On Radiative Density Limits and Anomalous Transport in Stellarators

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Abstract:

Density limits in stellarators are caused mainly by enhanced impurity radiation leading to a collapse of the temperature. A simple model can be established, which computes the temperature in the plasma with a fixed heating profile and a temperature-dependent radiation profile. If the temperature-dependent radiation function has one or several extrema, multiple solutions of the transport equation exist and radiative collapse occurs when the high temperature branch merges with the unstable temperature branch. At this bifurcation point the temperature decreases to a stable low temperature solution. The bifurcation point is a function of the heating power and the plasma density. Thus a density limit can be defined as the point where bifurcation occurs. It is shown that bifurcation and sudden temperature collapse does not occur below a power threshold. Anomalous thermal conductivity and the details of the impurity radiation, which in the present model is assumed to be in corona equilibrium, determine the scaling of the density limit. A model of the anomalous transport is developed, which leads to Gyro-Bohm scaling of the confinement time. The density limit based on this transport model is close to experimental findings in Wendelstein 7-AS.

1 Introduction

In Wendelstein 7-A [1] and Wendelstein 7-AS [2] with neutral beam heating the phenomenon of radiative collapse was found when the density in the plasma exceeds a certain value, which depends on the heating power and other parameters such as the magnetic field. The radiation power rises with rising electron density until the radiation power is a certain fraction of the heating power and the temperature strongly decreases.

The first theory of the density limit in stellarators was made by S.I.-Itoh and K. Itoh [3], who explained this limit as the result of a detachment process. The idea of a radiation-induced density limit has also been analysed by Sudo et al. [4], who, based on Itoh´s ideas, proposed a simple equation, which gives the maximum density in term of heating power and magnetic field. Details of the confinement properties and the plasma profiles do not occur in this equation. Recently Giannone et al. [5] have analysed data from Wendelstein 7-AS resulting in a density limit, which is similar to the Sudo-limit. On the other hand the heat balance equation contains the thermal conductivity, which is closely related to the energy confinement time. In the following we start from the heat conduction equation and – by dimensional analysis – derive a relation between anomalous thermal conductivity and energy confinement time as found in the experiment. Taking into account the bifurcation property of the solution with finite radiation losses the density limit is defined by a bifurcation point of the temperature, where stable and unstable solutions of the conduction equation merge and where the system makes a rapid transition to a low-temperature solution. From the scaling of this bifurcation point a scaling of the density limit can be obtained.

2 Thermal Conduction Equation

In general the temperatures of ions and electrons in a plasma are different and two separate transport equations need to be solved, however in a high density NBI heated plasma or in a fusion plasma the temperatures are about equal and by adding up the two transport equations we get one equation for a common temperature. This steady-state heat conduction equation is

$$\bullet n\chi(x,T)\bullet \quad T = h(x,T) \quad n(x)n_z(x)L_0L(T) \qquad \text{Eq. 1}$$

Here $h(x,T)$ is the heating term and x is the radial coordinate. The thermal conductivity is the sum of a neoclassical term and an anomalous term. n_z is the density of the impurities and n is the density of electrons. $L(T)$ is the dimensionless radiation function, which has a maximum at a certain temperature T_{max}. L_0 is the value at this temperature, which makes the dimensionless function $L(T)$ smaller than 1. These parameters L_0 and T_{max} depend on the impuritiy species. The heating function is redefined as $h \to h_{max}h(x)$, where $h(x)$ is a dimensionless heating profile and h_{max} is the maximum local heating power. The factor h_{max} is proportional to the total heating power. The ansatz of the heat conduction is $\chi=\chi_0 g(x)$ where $g(x)$ is a profile function, which does not depend on other parameters. However the factor χ_0 describes the dependence on magnetic field, geometric factors etc. The densities in this equation are a given functions and we normalise the densities to the maximum density $n \to n_0 n(x)$, $n_z \to N_z n_z(x)$, where now $n(x)$ and $n_z(x)$ are a profile functions. Normalising the spatial coordinates to the minor plasma radius a leads to the heat conduction equation in the form

$$-\nabla \bullet \frac{1}{H} n(x) g(x) \nabla T = h(x) - \lambda n n_z L(T_0 T) \qquad \text{Eq. 2}$$

The dimensionless parameters are

$$H = \frac{h_{max}}{n_0 T_0 \Omega} \frac{\Omega a^2}{\chi_0} \quad ; \quad \lambda = \frac{n_0 N_z L_0}{h_{max}} \qquad \text{Eq. 3}$$

T is dimensionless and normalized to T_0 and Ω is the gyro frequency. The reference temperature T_0 is either a fixed temperature or it can be defined by setting H = 1. H is the ratio between the heating power and the power loss by heat conduction and λ denotes the ratio between radiation loss and heating power. A general form of the thermal conductivity in non-dimensional form is

$$\frac{\chi_0}{\Omega a^2} = G\left(\beta, \frac{\rho}{a}, \frac{a}{R}, \iota\right) \qquad \text{Eq. 4}$$

ρ is the gyro radius and ι is the rotational transform. β is the ratio between plasma pressure and magnetic pressure. The function G depends on the physics of the anomalous transport. In a stellarator there might be more geometrical parameters like shear or the number of field periods, which are relevant for anomalous transport; here we have only retained aspect ratio and rotational transform. The gyro radius and β are computed with the temperature T_0.

3 Resistive turbulence model

Resistive ballooning modes in general toroidal equilibria have been investigated by D. Correa-Restrepo [6] and in a recent paper Correa-Restrepo has investigated resistive ballooning modes in the boundary region of toroidal plasmas [7] coming to the conclusion, that this region is resistively unstable. In particular, it was found that shear and magnetic well have little or negligible effect on the low-N ballooning modes. Numerical investigation of resistive ballooning modes in Wendelstein 7-AS and Wendelstein 7-X by R. Kaiser [8] showed overstable modes, which occur at any plasma β below the threshold of ideal ballooning modes. For these reasons it must be expected that MHD-turbulence exists in the boundary regions of any toroidal plasma experiment.

In the resistive turbulence model the anomalous perpendicular conductivity is the result of magnetic fluctuations and classical parallel conductivity, which in normalized form is

$$\frac{\chi_{\parallel}}{a^2\Omega} < \frac{(\delta B)^2}{B^2} > \;\sim\; \frac{T_0^{5/2}}{a^2\Omega n_0} < \frac{(\delta B)^2}{B^2} > \;\sim\; \frac{\rho_0\lambda_0}{a^2} F(\beta_0, R_m) \qquad \text{Eq. 5}$$

R_m is the Lundquist number. The details of this function F cannot be found without further numerical analysis. Here a few plausible arguments will be given, which are needed to justify a simple ansatz for the fluctuation level. The energy source of the fluctuations is the plasma pressure and therefore F will be a growing function of β_0. Furthermore, plasma resistivity is the reason for decoupling plasma and magnetic field and thus it is expected that the fluctuations grow with resistivity or – equivalent to that – they scale inversely to the Lundquist number. Resistive instabilities (interchange modes or ballooning modes) are driven by the curvature of the field lines. Measuring the curvature in terms of $1/R$ (R = major radius) we expect that the fluctuation level scales with the inverse aspect ratio. A simple ansatz exhibiting these features is

$$< \frac{(\delta B)^2}{B^2} > \; = F_0 \frac{\beta_0}{R_m} \frac{a}{R\iota} g(x) \qquad \text{Eq. 6}$$

where F_0 is a constant and $g(x)$ is a dimensionless radial function describing the increase of the magnetic field fluctuations towards the plasma boundary. This scaling can also be interpreted as follows: the energy reservoir for magnetic fluctuations is the thermal energy of the plasma, which explains why the energy density of the magnetic fluctuations is proportional to plasma beta. Resistivity opens this reservoir for instabilities, this is why the inverse Lundquist number occurs. The profile function $g(x)$ accounts for the increase of the turbulence level towards the plasma boundary, where the resistivity and hence the decoupling is large. The rest are geometrical factors like inverse aspect ratio or the rotational transform, which might also help to reduce the fluctuation level. In terms of the reference temperature the space-independent factor is

$$F_0 \frac{\beta_0}{R_m} \frac{a}{R} \sim \frac{n_0}{B^2 T_0 R} \qquad \text{Eq. 7}$$

Combining eqs. 5,6 and 7 shows that the anomalous thermal conductivity follows the Gyro-Bohm scaling. Putting all terms together yields a heating parameter, which is

$$H^{-1} = C_1 \frac{n_0 T_0^{5/2}}{PB^2\iota} \qquad \text{Eq. 8}$$

The constant C_1 absorbs all constants and profile factors. The transport equation is now

$$-\nabla \bullet \frac{1}{H}\; g(x) T^{5/2} \nabla_{\perp} T = h(x) \qquad \text{Eq. 9}$$

Setting H equal to unity yields the scaling of the reference temperature T_0 as follows

$$T_0 \sim \left(\frac{PB^2\iota}{n_0}\right)^{2/5} \qquad \text{Eq. 10}$$

With this definition of the reference temperature all parameters have been dropped from the transport equation and the solution of the dimensionless transport equation is a function $T(x)$ depending on the profile function $g(x)$ and $h(x)$ alone. The plasma energy is

$$E \sim a^2Rn_0T_0 \sim a^2Rn_0^{3/5}B^{4/5}\iota^{2/5}P^{2/5} \qquad \text{Eq. 11}$$

which leads to a scaling of the confinement time

$$\tau_E \sim a^2Rn_0^{3/5}B^{4/5}\iota^{2/5}P^{-3/5} \qquad \text{Eq. 12}$$

4 Bifurcation of temperature

Next we consider the transport equation with finite radiation losses. The solutions of the complete heat conduction equation with H=1

$$-\nabla \bullet n(x)g(x)\nabla T = h(x) - \lambda n n_z L(T_0T) \qquad \text{Eq. 13}$$

depends on two parameters T_0 and λ.

$$T = T(r, T_0, \lambda) \qquad \text{Eq. 14}$$

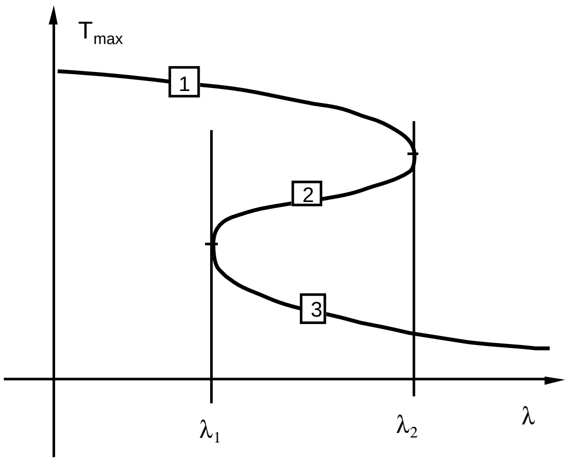

***Fig. 1**: Maximum temperature as function of the radiation parameter λ.*

Note that T_0 is a function of the heating power given in Eq.11. If the radiation parameter λ is small enough, only one solution exists. However, starting from a critical value of λ_1 more than one solution can exist [9]. A necessary condition for this effect is the existence of a temperature regime where the radiation function has a negative slope. Above a second critical $\lambda_c = \lambda_2$ only one low temperature solution can exist. These critical value of λ depend on the heating parameter and the can be utilized to define a density limit. A sketch of this situation is given in Fig. 1.

The critical radiation parameter λ_c where bifurcation occurs is a monotonously increasing function of the heating parameter $\lambda_c = \Lambda(T_0)$; or, going back to the definition of the parameters

$$\frac{C_h n_0^2 f_z L_0 R a^2}{P} = \Lambda\left(C_1\left(\frac{P\Omega^2\iota}{n_0}\right)^{2/5}\right) \qquad \text{Eq. 15}$$

C_h and C_1 are profile factors, f_z is the fraction of impurities. The function Λ is determined by the profile functions in the transport equation. Since the solution $T(x,\lambda,H)$ depends on the parameters λ and H, Eq. 15 provides a non-linear relation between the critical λ and the heating parameter H. The critical parameter λ_c increases with heating power. Let us assume that Λ obeys a power law with an exponent γ, then from Eq. 15 the result for the density is

$$n_0^{2+\gamma 2/5} f_z = C_2 \frac{1}{Ra^2} P^{1+\gamma 2/5} \Omega^{\gamma 4/5} \iota^{\gamma 2/5} \qquad \text{Eq. 16}$$

C_2 is a coefficient, which combines all profile factors. As an example we choose $\gamma = 5/2$, this yields

$$n_0^3 f_z = C_2 \frac{1}{Ra^2} P^2 \Omega^2 \iota \qquad \text{Eq. 17}$$

The density scales with $P^{2/3}$ $B^{2/3}$ and inversely with the cubic root of the plasma volume. Assuming γ= 3 would yield the scaling with $P^{11/16}$ $B^{3/4}$. As numerical computations have shown, the scaling of the density limit depends on the details of the radiation function and the localisation of the impurities in the radial direction [10].

5 Conclusions

In the present analysis an attempt has been made to understand the density limit and the radiative collapse in stellarator experiments on the basis of a combined effect of temperature dependent radiation power and anomalous thermal conductivity of the plasma. The transport equation has several solutions in a limited regime of the control parameter λ, which is proportional to the product of plasma density and the density of impurities. The radiative collapse of the temperature occurs when a bifurcation point λ_c has been reached, where a stable and an unstable solution merge. This bifurcation point provides a relation between magnetic field, heating power and plasma density and thus allows one to define a density limit. How the bifurcation point depends on heating power, magnetic field and plasma density is determined by the radiation function and the localisation of the impurities. For this reason it is not possible to formulate a scaling law of the density limit, which is valid for all impurity species.

Gyro-Bohm scaling of the anomalous transport coefficient can be derived from a resistive turbulence model leading to magnetic fluctuations and an effective perpendicular transport due to a large parallel thermal conductivity of electrons. The effect is related to the concept of Rechester and Rosenbluth, who explained anomalous transport by magnetic braiding and parallel transport of electrons. The transport model is based on dimensional analysis, which does not allow one to predict the exact scaling of the anomalous transport coefficient, however with some plausible arguments on β-dependence and dependence on Lundquist number the gyro-Bohm scaling of the energy confinement time can be derived.

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