Overview of Magnetic Structure Induced by the TEXTOR–DED and the Related Transport

S.S. Abdullaev 1), K.H. Finken 1), M.W. Jakubowski 1,3), M. Kobayashi 1), D. Reiser 1), A.M. Runov 2), D. Reiter 1)

1) Institut für Plasmaphysik, Forschungszentrum Jülich GmbH, EURATOM– Association, Trilateral Euregio Cluster, D–52425 Jülich, Germany
2) Max-Planck Institut für Plasmaphysik, D-17491, Greifswald, Germany
3) University of Opole, Institute of Physics, 45-052, Opole, Poland
e-mail contact of main author: s.abdullaev@fz-juelich.de

Abstract. The Dynamic Ergodic Divertor (DED), a new concept of the ergodic divertor, is presently installed for the TEXTOR tokamak. Beside the conventional ergodic divertor operation the DED also permits the operation with a rotating magnetic field which allows, in particular, to broaden the heat deposition pattern on the divertor plates. Since its first proposal of the DED in 1996 the structure of magnetic field, especially, the onset of ergodic zone of field lines and related transport in the DED operation has been extensively studied using different theoretical and numerical methods. New methods to study the magnetic field, in particular, the field line mapping have been developed. The presentation gives the overview of the studies on the structure of magnetic field in the DED, the formation of the ergodic and laminar zones of field lines at the plasma edge. It also includes studies on the modelling efforts of the transport of heat and particles in the ergodic and laminar zones.

1.Introduction

The concept of the Ergodic Divertor has been introduced for the control of the plasma edge by creating ergodic magnetic field lines there using the external coils. The TEXTOR tokamak is presently open for the installation of the Dynamic Ergodic Divertor (DED), a new tool for the control of the plasma edge. Beside the conventional concept of the ergodic divertor implemented in the tokamaks TEXT and Tore-Supra the DED also permits the operation with a rotating magnetic field which allows, in particular, to broaden the heat deposition pattern on the divertor plates. Since the first proposal of the DED in 1996 the structure of magnetic field, especially, the onset of ergodic zone of field lines and related transport in the DED operation has been extensively studied using different theoretical and numerical methods [1-11]. New mapping method to integrate Hamiltonian systems, in particular, the field line mapping have been also developed to study the chaotic magnetic field lines. Below we present the main results of these studies.

2. Magnetic field structure

The DED coils and spectrum of magnetic perturbations. The set of the DED coils which creates the external resonant magnetic perturbations at the plasma edge are located inside the vacuum vessel at the high field side (HFS) of the torus. It consists of 16 individual helical coils (four quadruples) plus two compensation coils, each winding once around the torus, following the equilibrium magnetic field of the plasma edge. The current distribution on the coils for the standard DED operation is $I_j = I_c \sin(\pi j/2 + \Omega t)$, where j ($j = 1, \ldots, 16$) stands for a coil number, I_c is a divertor current amplitude ($I_c \leq 15$ kA), and Ω is a rotation frequency of the perturbed field . Such a coil system creates magnetic perturbations given by the one component of the vector potential localized at the HSF, i.e., $A_\varphi(r, \theta, \varphi) = \sum_m A_m(r, \theta) \cos(m\theta - n\varphi - \Omega t)$ with Fourier components

$$A_m(r, \theta) \approx (-1)^{m+m_c} B_c r_c \sqrt{\frac{R_0}{R_0 + r\cos\theta}} \frac{\sin[(m - m_c)\Delta\theta_c/2]}{\pi m(m - m_c)} \left(\frac{r}{r_c}\right)^m , \tag{1}$$

where $B_c = 2\mu_o I_c n/\Delta\theta_c r_c$ is a characteristic strength of the magnetic perturbation determined by the divertor current I_c, the minor radius of coils r_c, and the poloidal angular extension of coil set $\Delta\theta_c$. Here R_0 is a major radius of the torus. The poloidal, m, spectrum of perturbation A_m is localized near the mode number m_c and possesses several major modes at each side of m_c. The perturbed field has the toroidal mode $n=4$, and it strongly decays along the radial coordinate: $A_\varphi \propto r^{m_c}$ ($m_c \approx 20$).

Hamiltonian field line equations. The Hamiltonian approach is the most natural and convenient method to study the stochastization of magnetic field lines. The magnetic field line equations $d\mathbf{x}/d\tau = \mathbf{B}$, where τ is length along a field line, can be formulated in Hamiltonian form introducing the a magnetic (Boozer) coordinate system $(\psi, \vartheta, \varphi)$: ϑ is an (intrinsic) poloidal angle, and ψ is a toroidal flux. In these coordinates the magnetic field has a canonical form: $\mathbf{B} = \nabla\psi \times \nabla\vartheta + \nabla\varphi \times \nabla H$, where the function $H = H(\psi, \vartheta, \varphi)$ is a Hamiltonian function of the canonical field line equations

$$d\vartheta/d\varphi = \partial H/\partial\psi, \qquad d\psi/d\varphi = -\partial H/\partial\vartheta. \tag{2}$$

Field lines of the equilibrium magnetic field are described by the axisymmetric Hamiltonian $H \equiv H_0(\psi) = \int d\psi q^{-1}(\psi)$, where $q(\psi)$ is a safety factor. In a magnetic coordinate system they are straight lines, $\vartheta = \varphi/q(\psi)$. For any non-axisymmetric magnetic perturbations in tokamaks the Hamiltonian function can be presented in the form

$$H = H_0(\psi) + \epsilon \sum_m H_m(\psi)\cos(m\vartheta - n\varphi - \chi), \tag{3}$$

where ϵ is a dimensionless perturbation parameter, determined by the ratio of the characteristic strength of the perturbed magnetic field, B_c to the toroidal magnetic field B_t, i.e., $\epsilon = B_c/B_t$. The terms $\epsilon H_{mn}(\psi)\cos(m\vartheta - n\varphi - \chi)$ correspond to the resonant magnetic perturbations, and $\chi = \Omega t$ is a phase.

Mapping method of integration of the field line equations. To study Hamiltonian field line equations a new symplectic mapping method has been developed [12,13]. It is much faster than the standard integration methods of ordinary differential equations, and constructed in a flux-preserving form. The mapping relates the cross-section points, (ϑ_k, ψ_k), of the field line $(\vartheta(\varphi), \psi(\varphi))$ with the poloidal section $\varphi = \varphi_k = (2\pi/ns)k$, $(k = 0, \pm1, \pm2, \ldots)$ with the ones $(\vartheta_{k+1}, \psi_{k+1})$ at $\varphi = \varphi_{k+1}$. The integer number s ($s \geq 1$) stands for the number of the map steps per one period $2\pi/n$ of the perturbation along the torus. The map has the following symmetric form

$$\Psi_k = \psi_k - \epsilon\frac{\partial S_k}{\partial\vartheta_k}, \qquad \Theta_k = \vartheta_k + \epsilon\frac{\partial S_k}{\partial\Psi_k}, \tag{4}$$

$$\Psi_{k+1} = \Psi_k, \qquad \Theta_{k+1} = \Theta_k + \frac{\varphi_{k+1} - \varphi_k}{q(\Psi_k)}, \tag{5}$$

$$\psi_{k+1} = \Psi_{k+1} + \epsilon\frac{\partial S_{k+1}}{\partial\vartheta_{k+1}}, \qquad \vartheta_{k+1} = \Theta_k - \epsilon\frac{\partial S_{k+1}}{\partial\Psi_{k+1}}, \tag{6}$$

where $S_k = S(\vartheta, \Psi, \varphi; \epsilon)|_{\varphi=\varphi_k}$ is a generating function. In the first order of ϵ it is determined by

$$S(\vartheta, \Psi, \varphi) = -(\varphi - \varphi_0)\sum_m H_m(\Psi)\left[a(x_{mn})\sin(m\vartheta - n\varphi - \chi) + b(x_{mn})\cos(m\vartheta - n\varphi - \chi)\right], \tag{7}$$

where $x_{mn} = (m/q(\Psi) - n)(\varphi - \varphi_0)$, $a(x) = (1 - \cos x)/x$, and $b(x) = \sin x/x$. The Poincaré map may be obtained by applying the map (4)-(6) s times.

Spectrum of magnetic perturbations in a toroidal system and the onset of the stochastic magnetic field lines. The Hamiltonian form of field line equations (2), (3) allows one to study directly the formation of the ergodic zone using, both the qualitative Chirikov's criteria and the symplectic methods of integration. Each of (m, n) - perturbations creates a magnetic island of the width $\Delta\psi_{mn} = 4|\epsilon H_{mn}(\psi)/(dq^{-1}/d\psi)|^{1/2}$ at the resonant magnetic surfaces $\psi = \psi_{mn}$ ($q(\psi_{mn}) = m/n$). For the certain level of perturbation ϵ the neighboring magnetic islands start to overlap that leads to the stochastization of field lines and to the formation of the ergodic zone at the plasma edge. The degree of ergodization can be regulated by the variation of the plasma current (or magnetic field), the plasma β_{pol} and the divertor current. The variation of the plasma current I_p (or toroidal magnetic field B_t) shifts the radial positions, $r_{mn} \approx R_0\sqrt{2\psi_{mn}}$, of resonant magnetic surfaces shown in Fig. 1 a.

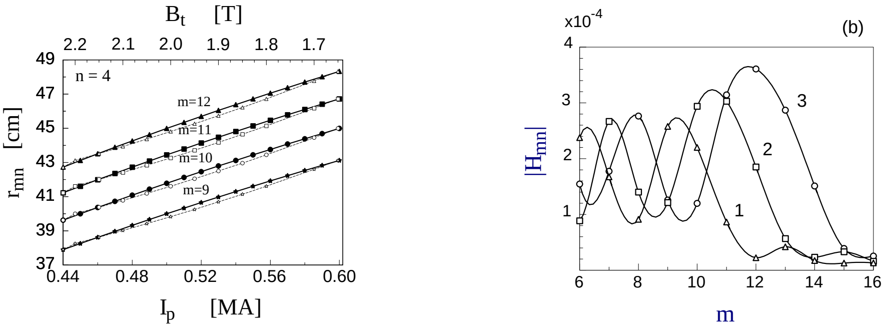

FIG. 1. a) Radial positions of r_{mn} vs I_p (solid curves) at a fixed $B_t = 2.25$ T and vs B_t (dashed curves) at the fixed I_p= 0.44 MA; b) Spectrum $|H_m|$ for different values of β_{pol}: 1 − 1.0, 2 − 0.6, 3 − 0.2, (I_p= 0.5 MA, $B_t = 1.875$ T, I_c= 15 kA).

By changing the plasma β_{pol} one can vary the spectrum of magnetic perturbations $H_m(\psi)$. The specific features of the relation of $H_m(\psi)$ with the vacuum magnetic perturbations, A_m (1), in a toroidal system have been established. It was found that the transformation matrix, $S_{mm'}$, i.e., $H_m(\psi) \approx \sum_{m'} S_{mm'}A_{m'}$, does not simply connect the neighboring $m' = m \pm 1$ modes, but it has general coherent features [5]. For the perturbation field located on the HFS it is determined by the Airy function $\mathrm{Ai}(x)$:

$$S_{mm'} \approx \frac{(-1)^{m+m'}}{(\beta_3 m'/2)^{1/3}}\mathrm{Ai}\left(\frac{\beta_1 m' - m}{(\beta_3 m'/2)^{1/3}}\right),$$

where $\beta_1 = d\theta/d\vartheta|_{\theta=\pi}$ and $\beta_3 = d^3\theta/d\vartheta^3|_{\theta=\pi}$ are the first and third derivatives of the poloidal angle with respect to the intrinsic angle ϑ, respectively, taken on the HFS. Since the derivatives β_1, β_3 mainly depend on the plasma β_{pol} it allows one to control the spectrum H_m by simple varying β_{pol}. Fig. 1 b shows an example of such a variation of H_m at the resonant magnetic surface $r = r_{mn}$ ($m = 10$).

The DED perturbation field creates the ergodic zone of field lines by overlapping of a several $m : n$ magnetic islands at the plasma edge near the resonant magnetic surface $q = 3$. At the standard operational regime the poloidal numbers m are in the interval: $10 \leq m \leq 14$ at the fixed toroidal mode $n = 4$. The degree of ergodization may vary by changing the positions of the resonant magnetic surfaces, r_{mn}, or the plasma parameter, β_{pol}. As seen from Fig. 1 one can increase the ergodization by decreasing β_{pol} at the fixed r_{mn}, or by outward shift r_{mn} at the fixed β_{pol}.

3. Properties of the ergodic and the laminar zones

The stochastization of field lines creates the region of open field lines at the plasma edge connecting to the wall. This region may be roughly divided into two zones, the ergodic and the laminar zones (see Figs. 2, 3). The ergodic zone consists of field lines with large connection lengths (more than six poloidal turns), while the zone of field lines with a few polodial turns is called a laminar zone. The variation of these zones by changing the plasma parameters, for example, the plasma current or the toroidal magnetic field, allows one to study the different regimes of the plasma edge extended from ergodic dominated edge to those similar to normal helical divertor structures. In Fig. 2 Poincaré sections of two type of the ergodized edge plasma are displayed for the plasma current: a) $-I_p = 420$ kA, b) $-I_p = 530$ kA. (The plasma parameters are: the major radius $R_0 = 175$ cm, the plasma radius $a = 46.7$ cm, the center of the last magnetic surface $R_a = 174$ cm, the plasma $\beta_{pol} = 1$. the divertor plate $r_d = 47.7$ cm, the divertor current I_c= 15 kA, the radial position of coils $r_c = 53.25$ cm). Fig. 2a describes the case with the dominated ergodic zone, while Fig. 2b shows the one with the dominated laminar zone. It is convenient to present the laminar zone by the contour plot of

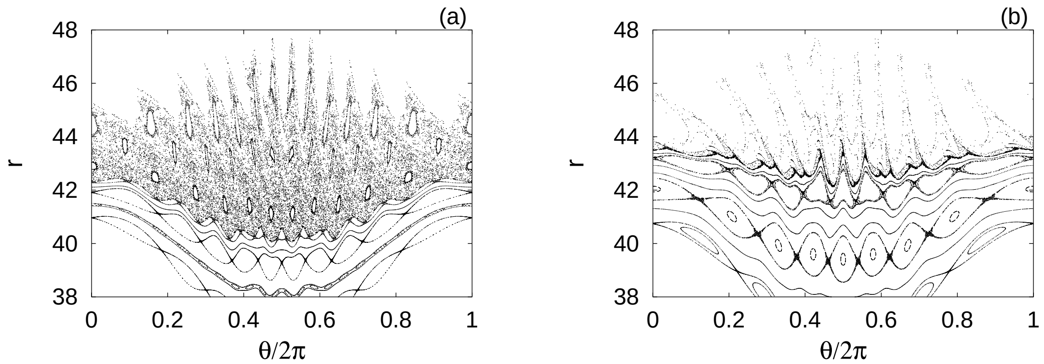

FIG. 2. Poincaré sections of magnetic field lines in the (r, ϑ)*:*
(a) $I_p = 420$ *kA; (b)* $I_p = 530$ *kA.*

regions (on the (ϑ, ψ)- plane) with the different wall to wall connection lengths (in a number of poloidal turns N_p). Such an example is shown in Fig. 3.

The magnetic field lines at the plasma edge may be viewed as chaotic scattering system whereby field lines enter into the plasma edge from the wall and leave when hitting the wall after a certain number of toroidal (or poloidal) turns [7]. The length of field lines inside the plasma region is very sensitive to their initial coordinates: a tiny change of the input conditions can produce drastic changes in the length of field lines.

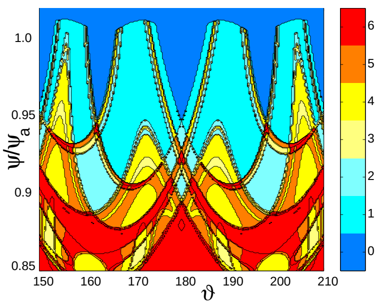

Fig. 3. Contour plot of the connection lengths in poloidal turns N_p*.*

4. Transport of heat in the ergodic and laminar zones

Several approaches to study the heat and the particle transport in the ergodic and the laminar zones have been developed. The first of these methods, the *three–dimensional Monte–Carlo code* for the heat balance equation, is based upon the "multiple local magnetic coordinate system approach" [10]. The main idea of this method is to solve the geometrical problem with the help of Lagrangian representation. Indeed, one can represent any diffusion-like equation as a random-walk process of the corresponding "test particles" (e.g., heat elements in case of temperature equation), and enforce that the motion of these particles follows the magnetic field. The details of the algorithm are described in Ref.[10].

It has been applied to the TEXTOR-DED with the partially ergodic magnetic field configuration. The plasma temperature fields and the profiles of the radial component of heat flux due to the classical parallel and anomalous perpendicular diffusion have been calculated. The results for realistic TEXTOR field obtained from DIVA-GOURDON [2] code are presented on the Fig. 4. For a parallel diffusion coefficient $\chi_{||}$ classical (Braginskii) expressions are used, and the anomalous cross-field diffusion coefficient $\chi_{\perp}$ is taken as 3 m^2/s. One can see the strong influence of the magnetic field on the temperature profile.

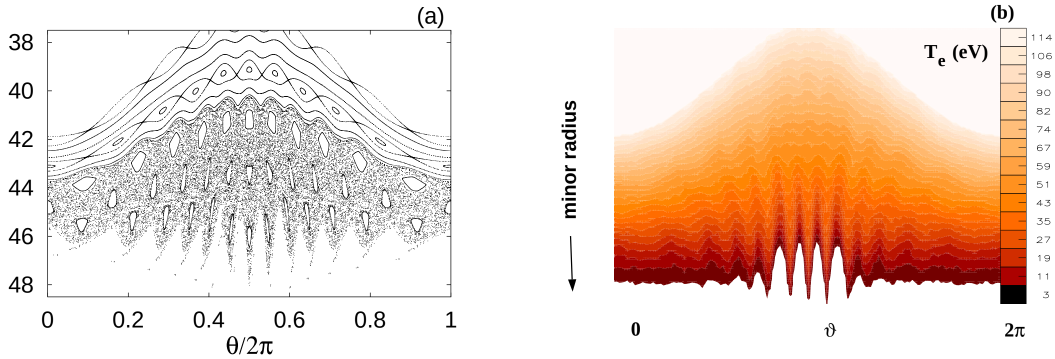

FIG. 4. a) Poincaré sections of magnetic field lines in the (r, θ):
(b) Corresponding temperature profile at the plasma edge.

An approach with a *finite element method* has been also under development. The idea of this modeling is that the flux tube of relatively short connection length would play a most important role in the transport at the edge, because the long connection length flux tubes are deformed into a very thin structure of order of ion Larmor radius, where the plasma easily diffuses to neighboring short connection regions.

Based on Fig. 3 the edge region is divided into areas with the flux tubes of one and two poloidal turns, and they are analyzed using 3D grids [11]. Regions with three and more poloidal turns are approximated as an ergodic region where simply the effective cross field transport coefficients are introduced. Because of the configuration of DED perturbation, the flux tube has a symmetric point at HFS for two-turn region and LFS for one-turn region, respectively. A SOL like transport is considered for these one and two turn regions, with the stagnation points at the symmetric planes. The numerical scheme is a splitting method. A finite element method solves the cross field transport on poloidal planes, and a finite difference method solves the parallel transport in volume cells between the poloidal cuts. The advantage of this approach will be the rather simplified picture based on the SOL model.

References

[1] DYNAMIC ERGODIC DIVERTOR (special issue) edited by FINKEN, K.H., Fusion Eng. Design **37** (1997) 335-448.

[2] KALECK, A., et al., Fusion Eng. Design **37** (1997) 353.

[3] FINKEN, K.H., et al., Nucl. Fusion **38** (1998) 515.

[4] ABDULLAEV, S.S., et al., Phys. Plasmas **5** (1998) 196.

[5] ABDULLAEV, S.S., et al., Phys. Plasmas **6** (1999) 153.

[6] FINKEN, K.H., et al., Nucl. Fusion **39** (1999) 637.

[7] ABDULLAEV, S.S., et al., Phys. Plasmas **8** (2001) 2739.

[8] EICH, T., et al., Contrib. Plasma Phys. **38** (1998) 112.

[9] EICH, T., et al., Nuclear Fusion **40** (2000) 1757.

[10] RUNOV, A.M., et al., Phys. Plasmas **8** (2001) 916.

[11] KOBAYASHI M., et al., Contrib. Plasma Phys. **42** (2002) 163.

[12] ABDULLAEV, S.S., J. Phys. A: Math. Gen. **32** (1999) 2745.

[13] ABDULLAEV, S.S., J. Phys. A: Math. Gen. **35** (2002) 2811.