Numerical Assessment of the Ion Turbulent Thermal Transport Scaling Laws

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Abstract. Numerical simulations of ion temperature gradient (ITG) driven turbulence were carried out to investigate the parametric dependence of the ion thermal transport on the reduced gyroradius and on the local safety factor. Whereas the simulations show a clear proportionality of the conductivity to the gyroradius, the dependence on the safety factor cannot be represented as a simple power law like the one exhibited by the empirical scaling laws.

1. Introduction

In this contribution, the numerical assessment of some of the scaling properties of ion thermal transport caused by ion temperature gradient (ITG) driven turbulence is presented. This is, in general, an important area of investigation, given the impact on the design of future machines. While most of the work is done by statistical analysis of empirical data [1] valuable contribution to the understanding of anomalous transport can come from the investigations of specific theoretical questions. In this respect, the analysis of the ITG turbulent dynamics is of high priority, given its recognized relevance in the ion thermal transport.

To this end, the simulation code ETAI3D that solves a fluid model of ITG turbulence has been developed [2]. The model equations, which contain the minimal ingredients necessary to investigate ITG turbulence in a torus, are

$$dw/dt + 2\epsilon\omega_d(\Phi + T_i) + A\nabla_{\|}v + (\kappa_n/r)\ \partial_\theta\Phi = D_w\nabla^2 w - \gamma_{\rm pfd}\rho_*^2\langle\Phi\rangle, \tag{1}$$

$$dv/dt + A\nabla_{\|}(\Phi + T_i) = D_v\nabla^2 v, \tag{2}$$

$$dT_i/dt + \Gamma\langle T_i\rangle A\nabla_{\|}v = -A\langle T_i\rangle^{1/2}|\nabla_{\|}|T_i + D_T\nabla^2 T_i, \tag{3}$$

where, $w = (\Phi - \langle\Phi\rangle)/T_e - \rho_*^2\nabla^2\Phi$ is the generalized vorticity (effectively the ion guiding center density), Φ is the electric potential, v the parallel ion velocity, T_i the ion temperature, $d/dt = \partial_t + \mathbf{v}_E\cdot\nabla$ the advection operator, $\omega_d = (1/r)\cos\theta\partial_\theta + \sin\theta\partial_r$ the curvature operator, $\nabla_{\|} = (1/q)(q\partial_\phi + \partial_\theta)$ the parallel derivative operator, $\langle\bullet\rangle$ denotes flux surface average, $A = \epsilon/\rho_*$, and Γ is a constant. Units of T_e for the temperature, T_e/e for the potential and $c_s = (T_e/M_i)^{1/2}$ for the velocity are employed. Note also that *large scale units* a (the minor radius) for the lengths and $a^2/(cT_e/eB)$ for the time are used.

Noteworthy features of this code are

1. Globality. The code solves the model equations, in toroidal geometry and in a domain comprised between two arbitrary circular magnetic surfaces, without using

the local or flux tube approximation. This is necessary to answer questions related to the $\rho_* = \rho_s/a$ scaling, without *a priori* assumptions about the scale separation. (ρ_s is the ion sound Larmor radius and a the minor radius.)

2. Flux boundary conditions. The input power is given as a control parameter by specifying either the heat flux through the inner boundary or by a given power deposition profile. The (fluctuating) ion temperature is the outcome of the simulations. This is primarily done to avoid any assumption about the (unknown) time-averaged profile and to allow quite naturally any possible phenomena of intermittency in the profile dynamics.

3. Emphasis on the mesoscale dynamics. As suggested by experiments, as well as by theoretical considerations, most of the turbulent energy accumulates in the spectral region around the poloidal and radial wavenumbers $k_\theta \rho_s \approx k_r \rho_s \sim 0.1$, which is where the turbulence is suppressed by Landau damping. This allows one to work with model equations which neglect most finite Larmor radius terms.

4. Emphasis on long time scales. Simulations are carried out for at least one *global* energy confinement time in most cases. Indeed all the profiles shown in this contribution are time averages over at least one confinement time.

The code has been employed to assess the dependence of the effective ion conductivity and on the reduced gyroradius ρ_* and on the local safety factor. All the studies presented in this communication were carried out in a toroidal annulus comprised between the magnetic surfaces at $r = 0.5$ and $r = 1$.

2. Gyroradius scaling.

The first application has been to the problem of the ρ_* scaling. In Bohm units ($\chi_B = cT_e/eB$) the ion conductivity χ_i scales like a power of ρ_*. In L-mode tokamak operations χ_i/χ_B seems almost ρ_*-independent (Bohm-scaling), while in H-modes one finds $\chi_i/\chi_B \sim \rho_*$ (gyro-Bohm scaling). In linear theory, radial eigenmode structures typically scale like $\rho_*^{1/2}$, which would give weak or null (Bohm) ρ_* dependence, whereas nonlinear considerations would suggest a gyro-Bohm scaling. Hence the need for an assessment. The finding that the actual ion conductivity scales indeed like gyro-Bohm, has been obtained by a series of numerical similarity experiments. In these experiments, the input power (in the form of a given heat flux $F_{\rm in}$ at the inner boundary) and ρ_* are varied proportionally, down to a value of $\rho_* = 1/200$, to test whether the profiles stay the same and the conductivity varies proportionally to ρ_*. The results of these experiments are summarized in Fig. 1 which shows the time-averaged temperature profile, temperature gradient, local effective conductivity and temperature fluctuations, obtained at progessively smaller ρ_* (1/50, 1/100 and 1/200), and proportionally reducing the input power $F_{\rm in}$ from $3. \times 10^{-2}$ to $7.5. \times 10^{-3}$. The pictures show that by reducing ρ_* one needs proportionally less power to sustain *the same gradient*, and thus the conductivity is proportional to ρ_*, when expressed in Bohm units (gyro-Bohm scaling). Consistently with this result, the radial correlation length and the correlation time were also found to scale like ρ_*. An example of potential isolines at $\rho_* = 1/100$ and $\rho_* = 1/200$ are shown on Fig. 2. Finally, this result has been found to hold independently of the normalized input power $F_{\rm in}/\rho_*$, which in our study was varied by a factor 100, from a minimum of $F_{\rm in}/\rho_* = 5. \times 10^{-2}$ (which leaves the gradient very close to the ITG threshold) up to $F_{\rm in}/\rho_* = 5$.

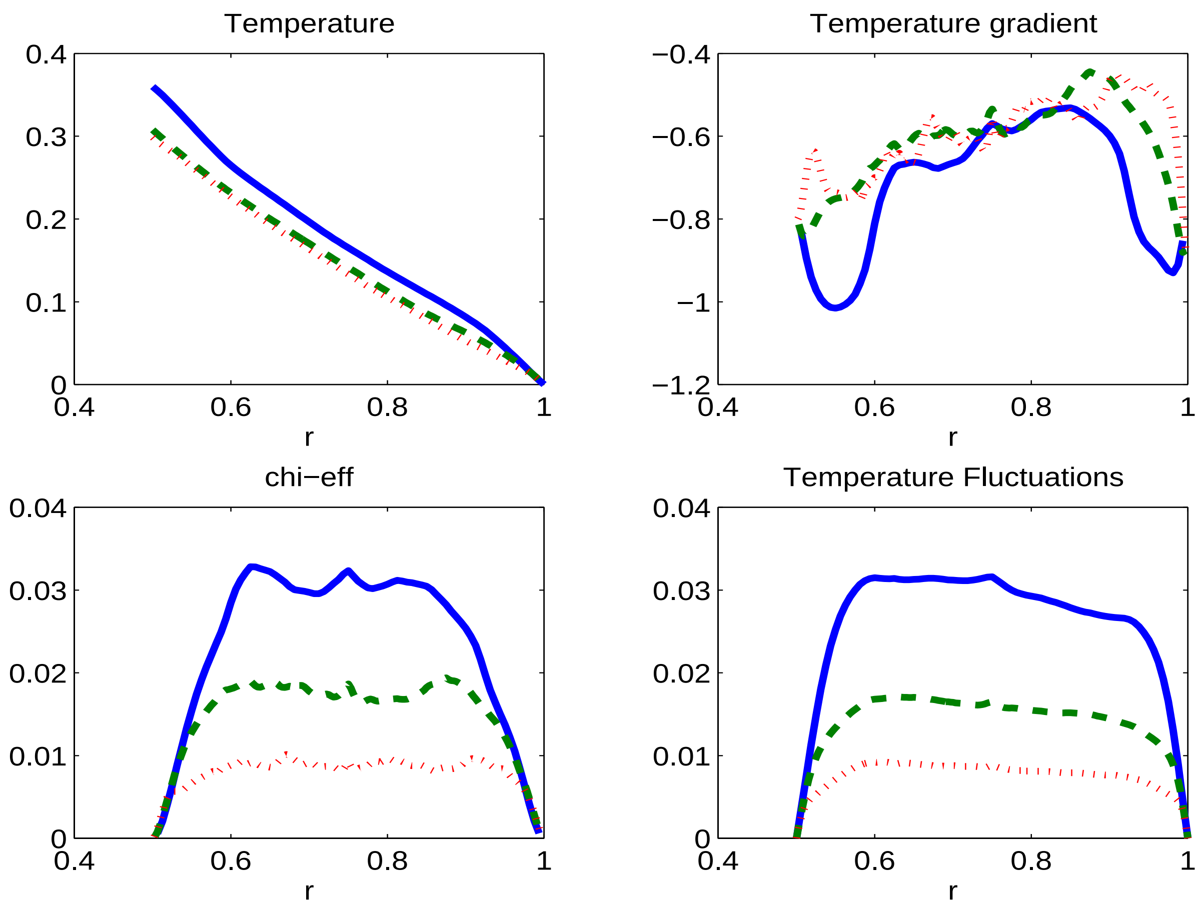

Figure 1: *ρ_* similarity runs; solid $\rho_* = 1/50$, dashed $\rho_* = 1/100$, dotted $\rho_* = 1/200$.*

3. Current scaling.

As a second application, the question of the plasma current scaling has been addressed. It is found experimentally that the confinement time is roughly proportional to the plasma current, which, when combined with the ρ_* scaling, is roughly equivalent to the conductivity being proportional to the square of the safety factor q. However, global current scaling experiments cannot easily distinguish between the dependence on q and on the magnetic shear, which change at the same time. A numerical assessment of the q scaling has been carried out for the ITG model using the similarity experiment technique, with q profiles of the form $q = q_a r^{\hat{s}}$, by changing q_a and keeping the magnetic shear $\hat{s}$ constant and uniform in space (here $\hat{s} = 1$). The main finding is that the q dependence *cannot* be cast in simple monomial form, as explained below.
At moderate input power, the similarity test consisted in running a simulation at $q_a = 4$ and $F_{\text{in}}/\rho_* = 1.5$, which we call here *the reference case* (run 1), and two simulations, both at $q_a = 2$, with $F_{\text{in}}/\rho_* = 0.75$ (run 2, to test whether $\chi_i \sim q$) and $F_{\text{in}}/\rho_* = 0.375$ (run 3, to test, as an alternative, $\chi_i \sim q^2$). The results are shown in Fig. 3, where the conductivity ratios of the reference run to either test runs are given. It is apparent that no similarity experiment gives satisfactory results, with $\chi_i \sim q$ being a too weak scaling and $\chi_i \sim q^2$ a too strong scaling. Thus a pure monomial scaling of the form $\chi_i \sim q^\beta$ would require $1 < \beta < 2$.
A second study was performed at a stronger normalized input power, with a new reference run with $q_a = 4$ and $F_{\text{in}}/\rho_* = 5$. Now the test case $q_a = 2$, $F_{\text{in}}/\rho_* = 2.5$ shows that $\chi_i \sim q$

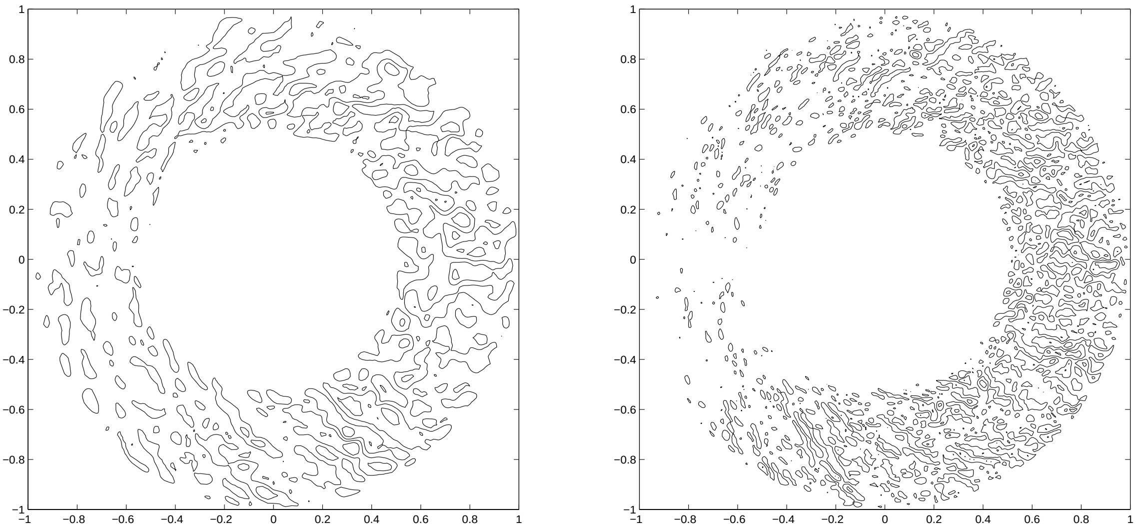

Figure 2: *Potential isolines,* $\rho_* = 1/100$ *(left) and* $\rho_* = 1/200$ *(right)*

is already too strong and thus one would have to take $\beta < 1$ at higher forcing (Fig. 4.) The conclusion is that no simple monomial dependence of the effective conductivity on q can be used to represent all the data.

On the other hand we note that, *qualitatively*, the numerical findings are consistent with the expression

$$\chi \sim \rho_* q^{\beta} (R/L_T)^{\gamma} f_{\rm thrs}[(R/L_T) - C_{\rm th}/q^{\beta'}] \tag{4}$$

where $f_{\rm thrs}[\cdot]$ is a monotonic threshold function. Note the double dependence on q characterized by two positive exponents β and β'. In general, a reduction of q (larger current) translates into a reduction of χ in two ways, via the factor q^{β} and through the increased threshold (factor $C_{\rm th}/q^{\beta'}$). At smaller input power, when one works closer to threshold, the latter is more effective and gives a *stronger apparent scaling* than when working farther from threshold. The fact that the threshold function depend on q is confirmed by running the code in linear mode, using a family of profiles with increasing temperature gradient and looking for the onset of the instability. The threshold was found to decrease when q is increased. It is interesting to note that a similar behavior is found experimentally in Tore Supra [3], where the measured electron conductivity is consistent with $\beta' = 1$, that is, an effective threshold depending inversely on q.

References

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[3] HOANG G. T., *et al.*, "Electron transport and improved confinemnet in Tore Supra", paper EX6/1, this conference.

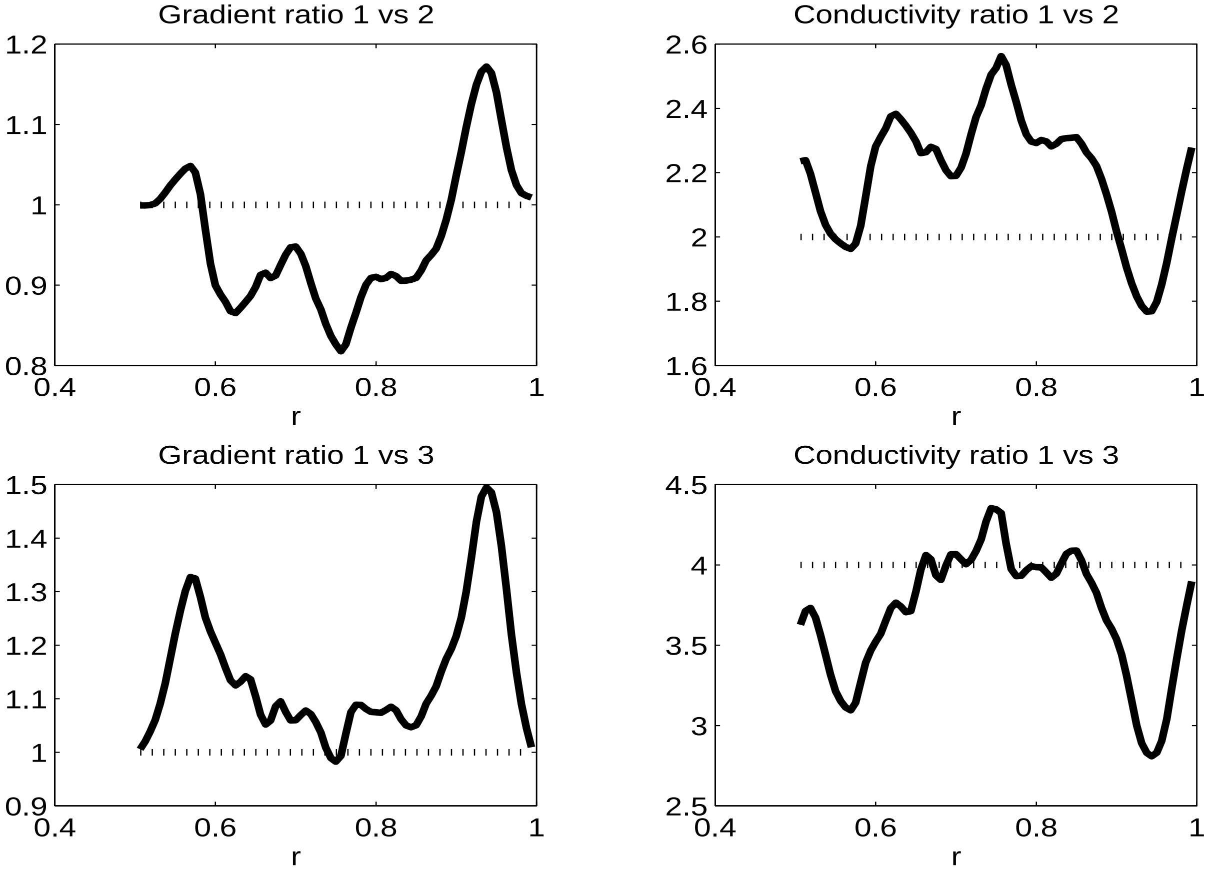

Figure 3: *Temperature gradient and conductivity ratios for the q scaling study at medium forcing*

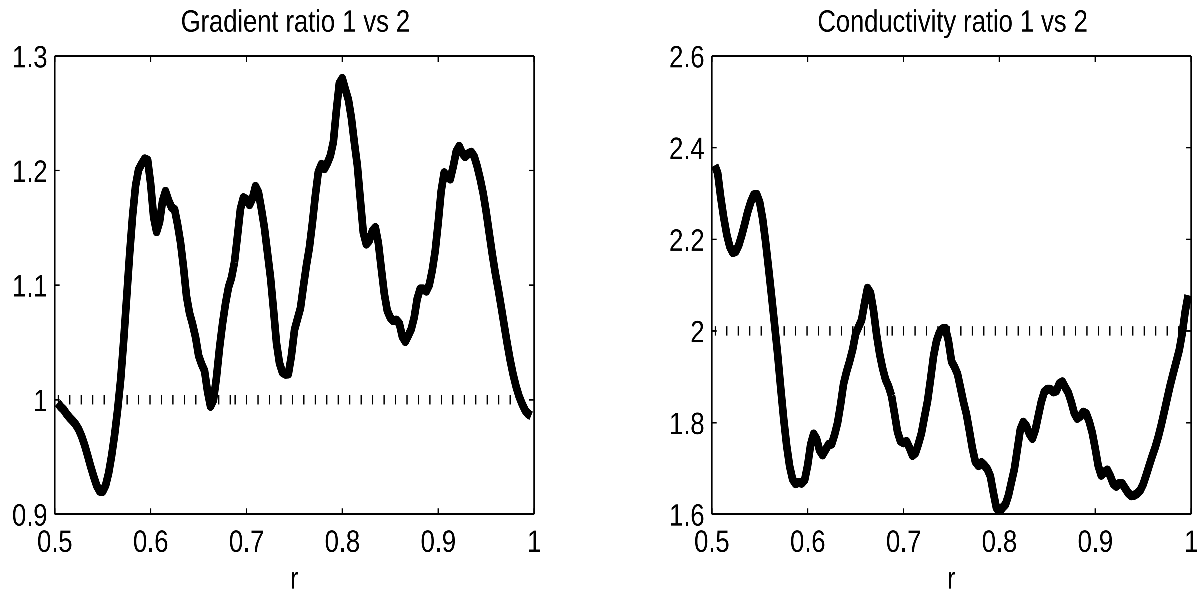

Figure 4: *Temperature gradient and conductivity ratios for the q scaling study at strong forcing*