. It decreases when E_Ψ increases.

IV. Radial Electric Field

The radial electric field in the vicinity of an island can be determined by the quasi-neutrality condition: $\Gamma_i = \Gamma_e$, or equivalently the momentum equation. Here Γ_i is the radial ion particle flux, and Γ_e is the electron particle flux. Combining Eqs. (2) and (3), we obtain an equation for the electric field [8]

$$m^2 (X/C)^3 + m^2 (X/C)^2 + [(M_i/M_e)^{1/2}(\nu_i/\varepsilon)^2 + (\nu_i/\varepsilon)^2](X/C^3)$$

$$- [(M_i/M_e)^{1/2}(\nu_i/\varepsilon)^2 - (\nu_i/\varepsilon)^2]/C^2 = 0, \qquad (4)$$

where $X = \omega_E (RB_P)(q_S' r_W / q_s)$, $C = (cT/|e|Br)(RB_P)(q_S' r_W / q_s)(N'/N)$. To obtain Eq.(4), we only use the $1/\nu$ flux for electrons and neglect the temperature gradient for simplicity. We have also neglected some form factors, *e.g.*, $G(\overline{\Psi})$ in Eq.(4), to keep only the salient nonlinear feature in X. Equation (4) can have multiple equilibrium solutions. Examples of the solutions are shown in Figs. 1 – 3 for the parameters $C = -0.5$, $m = 2$ and $(M_i/M_e)^{1/2} = 43$ with M_i the ion mass and M_e the electron mass. There is one equilibrium solution for $\nu_i/\varepsilon = 0.1$ as shown in Fig. 1. When ν_i/ε decreases to 0.0316, there are three equilibrium solutions as shown in Fig.2. The one in the middle is not stable. The new equilibrium solution has a larger value of the radial electric field and has the opposite sign. This new equilibrium solution is likely to have better plasma confinement. As ν_i/ε decreases further, the two roots on the left almost merge into one and there is one stable solution with better confinement. This is shown in Fig. 3. Thus, it is possible that in the vicinity of a magnetic island, the radial electric field can bifurcate to a large value. This in turn will suppress the turbulent fluctuations due to the radial gradients of the $\boldsymbol{E}\times\boldsymbol{B}$ drift and the diamagnetic drift, and improve the overall plasma confinement in the vicinity of a magnetic island. This mechanism for the confinement improvement is the same as the one employed in the H (high) -mode theory [9,10]. The difference is only in the bifurcation mechanism. It has been observed in tokamak experiments that plasma confinement improves in the vicinity of lower order rational surfaces [11-15]. The theory presented here may play a role in understanding that phenomenon even though the control parameter does not depend q because there can be magnetic islands centered on the lower order rational surfaces in tokamaks. The theory may be also applicable for stellarators. To check the theory, the electric field in the vicinity of a magnetic island needs to be measured.

V. Rotating Island

So far we have assumed that the island rotation frequency ω vanishes (in the laboratory frame) or is small. However, some islands do rotate. We extend the theory to a rotating island. The helical angle ξ is now defined as $\xi = m\ (\theta - \zeta/q_S) + \omega t$. The magnetic field model given in Eq.(1) implicitly depends on time. We assume that the electric field parallel to $\boldsymbol{B}$, $E_{||}$, vanishes. The electrostatic potential Φ has the form [16]

$$\Phi = -(\omega q/mc)\ (\psi - \psi_S) + F(\Psi), \tag{5}$$

where $F(\Psi)$ is an integration constant. We find the theory developed in Sec. III and IV remains valid for a rotating island if we replace $d\Phi/d\Psi$ in the non-rotating theory by $dF(\Psi)/d\Psi$. Thus, the bifurcation is in the quantity $dF(\Psi)/d\Psi$ when the island rotates. Note that $F(\Psi)$ does not have to be the same as the density profile $N(\Psi)$ or the temperature profile $T(\Psi)$ in the theory. The profiles of $N(\Psi)$ and $T(\Psi)$ are determined from the particle and energy transport equations, respectively .

Once the island is allowed to rotate, the theory becomes related to island rotation theory. In an island rotation theory, there are at least three unknowns that need to be determined: ω, island width r_w, and $F(\Psi)$. It is known that the island rotation frequency ω is determined from the $sin\xi$ component of the Ampere's law, and the island width is determined from the $cos\xi$ component of the Ampere's law [17,18,19]. In the past, the profile function $F(\Psi)$ has been determined using either a transport equation or a vorticity equation with a Braginskii viscosity [20-22]. Here, we determine it from the $(\boldsymbol{B}\times\nabla\Psi)/B^2$ component of the total momentum equation using the island induced nonlinear viscosity:

$$\langle \boldsymbol{J}\bullet\nabla\Psi\rangle = \langle \boldsymbol{B}\times\nabla\Psi\bullet\nabla\bullet\Sigma_j \boldsymbol{P}_j\ /\ B^2\rangle, \tag{6}$$

where $\boldsymbol{J}$ is plasma current density, and $\boldsymbol{P}_j$ is the pressure tensor for the species j. The equilibrium quasineutrality condition implies $\langle \boldsymbol{J}\bullet\nabla\Psi\rangle = 0$. An island rotation theory based on the theory developed here is under investigation.

VI. Toroidal Momentum Evolution within the Island Magnetic Surface

It has been shown that the fluxes shown in Sec.III are related to toroidal viscosity [23, 24] $\Gamma=(c/e)\langle\nabla\Psi\times\nabla\theta\bullet\nabla\bullet\boldsymbol{P})/\boldsymbol{B}\bullet\nabla\theta\rangle$. In Hamada coordinates $\boldsymbol{B}\bullet\nabla\theta$ is a flux function and $\boldsymbol{P}$ reduces to the plasma viscous tensor $\boldsymbol{\pi}$. (The plasma pressure term is averaged to zero in Hamada coordinates.) The toroidal momentum evolution on the island magnetic surface is thus

$$\partial\langle NM\boldsymbol{V}\bullet\nabla\Psi\times\nabla\theta/\boldsymbol{B}\bullet\nabla\theta\rangle\ \partial t = \langle \boldsymbol{J}\bullet\nabla\Psi\rangle - \Sigma_j\, e_j\, \Gamma_j\ /c, \tag{7}$$

where the subscript j indicates plasma species and the particle fluxes Γ_j are those given in Eqs.(2) and (3). The left side of Eq.(7) is approximately $\partial\langle NMRV_\zeta\rangle/\partial t$ where V_ζ is the toroidal flow speed. The convective inertia term is neglected in Eq.(7). Equations. (6) and (7) are equivalent if one neglects the inertia term.

The viscosity shown in Eqs.(2) and (3) is a result of the resonant modes that form the islands to remove the singularity at the rational surfaces. For modes that are not resonant on the rational surfaces, the viscosity calculated in [25] can be used to describe the momentum dissipation processes when these modes are present [26].

VI. Conclusions

We have developed a theory for the transport processes in the vicinity of a magnetic island in tokamaks where the toroidal symmetry in $|\boldsymbol{B}|$ is broken. This leads to enhanced transport fluxes that can be comparable to the anomalous ion transport flux. The radial electric field can now be determined from the quasineutrality condition, or equivalently the momentum equation on the island magnetic surface. We find that the equation that governs the radial electric field is nonlinear. It can have bifurcated solutions. After electric field bifurcation, turbulence can be suppressed and confinement can be improved [10]. This mechanism may play a role in the confinement improvement in the vicinity of the lower order rational surfaces observed in tokamak experiments [11-15]. The theory is extended to a rotating island. We find the non-rotating theory remains valid for a rotating island in tokamaks if we replace $d\Phi/d\Psi$ in the non-rotating theory by $d\, F(\Psi)/d\Psi$. The theory for a rotating island is also related to the island rotation theory. Our theory differs from the conventional island rotation theory in that we determine $F(\Psi)$ from the island-induced nonlinear viscosity instead of Braginskii viscosity. The theory will be incorporated in NCLASS [27] to simulate transport processes in tokamaks with islands.

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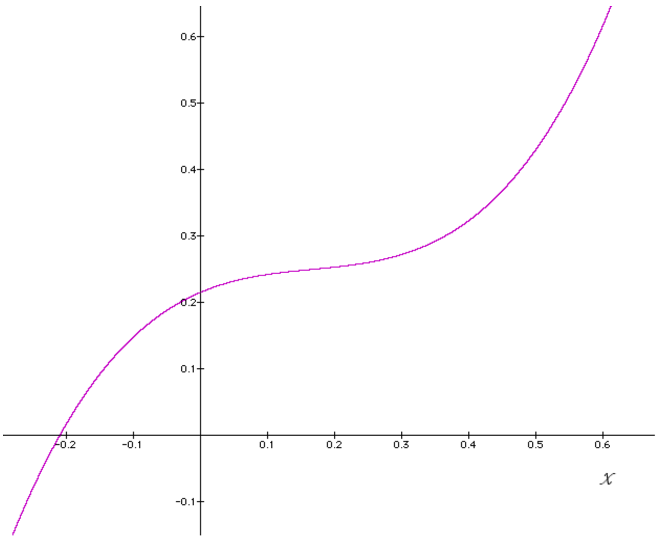

FIG. 1. The parameter $\nu_i/\varepsilon = 0.1$. There is one equilibrium solution.

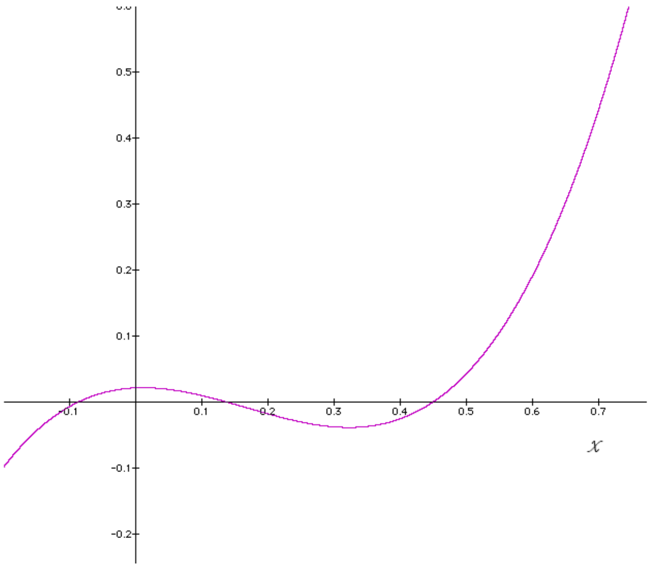

FIG. 2. The parameter $\nu_i/\varepsilon = 0.0316$. There are three equilibrium solutions. The one in the middle is unstable. The one on the right is the new solution.

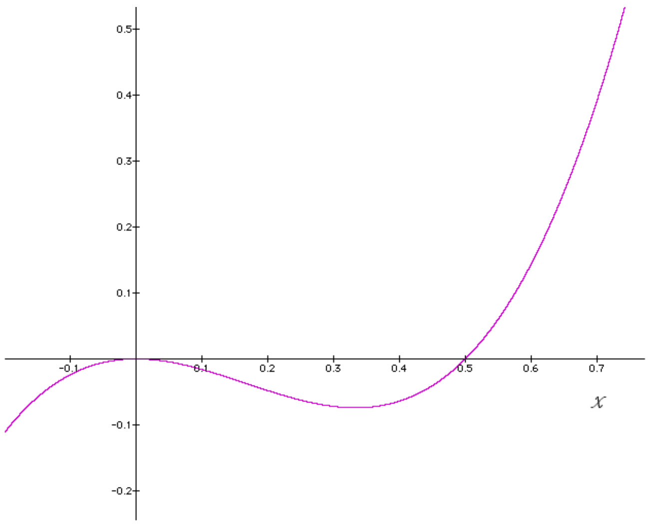

FIG. 3. The parameter $\nu_i/\varepsilon = 0.0001$. The two solutions on the left almost merge into one.