EFFECT OF ZONAL FLOWS ON DRIFT WAVE TURBULENCE THP1/05

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Abstract Nonlinearly generated zonal flows are obtained from ion temperature gradient driven (ITG) modes and electromagnetic drift interchange modes. It is found that in general the flow is not important at the correlation lengths but gives an absorbing boundary for long wavelengths. For ITG modes, however, resonant excitation is found close to marginal stability. This may explain the strong effects of zonal flows seen in some nonlinear gyrokinetic simulations.

1 Introduction

Recent experimental [1,2] and theoretical [3–6] results show that effects of background flows in tokamak often are important. In particular it seems that flows will always be important for the longest wavelengths in the plasma, effectively generating an absorbing boundary for these. When background flows stabilize the turbulence at the correlation length we get an internal transport barrier. If nonlinearly generated zonal flows stabilize the linear eigenmodes at the correlation length we either get complete stabilization in the collisionless case [3, 4] or a transport which is proportional to the collision frequency. In the present work we have found a resonant excitation of zonal flows close to marginal stability of ITG modes. We suggest this as the reason for the complete stabilization of ITG modes near linear marginal stability (nonlinear upshift) found in the Cyclone simulations [4].

2 Formulation

We have studied nonlinearly driven zonal flows in two cases, first due to ion temperature gradient driven modes in the electrostatic limit and then due to electromagnetic interchange modes. We use the reductive perturbative method [7] with the ansatz $f = \sum_m \epsilon^m f^m \;, \quad f^m = \sum_l f_l^m\,(x,\zeta,\tau)\exp\left[il\left(k_{\|}z + k_y y - \omega t\right)\right]$ where $\zeta = \epsilon(y - ut)$, $\tau = \epsilon^2 t$ and $\epsilon \sim e\phi/T_e = \hat{\phi} \ll 1$. For the x variation we assume a standing wave with wave number k_m.

To order ϵ, we obtain the linear dispersion relation. For ITG modes, it agrees with that in Ref. [8]. We include the nonlocal properties first derived in Ref. [9]. For electromagnetic drift interchange modes we ignore ion temperature effects. The linear dispersion relation then agrees with that in Ref. [10] if we ignore ion temperature effects. To order ϵ^2 we find in both cases that $u = \partial\omega/\partial k_y$ *i.e.,* the group velocity.

To order ϵ^3, we find that zonal flow potential $\hat{\phi}_0^{(2)}$. We here write it as

$$\hat{\phi}_0^{(2)} = k_m L_n k_y^2 a_s^2 T |\hat{\phi}_1^{(1)}|^2 \sin 2k_m x \tag{1}$$

This is a general form valid for both ITG and electromagnetic drift interchange modes. We here include the Reynolds stress for both modes, the convective ion temperature nonlinearity for the ITG modes and the nonlinearity of density convection and electron temperature convection (due to line bending) and the nonlinear kink term for the electromagnetic interchange mode. We note that $\hat{\phi}_0^{(2)}$ has $k_x = 2k_m$ *i.e.,* faster variation in x than the drift wave. This increases the shearing rate.

For the ITG mode, we obtain

$$T = \frac{\hat{u}(\hat{u}-\epsilon_n)}{\hat{N}}\left[2\Big(\hat{u}+\frac{5}{3\tau}\epsilon_n\Big) - \frac{\epsilon_n}{\tau}\Big(\eta_i - \frac{2}{3} - \frac{10}{3\tau}\epsilon_n\Big) \times Re\left\{\frac{\hat{\omega} + (\frac{1+\eta_i}{\tau})\epsilon_n}{\Big(\hat{\omega}(1+k_y^2 a_s^2) - \hat{\omega}_L\Big)\Big(\hat{\omega} + \frac{5}{3\tau}\epsilon_n\Big)}\right\}\right]$$

where $$\hat{N} = 4k_m^2 a_s^2 \hat{u}(\hat{u} - \Big(\hat{u} + \frac{5}{3\tau}\epsilon_n\Big)(\hat{u}+\frac{1}{\tau}) + \epsilon_n\left\{\Big(\hat{u}+\frac{5}{3\tau}\epsilon_n\Big)\Big((1+\frac{1}{\tau})(1-\epsilon_n)\hat{u} + \eta_e(\hat{u}+\frac{\epsilon_n}{\tau})\Big) + \frac{2}{3\tau}\hat{u}\Big((1-\epsilon_n)\hat{u} + \eta_e\epsilon_n\Big) - \frac{1}{\tau}\Big(\frac{2}{3}-\eta_i\Big)\hat{u}(\hat{u}-\epsilon_n)\right\}$$

and

$$\omega_L = \frac{\omega_n}{2}\left[1 - \epsilon_n\Big(1+\frac{10}{3\tau}\Big) - \frac{1}{\tau}\Big(1+\eta_i+\frac{5}{3}\epsilon_n\Big)k_y^2 a_s^2\right] - i\gamma_d\,; \quad \gamma_d = \Big(1+\frac{5}{3\tau}\Big)\frac{\epsilon_n|s|}{4q}\omega_n.$$

Strong Resonance

As it turns out, there will be a rather wide resonance for the real part of $\omega(1+k_y^2a_s^2)-\omega_L$ and it will then depend on the imaginary part how strong the resonance will be. We will now explore the limit when we need to solve the time dependent problem for $\phi_0^{(2)}$. For that, we extract the resonant denominator $\hat{\omega}(1+k_y^2a_s^2) - \hat{\omega}_L$ from T and multiply Eq.(1) by it. On transforming to the time domain *i.e.,* $\hat{\omega} \rightarrow (\hat{\omega_L}/(1+k_y^2a_s^2)) - i\partial/\partial t$, we obtain a resonant growth rate of $\phi_0^{(2)}$ when $\partial/\partial t$ exceeds $|\hat{\omega}(1+k_y^2a_s^2) - \hat{\omega}_L|$. As it turns out the resonance in the real part of ω is quite wide so the resonance condition typically reduces to

$$\frac{\partial\phi_0^{(2)}}{\partial t} > (\gamma + \gamma_d)\phi_0^{(2)} \tag{2}$$

Thus γ_d acts as an additional threshold for resonant excitation.

The electromagnetic result is similar but does not include ion temperature effects. This makes an important difference since ion temperature effects give a resonance close to marginal stability.

To the first order in ϵ we obtain in the electromagnetic case:

$$T_{el}^{(1)} = \eta_e \frac{\omega_n}{k_{||}c}\hat{A}_l^{(1)}$$

Here T_e is the normalized electron temperature perturbation, $\hat{A}$ is the normalized (by e/T_e) magnetic vector potential, $\omega_n = k_y V_n$ is the diamagnetic frequency, $\omega_d = k_y V_d$ is the magnetic drift frequency and $\omega_A = k_{||} V_A$ is the Alfvén frequency.

$$\hat{A}_l^{(1)} = \alpha \, \hat{\phi}_l^{(1)} \; ; \quad \alpha = k_{||} c \left[\frac{(\omega + \tau\omega_n)\, \hat{k}_\perp^2 + (1+\tau)\,\omega_d}{\omega_A^2 \hat{k}_\perp^2 + (1+\tau)\,\omega_d\,(\omega - \omega_n) - \eta_e \omega_n \omega_d} \right]$$

and the dispersion relation

$$(\omega + \tau\omega_n)\, \hat{k}_\perp^2 + \omega - \omega_n + (1+\tau)\,\omega_d - \frac{\alpha}{k_{||} c}\,(\omega - \omega_n)\,(\omega + \tau\omega_d) = 0. \tag{3}$$

Here $\hat{k}_\perp^2 = (k_m^2 + l^2 k_y^2) a_s^2$ To cubic order we find the denominator:

$$N = 4 k_m^2 a_s^2 u\,(u + \tau V_n)\,(u - V_d) + V_d V_n\,((1+\tau)\,(1 - \epsilon_n)\,u + \eta_e\,(u + \tau V_d))\,. \tag{4}$$

We have found that resonances coming from N are quite unlikely. Other possible resonances come from the denominator of α. This resonance can become important at high β i.e. of the order of the MHD β limit.

We can now write the shearing rate as:

$$\hat{\omega}_{rot} = \left| \frac{1}{\omega_n} \frac{\partial V_\theta}{\partial x} \right| = 4 k_m^3 L_n^2 k_y a_s^2 T \mid \tilde{\phi}_i^{(1)} \mid^2 \tag{5}$$

In the case, when shear flow is able to provide an absorbing boundary for the long wavelengths, the drift waves are saturated by the mode coupling leading to the saturation level [8]

$$\phi \approx \frac{\gamma}{\omega_n} \frac{1}{k_r L_n} \tag{6}$$

where $k_r > 2\pi/a$ and a is the minor radius of the torus.

Using the above estimate of ϕ and the fact that the nonlinearities isotropize the drift wave turbulence (*i.e.*, $k_r \sim k_m \sim k_\theta$), the magnitude of the flow shear rate in the nonresonant case can be estimated as:

$$\frac{dV_0}{dr} \simeq 4 k_m^2 a_s^2 \omega_n \left(\frac{\gamma}{\omega_n} \right)^2 \frac{L}{L_n} \tag{7}$$

Here, we use $\sin 2k_m x \sim 1$ and only the poloidal component of the flow is considered.

With $L \sim L_n$ and $\gamma \sim \omega_n$, the expression of shear flow rate further simplifies as

$$\frac{dV_0}{dr} \simeq 4 k_m^2 a_s^2 \gamma$$

At the correlation length with $k_m a_s < 1$, the flow shear rate is less than the growth rate. It essentially means that flow is not stabilizing the modes at the correlation length.

However, at long wavelengths, we expect that the Waltz's condition for the stability of the mode due to shear flow would be satisfied. With $\gamma = k_\theta V_n$, the condition for the stability is

$$k_\theta V_n \leq 4k_m^2 a_s^2 \omega_n \left(\frac{\gamma}{\omega_n}\right)^2 \frac{L}{L_n} \tag{8}$$

where the quantities on the right hand side of the above expression are evaluated at the correlation length (main drive for $\phi_0^{(2)}$). Thus, we expect stabilization of the long wavelength modes for which

$$k_\theta \leq 4k_m^2 a_s^2 k_{\theta c} \frac{L}{L_n}$$

or with $L \sim L_n$

$$k_\theta \leq 4k_m^2 a_s^2 k_{\theta c} \tag{9}$$

Here, subscript c indicates the correlation length. Since the flow will tend to tear apart vortices larger than the shear scale length, energy will go to modes with shorter wavelengths.

Eq.(9) further indicates that the wavelengths on which the flow has a stabilizing effect is longer than the correlation length, or

$$k_{\theta s} < k_{\theta c} \tag{10}$$

where subscript s represent the flow stabilized critical length.

The above result also indicates that modes at the correlation length are not affected by the flow. With $k_m^2 a_s^2 \sim 0.1$, the typical magnitude of $k_{\theta s}$ turns out to be approximately $k_{\theta s}/2$ which means that the longer wavelengths in the system will be stabilized by the flow. This further confirms that the flow can provide an absorbing boundary for long wavelength modes as assumed in Eq.(6).

3 Discussion

In the nonresonant case we find that typically $\hat{\omega}_{rot} < 0.5\hat{\gamma}$ at the correlation length. This means that the stabilizing effects are small at the correlation length. When we consider the influence of zonal flow, generated at the correlation length, on longer wavelength modes, we find that the stabilization sets in at about twice the correlation length. This means that zonal flows can give an absorbing boundary for long wavelengths. The nonresonant case is the most typical. For the electromagnetic case without ion temperature effects, resonance seems to be quite unlikely and sets in only close to the MHD β limit. For the ITG modes, however, we find resonance close to marginal stability. This is in agreement with the particle code results in the Cyclone project [4]. We have also found that magnetic shear damping can detune the resonance. This may be the reason for the difference between global and flux tube results in the Cyclone simulations [4] since global simulations have a space dependent drift frequency which leads to an enhanced magnetic shear damping [11]. To fully explore the resonance, however, we need to study the time dependent problem for the zonal flow.

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