Effects of Tokamak Flux Surface Non-Circularity on Charged Particle Transport Processes in the Weak Collisionality Regime

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Abstract. Introducing a novel analytical model for tokamak fields with prescribed flux surface shapes into a 3D Fokker-Planck simulation, the influence of parameters determining the non-circularity of flux surfaces on the longitudinal transport of charged particles as well as on their radial fluxes is investigated for weak collisionality. It is demonstrated that a high triangularity can decrease the trapped particle fraction by up to 30%. An increase of elongation and triangularity at fixed plasma current and fixed B was found to reduce the effective radial excursion of charged particles thus resulting in the improvement of plasma confinement. An extreme sensitivity of the NBI induced current to the flux surface non-circularity was seen for $Z_{beam} \approx Z_{eff}$. Another interesting result is that the triangularity and up-down asymmetry of flux surfaces strongly affect the slowing-down induced radial convection of high-energy toroidally trapped charged particles being in resonance with the TF ripple perturbation. This transport mechanism is deemed to be consequential for the confinement of charged fusion products in tokamaks. Further the ripple-induced stochastic transport domains are shown to vary with flux surface shapes.

1. Introduction

The non-circularity of flux surfaces (FS's) is known to play an important role for the equilibrium, the MHD stability and for transport processes in tokamak plasmas. According to present-day tokamak experiments an elongated and triangular plasma shape (e.g. see [1,2]) is beneficial for sustaining a high operational performance required in next-step experimental reactors. To establish the physical basis for understanding the FS non-circularity effects, the theoretical examination of the influence of FS geometry on basic plasma parameters such as confinement time, poloidal beta, the fractions of bootstrap current and of non-inductive driven current to the plasma current is of apparent importance. Though most of relevant theoretical considerations of transport processes in a multispecies plasma [3-6] are, in general, valid for arbitrary tokamak magnetic configurations, the corresponding specifications to real, actual FS shapes is a rather complex problem involving a large number of calculations [7]. The only case in which the analysis can be completed is the one associated with elliptic FS's [8, 9]. However, the consideration of complex FS geometry can be essentially simplified by using analytical models for the tokamak magnetic field based on prescribed FS shapes [10]. Thus, for instance, the sensitivity of ripple-induced fast ion transport to FS triangularity could be demonstrated [11].

Here we examine how non-circular FS's affect the charged particle transport properties in tokamaks in the low-collision-frequency regime. Our study is based on the above mentioned analytical model [10] of axisymmetric toroidal magnetic configurations with prescribed flux surfaces using only four geometric parameters determining the shape and position of FS's. These are the Shafranov shift $\Delta(r)$, the elongation $k(r)$ as well as the parameters of triangularity $\Lambda(r)$ and of up-down asymmetry $\eta(r)$, where r denotes the FS radius at the midplane. The longitudinal transport and the radial fluxes of ions are studied as they vary with these geometric parameters. We investigate the radial profiles of the fraction of toroidally trapped particles, f_t, of the plasma bootstrap current and of the NBI induced current as depending on the non-circularity parameters. Finally, the influence of FS non-circularity on the ripple-induced transport is considered.

2. Magnetic Field Model

We refer to a prescribed axisymmetric magnetic field $\mathbf{B}(R,Z)$ with the FS's determined by the following parametric dependences: $R = R(\chi, r, \Pi(r))$; $Z = Z(\chi, r, \Pi(r))$. Here R and Z are the spatial variables of the cylindrical coordinate system $[R, Z, \varphi]$, χ is the poloidal angle and $\Pi(r)$ represents the set of parameters describing the position and shape of FS's [10]. The magnetic field and the flux coordinates $[\Phi, \vartheta]$ are related to r and χ by

$$\mathbf{B}_t \equiv J\nabla\varphi = \nabla\Phi \times \nabla\vartheta = \hat{\vartheta}(r,\chi)\nabla\Phi \times \nabla\chi\,,\quad \mathbf{B}_p = q^{-1}(r)\nabla\varphi \times \nabla\Phi\,, \tag{1}$$

$$d\vartheta/d\chi \equiv \hat{\vartheta}(r,\chi) = Y(r,\chi)/\langle Y\rangle,\ d\Phi/dr = J\langle Y\rangle,\ Y(r,\chi) := \{R,Z\}/R\,,\ \langle \ldots \rangle \equiv \frac{1}{2\pi}\int d\chi(\ldots), \tag{2}$$

where $\Phi(r)$ denotes the toroidal magnetic flux (divided by 2π), ϑ the poloidal flux coordinate, $q(r)$ the safety factor and $J(r)$ the total poloidal current (divided by 2π) outside a given FS.

3. Longitudinal Transport in Noncircular Tokamaks

As shown in [3,6,12,13] the longitudinal ion transport in low-collision tokamak regimes depends essentially on the fractions of trapped (f_{tr}) and of circulating particles (f_c), which vary with FS shape. Hence, obviously, FS noncircularity will affect the longitudinal transport. Demonstrating this we consider the influence of FS shape on both the beam-induced and the bootstrap current.

To describe the electron current induced by injected ions the electron-electron collision operator was used in the form given in Ref.[12] which is more general in comparison with that employed in [13]. Amending thus the neoclassical approach of [13] the flux-surface-averaged ratio F_e of electron current j_e to beam current j_b was obtained as

$$\left|\frac{j_e}{j_b}\right| \equiv F_e = \frac{Z_b}{Z_{eff}} f_c G(Z_{eff}, f_c)\,, \tag{3}$$

with

$$G = \frac{4}{3\sqrt{\pi}}\left\{\int_0^\infty \frac{\nu_{ei} t^{3/2} e^{-t}}{\nu_\Sigma}dt + f_c \frac{\int_0^\infty \frac{\nu_{es} t^{3/2} e^{-t}}{\nu_\Sigma}dt \cdot \int_0^\infty \frac{\nu_{es} e^{-t}}{\nu_\Sigma}dt}{\int_0^\infty \frac{\nu_{es}(\nu_{ei} + f_{tr}\nu_{ee})e^{-t}}{\nu_{ei}\nu_\Sigma}dt}\right\}; \tag{4}$$

Here Z_b and Z_{eff} are the beam ion charge number and the effective plasma charge, respectively, and $f_c = \frac{3}{4}\langle B\rangle \int_0^{B_{\max}^{-1}} \frac{\lambda d\lambda}{\langle\sqrt{1-\lambda B / B}\rangle}$. In Eq.(4) ν_{ee}, ν_{ei} and ν_{es} denote the deflection and slowing down frequencies for electrons; further is $\nu_\Sigma = \nu_{ei} + f_{tr}\nu_{ee} + f_c\nu_{es}$, $f_{tr} = 1 - f_c$, $B_{\max}$ the maximal magnetic field strength on the flux surface and <…> indicates FS averaging. Note that, for the case $\nu_{ee} = \nu_{es}$, Eq.(3) yields the results of Ref.[13]. To calculate f_c, which acts on the electron current, we employ the analytical approximation of [10] for tokamak magnetic fields with noncircular FS's. Radial profiles of the ratio f_{tr}/f_c were numerically evaluated for different triangularities and are presented in Fig.1. On the other hand, using the analytical B-model of Eqs. (1) and (2) the fraction of toroidally trapped particles may be as well calculated analytically as

$$f_{tr} = \frac{2\sqrt{2\varepsilon}}{\pi}\left(1 + \frac{1}{30}\frac{d\ln k}{d\ln r} + \frac{1}{3}\frac{d\Delta}{dr} - \frac{9}{35}\Lambda + \frac{1}{6}\varepsilon\right) \tag{5}$$

being in agreement with the numerical result $1\text{-}f_c$. From Eq. (5) and Fig. 1 it becomes evident that it is the triangularity which essentially affects the ratio f_{tr}/f_c, at least at the plasma periphery. So we may expect that triangularity will impact also on the net beam driven current $j_{net}=j_b+ j_e$. Numerically calculating $j_{net}/j_b=1-F_e$ from Eq.(3) we display the corresponding results in Fig.2.

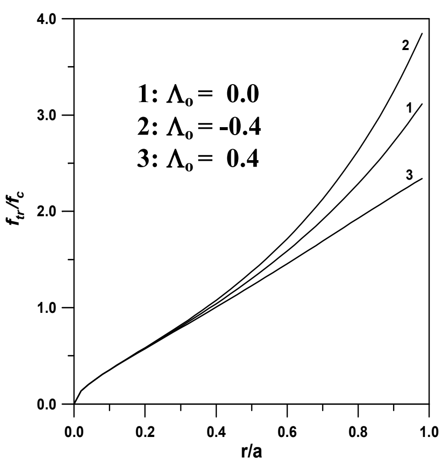

FIG. 1. Ratio of trapped and circulating particles as a function of normalized FS radius for a tokamak with $A=3$, $\Delta(r)=0.07(1-r^2/a^2)$, $k=1.4+0.3r^2/a^2$, $\eta=0.05\ r^2/a^2$ for various triangularity parameters $\Lambda=\Lambda_o\, r^2/a^2$.

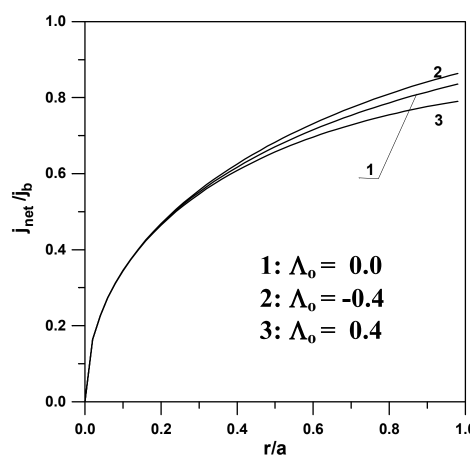

FIG. 2. Radial profile of the net current $j_{net}=j_b+j_e$, generated by beam ions in a tokamak (with same parameters as in Fig.1) for different values of triangularity.

Further, also the bootstrap current j_{bs} in a tokamak depends on the ratio of trapped and circulating particles *($x=f_{tr}/f_c$)* as can be concluded from the expansion [6]

$$\langle \mathbf{j}_{bs} \cdot \mathbf{B}\rangle = L_{31}\left[A_1^e + Z_{eff}^{-1} T_i / T_e \left(A_1^i + \alpha_i A_2^i\right)\right] + L_{32} A_2^e \tag{6}$$

where $L_{31} = L_{31}(x, x^2, Z_{eff}, Z_{eff}{}^2)$ and $L_{31} = L_{31}(x, x^2, Z_{eff}, Z_{eff}{}^2)$ are distinct functions of x (explicit expressions can be found in [6]), $\alpha_i = -1.172/(1+0.462x)$ and $A_1^{\alpha} = p_{\alpha}'/p_{\alpha}$ and $A_2^{\alpha} = T_{\alpha}'/T_{\alpha}$ are the thermodynamic forces. To clarify the influence of triangularity on j_{bs} two different cases were investigated numerically as presented in Fig.3. In the first case, which corresponds to a flat temperature profile model, triangularity can only moderately affect j_{bs}. In contrary, in the second case with a flat plasma density profile, triangularity exhibits a significant influence on the bootstrap current, which increases towards the plasma periphery. This different behaviour arises from the unlike dependence of the coefficients L_{31} and L_{32} on Λ via x.

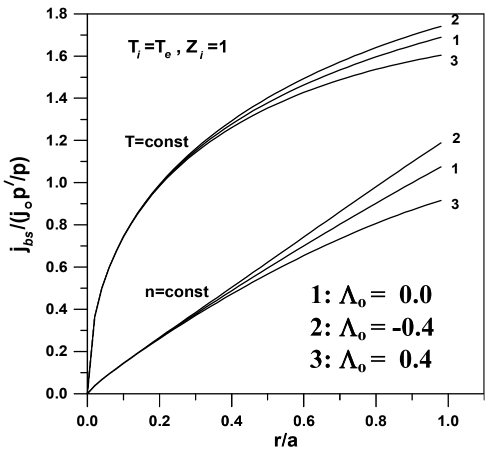

FIG. 3. Radial profiles of the normalized bootstrap current in a tokamak (with same parameters as in Fig.1) for different triangularities and plasma density and temperature profiles. Calculations were carried out for equal electron and ion temperatures and Z_{eff}=1.

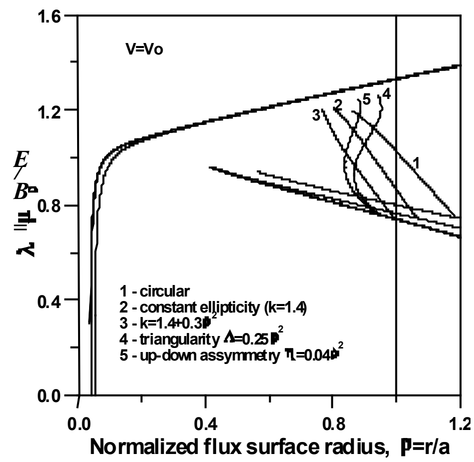

FIG. 4. Curves 1-5 represent the stochasticity domain boundaries for various FS shapes. The upper contours show the boundary of the confinement domain of co-moving particles. The unmarked lines within the confinement domain correspond to the separatrix between trapped and untrapped particles.

4. Influence of FS Non-Circularity on the Ripple Induced Transport

Stochastic collisionless transport is known to contribute substantially to the loss of energetic ions in tokamaks [14]. Here we investigate the effect of non-circularity of FS's on the Goldston-White-Boozer stochasticity threshold δ_{GWB} [14] modified for noncircular geometry. Examining the confinement domains of fast ions we evaluate the above stochasticity criterion for different analytical B-models and various shapes of FS's and demonstrate how the FS configuration may essentially affect the origin of ripple-induced stochastic radial diffusion. As seen in Fig. 4, turning from circular to elliptic flux surfaces will enhance the stochastic transport region, whereas a superimposed triangularity and up-down asymmetry can somewhat reduce this effect. Another interesting result is that the triangularity and up-down asymmetry of FS's affect strongly the slowing-down induced radial convection of high-energy toroidally trapped charged particles being in resonance with the TF ripple perturbation [15]. For *MeV* alphas in JET-like tokamaks this yields a high radial convection rate of resonant particles (superbananas), $|dr/dt| \sim 10^2\ cm\ s^{-1}$. This transport mechanism may be consequential for the confinement of charged fusion products in tokamaks. The quantitative evaluation of the above effect on the overall loss and distribution of charged fusion products constitutes a rather complex numerical problem and is beyond the scope of the present paper.

5. Summary

Fast ion transport in tokamaks was demonstrated to be substantially influenced by FS shapes. High FS triangularity ($\Lambda \approx 0.5$) together with $d\Delta/dr \approx -0.5$, as typical at the plasma periphery, was shown to decrease f_t by up to 30%. We found that positive triangularity weakens the longitudinal plasma transport whereas negative triangularity enhances it. Further, the NBI induced current appeared to be extremely sensitive to FS non-circularity, when the charge number Z_b of beam ions is approximately equal to the effective plasma charge Z_{eff}. The strongest effect of FS non-circularity on the bootstrap current was deteceted in the case of flat plasma density profiles. Also the ripple-induced stochasticity domain was shown to be essentially affected by FS geometry.

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