Scaling and Modeling Studies of High-Bootstrap Fraction Tokamaks

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Abstract. A theoretical framework is developed to generate tokamak equilibrium configurations for which, on one hand, the current results entirely from the bootstrap current source driven by the pressure gradient while, on the other hand, the pressure gradient is determined from the thermal conduction equation with a thermal diffusivity constructed to have properties observed in confinement experiments: gyroBohm confinement, gradients only with respect to the poloidal flux, global confinement depending only on plasma current and independent of toroidal field, a critical temperature gradient, and an overall confinement improvement with negative shear. The nondimensional method used yields eigenvalues composed of a collection of physics quantities, resulting in scaling relations among physics variables. It is found that the the plasma temperature scales as $T \propto P^{2/3} \varepsilon^{-1/3}$, while $I_p \propto n^{1/2} P^{1/3} a\, \varepsilon^{1/12}$. The system has a solvability criterion which does not permit solutions when the confinement improves rapidly with increasing negative shear. A simplified 1-D model captures the essential physics of the coupling between bootstrap currrent generation and thermal conduction.

1. Introduction

Our ultimate vision of a tokamak based fusion power system has both a plasma current that arises almost entirely from bootstrap current because thermonuclear reactions drive no current and a reasonable β_{tor} limit ($\beta_{tor} \geq 0.04$). Economic and efficiency arguments restrict any current arising from external auxiliary power to be small relative to the plasma current. Seed currents in the immediate vicinity of the magnetic axis may well be possible. Thus, an essential step in advanced tokamak research is to understand whether a 100% bootstrap fraction discharge actually exists, what its properties will be, and what seed current is necessary.

Experimentally, there are many discharges that are fully non-inductive [1–7] for which auxiliary power sources drive a significant fraction of the plasma current. By definition, such discharges cannot be 100% bootstrap fraction discharges. Attainment of high-bootstrap-fraction requires that *heating sources drive no current directly*. Perpendicular ECH heating, fast-wave heating with a symmetric $k_{||}$-spectrum, and thermonuclear reactions all fulfill this requirement. Figure 1 illustrates the coupling between transport, equilibria, and bootstrap current. This work develops a framework to solve the coupled steady-state thermal conduction and Grad-Shafranov equations.

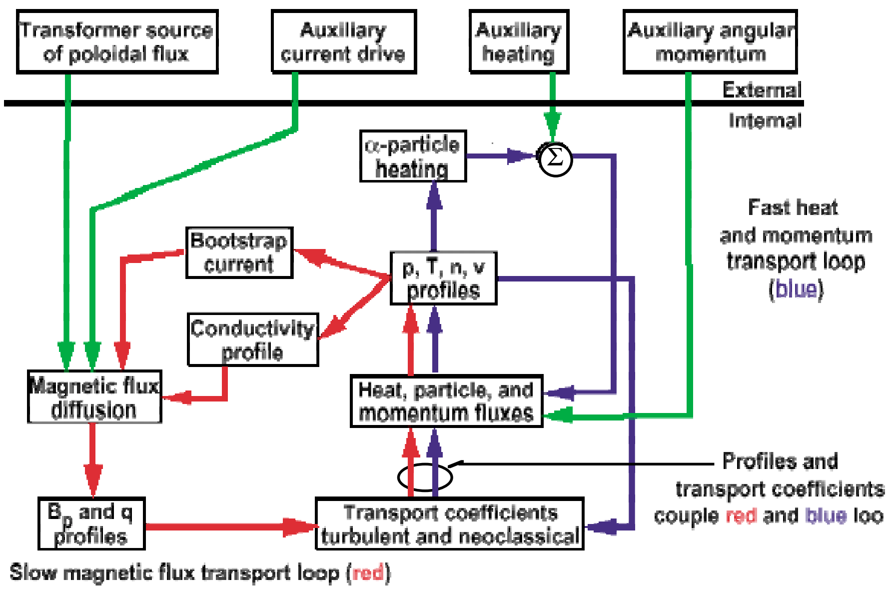


Fig. 1. Coupling between transport, equilibria, and bootstrap current physics.

Previous modeling studies have used time-dependant codes in physics variables [8,9]. This approach can handle more general heat conductivity models than our approach which is restricted to analytic repesentations of the diffusivity and its scaling properties.

These studies follow the approach to equilibria while the framework developed here deals with the eventual steady-state. Note that the 100% bootstrap discharge is a self-organized system. Given the heating source and other evident parameters such as the toroidal field, size, plasma shape, and density, the self-organized system will determine the plasma current, temperature, and q profiles.

Section 2 develops our nondimensional steady-state model and Section 3 extends this to a simplified 1-D version. Section 4 presents results. Section 5 shows data from DIII–D with ECCD current drive and compares them with modeling studies of ECH-heated plasmas with and without ECCD.

2. Nondimensional Transport Modeling

The starting point is the familiar Grad-Shafranov and flux-surface-averaged heat transport equations [10]. The first step is to convert these equations to nondimensional form based on a nondimensional poloidal flux independent variable $\tilde{\psi} = \psi/\psi_o$ where ψ_o is the poloidal flux at the discharge boundary. The following nondimensional variables are defined: $dR = (A/\pi)^{0.5}du$, $dZ = (A/\pi)^{0.5}dv$, $P = P(0)\, p(\tilde{\psi})$, $n=n_o\, \rho(\tilde{\psi})$, $T=T_o\tau(\tilde{\psi})$. Here A denotes the area within the separatrix. With these definitions, the Grad-Shafranov equation takes the form

$$u\frac{\partial}{\partial u}\frac{1}{u}\frac{\partial\tilde{\psi}}{\partial u} + \frac{\partial^2\tilde{\psi}}{\partial v^2} = = -\lambda_1\frac{dp}{d\tilde{\psi}}\left\{\left[\frac{u^2}{u_o^2} - \frac{1}{\left\langle\frac{u_o^2}{u^2}\right\rangle}\right] + \frac{f_t\, C_{bs}}{\left\langle\frac{u_o^2}{u^2}\right\rangle}\right\} \tag{1}$$

where $u_o = R_o/(A/\pi)^{0.5}$ is the nondimensional geometrical axis. The first term with the [ ] brackets is the Pfirsch-Schluter currents which have no net toroidal current. The second term is the bootstrap contribution. Since the only net current source is the bootstrap current, the resulting equilibria represent fully-aligned, 100% bootstrap fraction discharges [11]. All physics quantities in Eq. (1) are collected into an eigenvalue $\lambda_1 = (A/\pi)\,(\mu_o\, P_o\, R_o^2)/\,(\psi_o^2)$. The value of λ_1 is that needed to make the solution $\tilde{\psi}=1$ at the plasma boundary.

It is well known that density gradients are more effective than temperature gradients in creating bootstrap current. Our framework does not provide for a transport equation for plasma density so we shall adopt a simple model relating the pressure, density and temperature profiles by assuming $\rho = p^\gamma$, with $\gamma \le 0.25$, which corresponds to a weak density gradient. In this case, the contribution to the bootstrap current drive takes the form

$$\left\{2\gamma L_{31} + \left(2L_{31} + L_{32} + \alpha L_{34}\right)(1-\gamma)\right\} \equiv C_{BS} \quad , \tag{2}$$

where the L_{nm} can be taken from the bootstrap current literature [12,13]. The parameter C_{bs} will be retained (but considered a constant) to assess the relative roles of temperature and density gradients. Reference [13] suggests that $C_{bs} \approx 0.6$ for $\gamma = 0.25$.

The Grad-Shafranov Eq. (1) is solved with the right-hand side is regarded as known and with a separatrix , single-null outer shape. From this solution, a number of useful surface functions are evaluated. Other, similar functions are evident.

$$\Sigma = \frac{\oint d\ell\, 2\pi R}{(2\pi)^2 R_o \left(A/\pi\right)^{0.5}} \qquad A^* = \oint d\ell\,\frac{\nabla\psi}{R}\oint d\ell\,\frac{R}{\nabla\psi}\left(\frac{1}{4\pi A}\right) \quad s = 2\left(1-\frac{e}{i}i'\right) \quad e = \tilde{A}/\tilde{A}' \tag{3}$$

where i and $\tilde{A}$ are the nondimensional plasma current and area within a flux surface and $'$ denotes a derivative with respect to poloidal flux $\tilde{\psi}$. The definition of s makes s a flux function, reduces to an appropriate cylindrical limit, and permits modeling of magnetic shear effects on confinement.

Next, we turn to the heat transport loop and calculation of the thermal diffusivity, which will provide the density and temperature gradients needed by the Grad-Shafranov equation. The crucial and poorly known physics of the thermal diffusion loop concerns how the poloidal field affects the thermal difffusivity. By construction, the thermal diffusivity properties are: 1) overall, dimensionally-correct gyroBohm scaling, 2) gradients only with respect to poloidal flux, reflecting the fact that density, temperature and presure are flux functions, 3) global confinement that depends only on the plasma current (and not on the toroidal field) as supported by global databases [14], 4) a critical temperature gradient $(dT/d\psi)_c = T/\delta\psi$, 5) no dependence of the thermal diffusivity on β, again in accordance with tokamak data [15]. And 6) an overall dependence on magnetic shear given by $e^{\alpha s}$ with the shear s being defined in Eq. (3) above. The thermal conduction equation is

$$\frac{P_{heat}}{S} = C\, e^{\alpha s} \frac{n\, M^{0.5} A^* T^{0.5}}{(2\pi)^3 e^2} \left(\frac{\partial T}{\partial \psi}\right)\left(\frac{\partial T}{\partial \psi} - \frac{T}{\delta\,\psi}\right) \quad , \tag{4}$$

where $\delta\psi$ is a finite increment in poloidal flux. All functions in Eq. (4) are flux functions and consequently the equation is one dimensional. The constant α has been introduced by construction as a measure of the importance of magnetic shear in determining the thermal conductivity. This is motivated by the improvement in confinement in negative magnetic shear regions. Cast in nondimensional terms, the thermal conduction equation reads

$$\tau' \left(\tau' + \tau/\delta\psi\right) = \lambda_2 \frac{\Pi}{\Sigma} \frac{2\pi}{\tilde{A}\, \rho\, \tau^{0.5} e^{\alpha s}} \quad , \tag{5}$$

where $\rho = \tau^{\gamma/(1-\gamma)}$ and the nondimensional surface area Σ and power flow Π are defined by

$$S = \Sigma\, (2\pi)^2 R_o \left(A/\pi\right)^{0.5} \qquad P_{heat} = \Pi\, P_o \qquad \tilde{A} = A^* / (4\pi A) \quad , \tag{6}$$

and the eigenvalue λ_2 is

$$\lambda_2 = \frac{P_o\, e^2\, \psi_o{}^2 \sqrt{\pi}}{C\, (2\pi)^3 A^{1.5} R_o\, n_o\, M^{0.5}\, T_o{}^{2.5}} = \frac{1}{\lambda_1 C\, 4\pi^4} \left(\frac{P_o\, e^2 \mu_o}{T_o{}^{1.5} M^{0.5}}\right) \left(\frac{R_o^2\, \pi}{A}\right)^{0.5} \quad . \tag{7}$$

One then solves Eq. (5) for p´, and integrates Eq. (5) from 0 to 1. The correct value of λ_2 gives p (1.0) = 1.0.

$$p' = -\frac{\left(p/\delta\psi\right) + \sqrt{\left(p/\delta\psi\right)^2 + \lambda_2 \left(\frac{\Pi}{\Sigma}\right) \frac{8\pi\, p^{(3\gamma-1)/2}}{\tilde{A}\ \ e^{\alpha s}}}}{2\,(1-\gamma)} \quad . \tag{8}$$

It remains to calculate the magnetic shear parameter. The plasma current can be readily calculated as well. Eq. (9) describes the increase of plasma current with poloidal flux

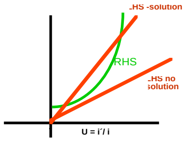
$$\frac{\partial I}{\partial \psi} = \oint \frac{j_\phi}{\nabla\psi} d\ell = \oint \frac{d\ell\, R}{\nabla\psi}\, \mu_o\, f_t\, P_o \frac{\partial p}{\partial \psi} \qquad \mu_o I = \oint \frac{\nabla\psi}{R}\, d\ell \quad . \tag{9}$$

Combining both of the Eq. (9) leads to a differential equation for I^2 and its nondimensional form

$$\frac{\partial I^2}{\partial \psi} = -2\, A^* \mu_o\, f_t \frac{dP}{d\psi} \qquad \frac{\partial}{\partial \tilde{\psi}}\, i^2 = -\lambda_3\, \tilde{A}^*\, f_t \frac{\partial p}{\partial \tilde{\psi}} \quad , \tag{10}$$

where $\lambda_3 = 8\pi A\ C_t \mu_o P_o / I_o{}^2$ and the flux function A^*, which is well fit by $A^* = 4\pi\ A = (2\pi)^2 r^2 \kappa$. It is also useful to introduce an approximate expression for the trapped particle fraction.$f_t = C_t(r/R)^{0.5}$. The next step is to use Eq. (10) to evaluate the LHS of Eq. (8), resulting in an equation for U = i′/i

$$\left(\frac{2i^2}{\lambda_3 \tilde{A}^* f_t}\right) U = -p' = \frac{\left(p/\delta\tilde{\psi}\right) + \sqrt{\left(p/\delta\tilde{\psi}\right)^2 + \lambda_2 \left(\frac{\Pi}{\Sigma}\right) \frac{8\pi\, p^{(3\gamma-1)/}}{\tilde{A}\;\; e^{-\alpha U}}}}{2\,(1-\gamma)} \qquad (11)$$



As the sketch indicates, this equation has either two solutions or none for U > 0, indicating a potentially fundamental incompatibility of the effect of a magnetic shear on high bootstrap fraction plasmas under circumstances where an increasing outward plasma current creates negative shear and low thermal diffusivity.

3. One-Dimensional Model

It is elucidating to develop a one-dimensional model which captures the physics of the coupled magnetic and heat diffusion loops yet permits easy variations in the thermal diffusivity and other coupling terms to ascertain how profiles will respond. The key to the one-dimensional model is Eq. (10) which can be integrated using r as an independant variable and approximating $A^* = 4\pi A = (2\pi)^2 r^2 \kappa$. One then uses the nondimensional pressure profile determined by Eq. (8) and its corresponding eigenvalue in Eq. (11) to complete the solution.

4. Results

Our principal result is construction of a framework which successfully accounts for plasma current generated by pressure gradients via the bootstrap mechanism, the resulting magnetic field, its effect on flux-surface-averaged thermal conductivity and hence on what must be a self-consistent pressure gradient driven by a plasma energy source. The framework produces nondimensional profiles and eigenvalues. Figures 2 and 3 are examples which give the nondimensional temperature and q-profiles for a case with no magnetic shear dependence and the value of $\delta\psi = 1.0$ for the critical gradient. For nested circular cylinder geometry, a simple 1-D model mode is available.

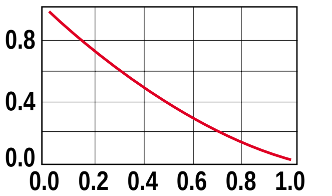


Fig. 2. Nondimensional temperature.

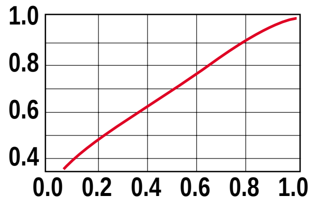


Fig. 3. Relative q profile.

Evaluating the eigenvalues and normalizing the thermal diffusivity to DIII-D shot 10773 discussed below gives:

$$T_{keV}(0)^{3/2} = 0.6\,P_{MW}\left(\frac{R}{a}\right)^{1/2} C_{bs} \quad \text{and} \quad I_{p,MA} = (0.15)\left(C_{bs}\right)^{5/6}\left(n_{19}\right)^{1/2}\left(P_{MW}\right)^{1/3} a_m \left(\frac{a}{R}\right)^{1/12}.$$

It is interesting that the central temperature depends only on the heating power and not at all on plasma density and size.

5. DIII–D

Plasmas with high but not unity values for the bootstrap fraction have been produced on DIII–D. Discharges 107736 had both directed NBI heating and the gyrotron systems set for ECCD. Figure 4 shows that these discharges came close to fully noninductive performance.Modeling studies (Figs. 5 and 6) utilizing a diffusivity normalized to shot 107736 indicate that 5 MW ECCD should suffice to increase plasma current but is insufficient to maintain a 200 kA discharge by bootstrap ECH alone.

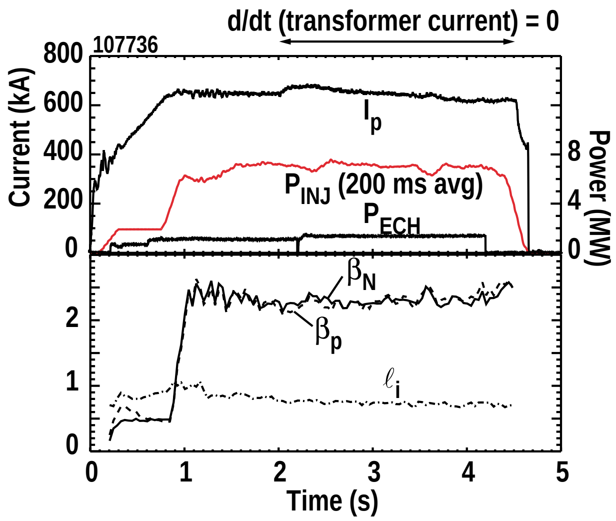


Fig 4. Nearly stationary discharge for over 2 s at high beta (107736). (a) Plasma current, NB power (200 ms average; the power is modulated to maintain constant stored energy), and EC power. The transformer current is fixed from 2.0 s onward. (b) β_N, β_p, and ℓ_i. β_N is held constant by the NB feedback control. There is a very slow broadening of the current profile indicated by the decreasing ℓ_i.

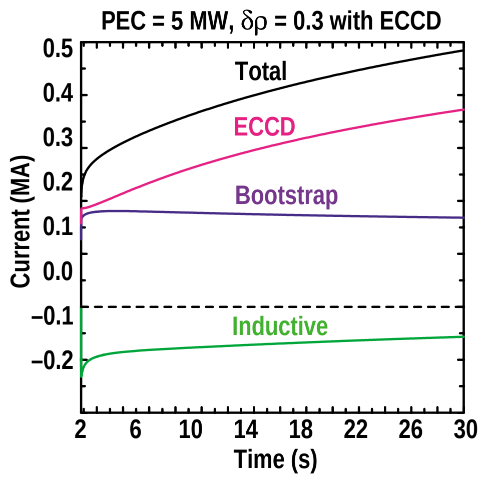


Fig. 5. Plasma Current with 5 MW ECH oriented for ECCD.

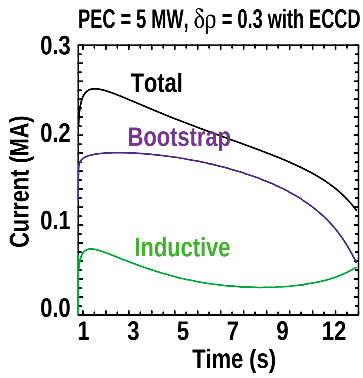


Fig. 6. Plasma current with 5 MW ECH and no ECCD.

6. Conclusions

A framework has been developed to construct plasma pressure profiles consistent with 100% bootstrap-fraction plasma current and which satisfy a flux-surface-averaged heat conduction equation. Results are found to depend on the form chosen for the heat diffusivity. When confinement increases strongly with negative shear, steady-state solutions can no longer satisfy the coupled Grad-Shafranov and thermal conduction equations with only a bootstrap current source. The nondimensional approach produces scaling relations for discharges with 100% bootstrap fraction and they predict plasma currents of ~200 kA and T~ 2 keV for the capabilities of planned experiments.

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