Advances in Global Linear Gyrokinetic Simulations

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Abstract. Substantial advances have been made in the global calculation of microinstabilities. First, we present results of the world's first global linear gyrokinetic code in fully 3D configurations. We show that the unstable Ion Temperature Gradient (ITG) modes in quasi-axisymmetric and quasi-helical configurations exhibit 3D features, while the global stability properties seem rather insensitive to the three-dimensionality of the configuration. Second, the inclusion of equilibrium radial electric fields in both finite element particle-in-cell (PIC) and spectral global codes yield the remarkable result that the value of the $E \times B$ flow can be as effective as its shear in stabilizing toroidal ITG modes. Third, electron dynamics and finite beta effects are addressed by including electromagnetic perturbations with a two-potential formulation.

1. Introduction

The present understanding of anomalous transport in magnetically confined plamas is based on various underlying microinstabilities driven by equilibrium gradients. While nonlinear simulations are necessary for the determination of the heat flux generated by the microturbulence, linear simulations are useful in identifying the physical and geometrical parameters influencing the stability. In this paper a global approach is adopted, in which the relevant gyrokinetic equations are solved in the full plasma domain in the appropriate geometry. Throughout this paper the usual gyrokinetic ordering is used: $O(k_\perp \rho) \approx 1$, $\omega/\Omega \approx k_\| / k_\perp \approx e\phi/T_e \approx \rho/L_n \approx \rho/L_T \approx O(\epsilon)$ and $\rho/L_B \approx O(\epsilon_B)$, where ρ is the ion Larmor radius, Ω is the ion cyclotron frequency, ϕ is the perturbed electrostatic potential, L_n, L_T and L_B are the equilibrium scale lengths of density, temperature and magnetic field, respectively. The gyrokinetic equations are valid to the order $O(\epsilon^2)$, $O(\epsilon\epsilon_B)$ and $O(\epsilon_B)$. Two different methods are used: one is a time-evolution, Particle In Cell (PIC) - finite element method [1], the other is a spectral approach [2].

2. ITG modes in stellarator configurations

The next generation of stellarators will be characterized by much smaller neo-classical transport than previously and by particle confinement close to that of tokamaks. The question naturally arises as to whether anomalous transport in these devices will show up. As a first step in this direction we have developed a global gyrokinetic code, EUTERPE, aimed at the investigation of ITG modes in general 3D geometry. The essential ingredients used in the model and its numerical implementation are summarized here. More details can be found in [3]. We consider general 3D MHD equilibrium configurations with nested flux surfaces provided by the VMEC code [4]. The VMEC coordinates are mapped into straight-field-line magnetic coordinates. We use PIC with 3D quadratic splines finite elements on the linearized δf method [5] in which only the perturbed part of the distribution function is discretized. We take advantage of the interchange nature of the ITG mode ($k_{\|} \approx 0$) by extracting the fast phase variation across field lines using the method presented in [1], thus making simulations with large mode numbers possible with virtually no increase in the required computing power. To reduce numerical noise a Fourier filter is applied that keeps m and n components with small $k_{\|}$ and those resulting from the coupling induced by the 3D magnetic field configuration. The resulting EUTERPE code has been successfully validated against the 2D GYGLES code [1] in its toroidal axisymmetric and helically symmetric versions [6].

We have investigated the global linear stability of ITG modes in the QAS3 configuration which is under consideration at PPPL [7]. QAS3 is characterized by three field periods, a dominant tokamak-like $B_{m=1,n=0}$ component, but also a strong "mirror-field" $B_{m=0,n=1}$ component. Its shear is negative. Fig.1 shows the ITG mode $m_0 = -24$, $n_0 = 8$ as computed by EUTERPE. (In a 3D configuration the eigenmodes of the system have several m and n, so m_0, n_0 refer here to the dominant mode numbers.) The ITG mode is ballooning all along the torus on the low field side, with only a slight variation in amplitude along the toroidal direction. Thus this mode is almost characterized by a single toroidal mode number as in a tokamak. The similarity of QAS3 with a tokamak is further exemplified by comparing the frequencies and growth rates of QAS3 and an "equivalent tokamak" having only the axisymmetric components of the plasma surface $R_{m,n=0}, Z_{m,n=0}$ but otherwise the same parameters (Fig.2). We have checked that this similarity of ITG behaviour is not a coincidence by considering a sequence of equilibria that vary continuously from the pure axisymmetric case to the QAS3 case. In contrast to the balloning model which predicts a strong influence of the local magnetic shear on drift wave stability [8] we have found virtually no influence, even though the local shear in QAS3 is strongly modulated: the ITG mode amplitude is only slightly modulated and the growth rate is not affected. We conclude that the stability properties of the global toroidal ITG mode in QAS3 are indeed virtually identical to those obtained in a tokamak.

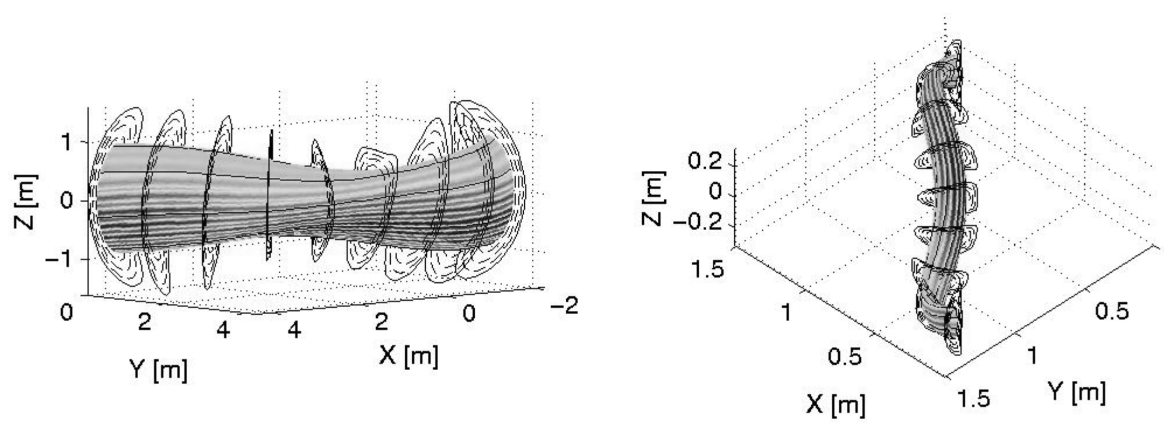

Figure 1: *ITG mode in QAS3 (left) and in HSX (right) as computed with the EUTERPE global gyrokinetic code.*

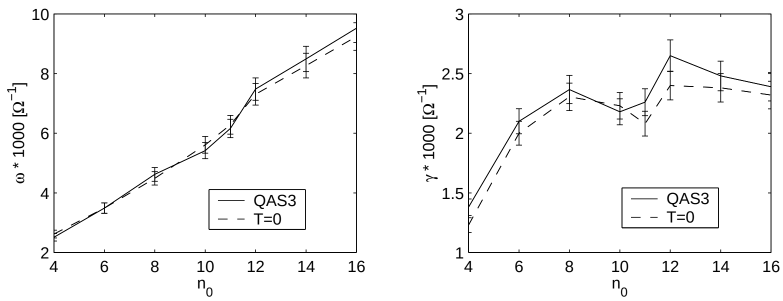

Figure 2: *Frequencies (left) and growth rates (right) of ITG modes vs dominant toroidal mode number in QAS3 (continuous line) and an equivalent tokamak (broken line).*

The HSX device is a helical axis stellarator with 4 field periods, a dominant helical-like $B_{m=1,n=1}$ component and a rather flat safety factor profile, $q_{edge}/q_{axis} \approx 0.95$. It has recently started operation at the University of Wisconsin. We show in Fig.1 the mode $m_0 = -9, n_0 = 8$ as computed with the EUTERPE code. The ITG mode amplitude shows, in addition to a tendency to alignment with the quasi-helical symmetry, a toroidal modulation of about 50%. This amplitude modulation has been found to correlate with the variation of the Jacobian: $|\tilde{\phi}|$ reaches its maximum where the Jacobian is minimum. Because of its helical magnetic axis, the structure of the Jacobian results from the variation of the major radius along the toroidal direction. In HSX it varies from $R = 1.45$m at the beginning of a field period to $R = 1.05$m in the middle of a field period. The curvature drifts are then stronger in the middle of a field period and this pushes the ITG mode amplitude higher there. In order to quantify the influence of plasma shape, we consider a sequence of equilibria varying from the quasi-helical HSX configuration to a helically symmetric configuration, increasing the number of field periods in proportion to the aspect ratio. The modulation of the ITG amplitude decreases until perfect alignment with the helical symmetry is reached. However, we have found that the growth rate varies by less than 10%. This indicates that global ITG mode stability properties in HSX are essentially similar to those of a helically symmetric configuration.

3. Effects of $E \times B$ flows

In tokamaks, $E \times B$ sheared flows have been observed to be related to the appearance of improved confinement regimes, hence reduced anomalous transport. Theories of flow shear fluctuation supression [9, 10] show that if the $E \times B$ flow is sheared, then it should decorrelate the turbulence and therefore stabilize it when the shearing rate $\omega_{E\times B}$ approximately equals the growth rate γ. In [9] $\omega_{E\times B}$ is proportional to $\partial/\partial\rho(u/\rho)$, where $u = E\times B/B^2$ and ρ is the minor radius, whereas in [10] it is proportional to $d^2/d\psi^2\Phi_0(\psi)$, which reduces in the circular, large aspect ratio limit to $\partial/\partial\rho(qu/\rho)$. Here, an equilibrium radial electric field is assumed and its effect on ITG modes global linear stability is studied. Both PIC-finite element and spectral methods have been applied. We consider a tokamak with circular cross-section, and a T_i profile peaking at mid-radius with R/L_{Ti}=7.4. The $E\times B$ profile is specified in three different ways: (1) a constant profile of u/ρ; (2) a linear profile of u/ρ; and (3) a constant profile of $d\Phi_0/d\psi$, where $\mathbf{E} = -\nabla\Phi_0(\psi)$ and ψ is the poloidal flux. Our results in Fig.3 show that indeed a sheared $E \times B$ flow is

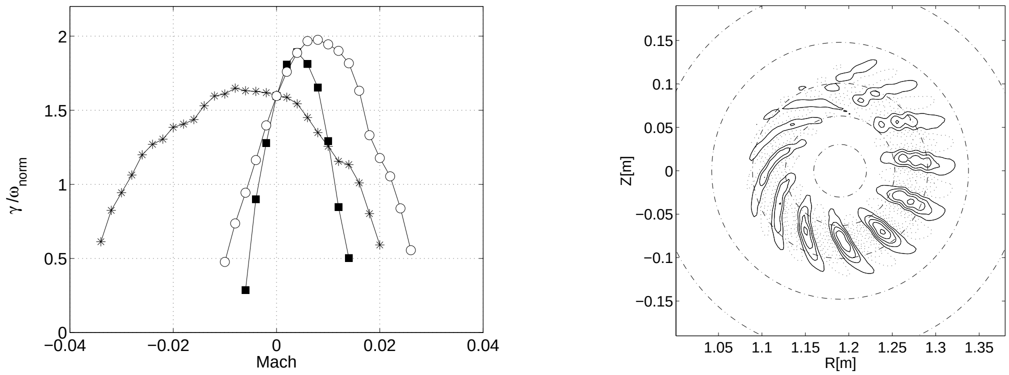

Figure 3: *Left: growth rates of the most unstable toroidal ITG mode vs Mach number for a sheared profile of* $u = E \times B/B^2$ *(open symbols), for a shearless profile of* u/ρ *(filled symbols) and for a shearless profile of* $d\Phi_0/d\psi$ *(stars). Right: perturbed* ϕ *for a shearless profile of* $d\Phi_0/d\psi$ *with Mach=-0.034.*

stabilizing the toroidal ITG mode (open symbols). But the shearless cases are interesting: the shearless u/ρ (filled symbols) provides even stronger stabilisation than the sheared case, while the shearless $d\Phi_0/d\psi$ is more weakly stabilized. We conclude that, in addition to the commonly admitted shearing rate criteria, there is another stabilizing mechanism: the value of flow is as important as its shear. From an analysis of the eigenmode structure we interpret it as follows: the toroidal ITG mode balloons in the unfavourable grad B drift region; as the value of $E \times B$ flow is increased it pushes the maximum mode amplitude poloidally away from the unfavourable grad B drift region, thereby decreasing the instability drive. A heuristic criterion can be derived: there is stabilization when the poloidal $E \times B$ rotation frequency becomes comparable to the linear growth rate without rotation. Note that the profile of constant u/ρ is sheared in the sense of Ref.[10], so the two effects of shearing and rotation value add up for this particular profile.

Taking parabolic profiles of u/ρ with zero first derivative at the maximum gradient position, we have shown that the effect of the so-called "flow curvature" is negligible [11]. Also, we have analyzed an ASDEX-Upgrade case with improved confinement. Our results show that ITG modes are stabilized for values of $E \times B$ flow in reasonable agreement with the experiment [11]. We have also shown that large T_i/T_e is stabilizing, whereas the inclusion of trapped electron dynamics is destabilizing.

4. Electromagnetic effects on microinstabilities

Electromagnetic perturbations and electron dynamics can strongly affect the behaviour of microinstabilities: finite plasma β can stabilize ITG modes. The model used describes the perturbation with a two-potential approximation ($A_{||}, \phi$). The same spectral approach as in [2] is adopted. We show in Fig.4 the growth rates as a function of β. Neglecting trapped electron effects, the ITG mode is quickly stabilized at $\beta \approx 4\%$. Including trapped electrons, we show different cases for different values of magnetic shear. Trapped electrons are overall destabilizing, but the most remarkable feature is the effect of shear coupled to β. For small β values, reversing the shear is stabilizing, whereas for $\beta > 3\%$, zero or negative

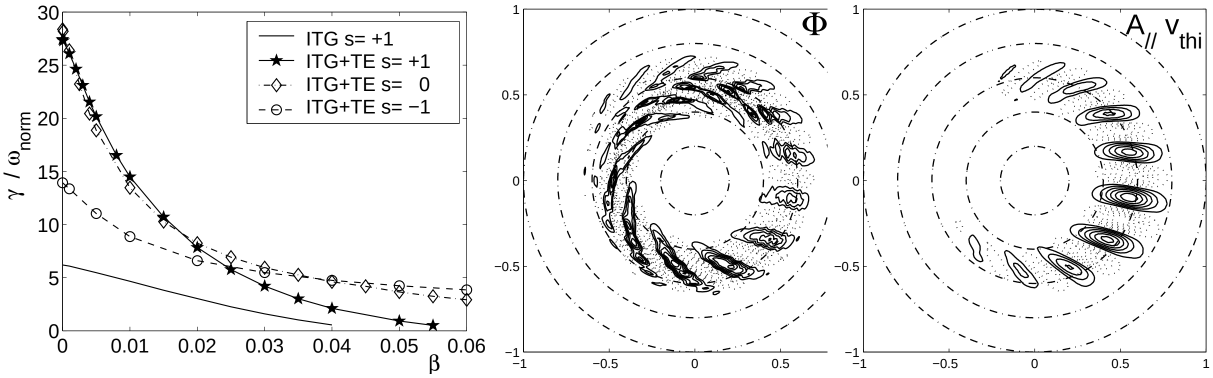

Figure 4: *Left: growth rates of most unstable ITG modes vs β, for different values of magnetic shear (plain line: without trapped electrons, other curves with). Perturbed ϕ (middle) and $A_{||}$ (right), for the case β=4%, shear=+1.*

shear cases are more unstable and complete stabilization is prevented at least until β=6%. For zero or positive shear, the electromagnetic component of the perturbation, measured as $v_{thi} < |A_{||}| > / < |\phi| >$, where $<>$ denotes the spatial average, is increasing from about zero at β=0 to about 0.4 at β=4%. Fig. 4 shows a ballooning-like mode structure for $A_{||}$ which varies very little with β, whereas ϕ is more affected. For negative shear, ITG modes are more slab-like and the electromagnetic component does not exceed 3×10^{-3}.

Trapped electron modes, on the other hand, are insensitive to finite β effects, and reversing the shear is always stabilizing. The insensitivity to β is confirmed by the fact that these modes are almost purely electrostatic: $v_{thi} < |A_{||}| > / < |\phi| > < 10^{-3}$ for all β values.

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