Summarized General Scaling Laws Covering Over the Representative Tandem-Mirror Operations in GAMMA 10

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Abstract. *Generalized scaling laws* covering over the representative tandem-mirror operational modes from 1979 to 2000 in GAMMA 10 [*i.e.*, characterized in terms of (i) *a high-potential mode* having kV-order ion-confining potentials (ϕ_c) and thermal-barrier potentials (ϕ_b), and (ii) *a hot-ion mode* yielding fusion neutrons with 10-20-keV bulk-ion temperatures] are found and summarized for providing the possibilities of combining the characteristics from each mode into novel extended operational modes. The physics scalings of the formation of the plasma confining potentials as well as the associated effects of the potentials on plasma-parameter improvements are systematically investigated as follows: (i) The *potential-formation scalings* in the two representative modes are consolidated and generalized on the basis of the consistency with the novel findings of wider validity of *Cohen's strong electron-cyclotron heating (ECH) theory* covering over both modes. (ii) The *produced potentials*, in turn, provide a favorable *novel scaling of the increase in the central-cell electron temperatures T_e with increasing ϕ_b*, limited by the available ECH powers. The scaling of T_e with ϕ_b is well interpreted by *the Pastukhov theory* of plasma potential confinement, as a similarly reported scaling of potential-confined ion temperatures with ϕ_c. (iii) A scaling of ϕ_c [kV] with ECH powers in a plug region, P_{ECH} [kW], and the central-cell densities, n_c [10^{18} m^{-3}], is summarized as $\phi_c = 1.73\times10^{-4} P_{ECH}^{1.73} \exp(-0.33 n_c)$. (iv) Consequently, under the assumption of the validity of the extension of these theoretically well interpreted scaling data, the formation of Pastukhov's predicted ϕ_c of 30 kV for confining Q=1 plasmas is scaled to require 5-MW P_{ECH}.

1. Introduction

Generalized and extended scaling laws covering over the representative tandem-mirror operational modes are summarized for clarifying the present status in GAMMA 10 and for providing innovative approaches to alternative fusion-energy achievement. The main tandem-mirror operations from 1979 to 2000 in GAMMA 10 are characterized in terms of (i) a high-potential mode having kV-order plasma-confining potentials [1,2], and (ii) a hot-ion mode yielding fusion neutrons with 10-20 keV bulk-ion temperatures [3,4]. In this manuscript, a large number of database obtained from these two main GAMMA10 operations is intensively investigated and summarized to highlight the common essential physics bases underlying in both modes. In particular, systematic researches into the formation of plasma-confining potentials and the associated effects of the potential formation on the improvements in plasma parameters are carried out.

2. Experimental Apparatus

GAMMA 10 is a minimum-B anchored tandem mirror with outboard axisymmetric plug and barrier cells [1-4]. It has an axial length of 27 m, and the total volume of the vacuum vessel is 150 m^3. The central cell has a length of 6 m and a fixed limiter with a diameter of 0.36 m, and the magnetic-field intensity at the midplane B_m is 0.405 T with a mirror ratio R_m of 5.2. Ion-cyclotron heatings (ICH) (200 kW at 4.47 or 6.36 MHz, as well as 100 kW at 9.9 or 10.3 MHz) are employed for the central-cell hot-ion production and the anchor stabilization, respectively. The plug and barrier cells are axisymmetric mirrors; they have an axial length of 2.5 m (B_m=0.497 T, and R_m=6.2). Microwaves (150 kW at 28 GHz) are injected in the

extraordinary mode into the plug and the barrier regions to produce an ion-confining potential ϕ_c, and a thermal-barrier potential ϕ_b, respectively. Plug potentials Φ_P are measured with originally developed electrostatic spectrometer arrays for end-loss-ion energy analyses (ELA). Central-cell potentials Φ_C and barrier potentials Φ_B are directly measured with heavy-ion (Au^0) beam probes (HIBP). Therefore, one can obtain ϕ_c and ϕ_b, as Φ_P-Φ_C and Φ_C-Φ_B, respectively. X-ray diagnostics give profiles of electron temperatures T_e along with detailed electron-energy spectra in each region [2].

3. Scaling Law of the Formation of Thermal-Barrier and Ion-Confining Potentials

Figure 1 stands for one of *the most essential scalings of potential formation*. The scaling law of ion-confining potentials ϕ_c with thermal-barrier potentials ϕ_b in the high-potential mode [Fig. 1(a)] is compared to the scaling in the hot-ion mode [Fig. 1(b)]. Both scalings show favorable similar characteristics of remarkably growing ϕ_c with increasing thermal transport-barrier depth ϕ_b.

Careful notice, however, finds out the different dependence on the ratio of the plug to central-cell densities, n_p/n_c, despite the similarly behaved relations between ϕ_c and ϕ_b. This is contrary to the standard prediction from Cohen's theory of potential formation [5]. (Plotted data have the parameter regime of n_p/n_c labeled with the curves.) Existence of different underlying physics for these two typical operations could result in no chances of the upgraded innovations of combined modes.

However, *a new finding* of the synthesized generalization of these two scalings of ϕ_c, ϕ_b, and n_p/n_c is achieved due to the inclusion of the scaling of central-cell electron temperatures T_e [keV] with ϕ_b [kV]. The empirical scalings of T_e=0.23ϕ_b+0.03 and T_e=0.16ϕ_b+0.01 in the high-potential and hot-ion modes, respectively, (see Fig. 2) are introduced to reduce T_e in Cohen's strong ECH theoretical formula of ϕ_c=T_e[0.665(n_p/n_c) exp(1.19ϕ_b/T_e)]$^{2/3}$-ϕ_b [2,5] for plotting the solid curves labeled n_p/n_c in Figs 1(a), and 1(b), respectively. The curves well trace the data in Figs 1(a), and 1(b). In Fig. 1(c), the data sets of ϕ_c, ϕ_b, and n_p/n_c in Figs 1(a) and 1(b) are replotted using the same symbols. It is found that the data lie well on each curved surface labeled (a) and (b) calculated from the strong ECH theory along with the empirical scalings of T_e with ϕ_b, respectively. These methods and plots in *the proposed novel three-dimensional space (ϕ_c, ϕ_b, and n_p/n_c)*, thus, address the misunderstanding of no dependence of the relation between ϕ_c and ϕ_b on n_p/n_c. Therefore, *the new finding of the existence of common physics relation among ϕ_c, ϕ_b, and n_p/n_c in*

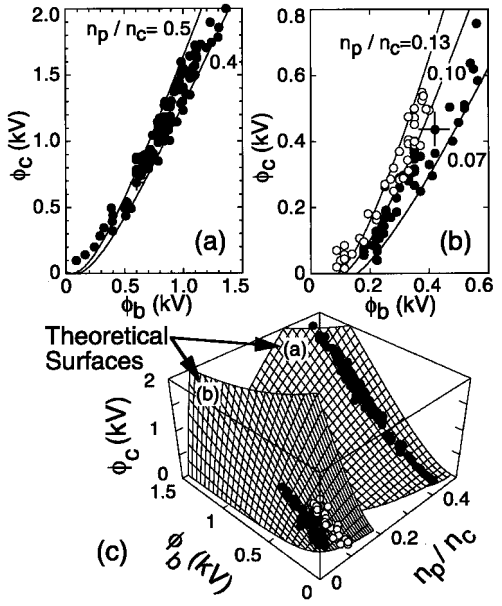

FIG. 1. The Scaling laws of ion-confining potentials ϕ_c with thermal-barrier potentials ϕ_b in (a) the high-potential and (b) the hot-ion modes. The data having the ratio of the plug to the central-cell densities n_p/n_c in 0.4-0.5 for (a), as well as in 0.07-0.10 (filled circles) and 0.10-0.13 (open circles) for (b) are plotted. In (c), the generalized scaling surfaces (a) and (b) in the proposed three-dimensional space of ϕ_c, ϕ_b, and n_p/n_c are calculated from Cohen's strong ECH theory in combination with the scaling of T_e [keV] with ϕ_b [kV] [i.e., (a) T_e=0.23ϕ_b+0.03, and (b) T_e=0.16ϕ_b+0.01 (see FIG. 2)]. The data in (a) and (b) are well replotted using the same symbols on the theoretical surfaces of (a) and (b) in (c), respectively.

both modes encourages us to realize the possibilities of combining the characteristics in each mode for opening an innovative mode having high-potential and hot-ion properties simultaneously. Furthermore, the validity of this extended scaling theory is supported by *a new finding* of the theoretically predicted plateau-electron formation [5] measured with plug X-ray energy spectra for *both* modes.

4. Scaling Law of Central-Cell Electron Temperatures with Thermal-Barrier Potentials

The scalings of T_e increasing with increasing the thermal-transport barrier depth ϕ_b are shown in Figs 2(a) and 2(b) for the high-potential and hot-ion modes, respectively. In Fig. 2(a), data on axis with the warm-electron temperatures T_{ew}=1-2 keV and the density ratio n_{ew}/n_c= 0.01-0.05 to n_c=(4-6)×10^{17} m^{-3} are plotted. Ion temperatures T_i=2 keV, and the neutral-particle populations n_0=(1-4)×10^{15} m^{-3} are observed. Here, n_0 decreases with increasing T_e. In Fig. 2(b), data having T_i=1-5 keV, n_c=(1-2)×10^{18} m^{-3}, n_0=(1-4)×10^{15} m^{-3}, and T_{ew}=1 keV with n_{ew}/n_c=0.005-0.01 are plotted. At this time, T_e increase up to 250 eV with increasing ϕ_b is limited by ECH powers for potential formation (150 kW in Fig. 2) and by no direct auxiliary central-cell electron heatings.

In order to find out the common physics interpretations covering over the scaling data in Figs 2(a) and 2(b), theoretical analyses are carried out by the use of the power- and particle-balance equations (1) and (2), respectively.

$$\frac{dWV_{BB}}{dt} = P_{wb}V_{BB} + P_{hb}V_h - \frac{WV_{BB}}{\tau_E}, \qquad (1)$$

and

$$\frac{dn_c}{dt} = n_0 n_c \langle \sigma v \rangle - \frac{n_c}{\tau_p}, \qquad (2)$$

where W and σv denote the bulk-electron energy density of $3/2n_cT_e$, and the ionization cross section, respectively. Slowing-down power densities to the bulk electrons from the warm electrons, and the hot ions are defined as P_{wb} and P_{hb}, respectively. The volume of the warm electrons, flowing from the plug region and thus existing between both barrier regions, is denoted by V_{BB}. Diamagnetic-loop-array signals for hot-ion profile measurements are analyzed to identify the axial profile and the volume of hot ions V_h.

In Fig. 2(a), the solid curves labeled n_{ew}/n_c=1 and 4% having T_{ew}=1 keV are calculated from Eq. (1) with the substitution of Pastukhov's energy-confinement time [6] into τ_E. Here, the second term of the right-hand side of Eq. (1) is one-order-of magnitude smaller than the first term (2 keV ions with V_h/V_{BB} of 0.39). The dashed curves are simulated from Eq. (2) with

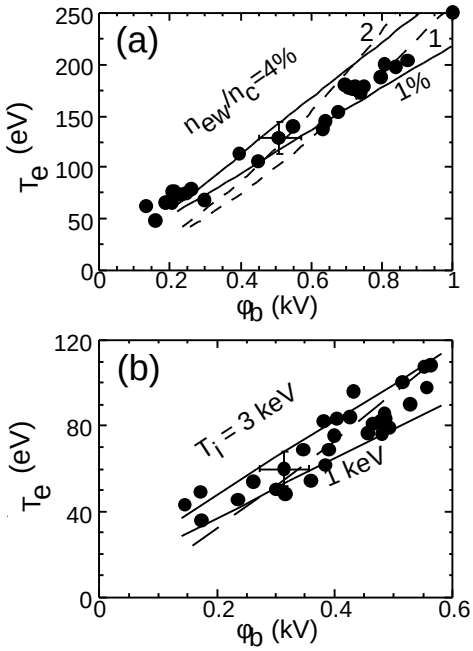

FIG. 2. The scalings of T_e with ϕ_b in (a) the high-potential and (b) the hot-ion modes are fitted by the solid and dashed curves from Eqs. (1) and (2), respectively, using Pastukhov's energy and particle confinement times along with the labeled values of the ratio of the warm to bulk-electron densities n_{ew}/n_c, T_i, or neutral-particle populations n_0 [1 and 2 in (a) corresponding to 1 and 2×10^{15} m^{-3}, respectively, and in (b) being 2×10^{15} m^{-3}] (see text). In (a), data with T_{ew}=1-2 keV, n_{ew}/n_c=1-5% [n_c=(4-6)×10^{17} m^{-3}], T_i=2 keV, and n_0=(1-4)×10^{15} m^{-3}; and in (b), data with T_i=1-5 keV, n_c=(1-2)×10^{18} m^{-3}, and n_0=(1-4)×10^{15} m^{-3} are plotted.

Pastukhov's particle-confinement time [6] into τ_p. The tendency of increasing neutral-shielding effect with increasing T_e or ϕ_b is seen by the data from $\phi_b \approx 0.8$, through 0.5, and then to 0.2 kV, since these data are traced by the dashed curves labeled 1 and 2 [corresponding to n_0=1, and 2×10^{15} m^{-3}, respectively], and then the calculations with poorer shielding with $n_0 \approx 5\times10^{15}$ m^{-3}, respectively. This behavior is consistently understood by the dependence of σv on T_e. Similarly, data in Fig. 2(b) are also fitted by the calculated solid curves from Eq. (1) with T_i=1 and 3 keV along with $n_c=1.5\times10^{18}$ m^{-3}. In contrast to the parameter regime in Fig. 2(a), $P_{hb}V_h$ dominates over $P_{wb}V_{BB}$. In Fig. 2(b), the dashed curve is similarly estimated from Eq. (2) by the use of the averaged data of $n_0=2\times10^{15}$ m^{-3} and the above-described parameters.

Good agreement between the data and the calculated results in each different parameter regime with the different dominant-heating source in Figs 2(a) and 2(b) provides *a novel finding of the validity of the Pastukhov theory on electron energy and particle confinement*. The scaling validation *confirms the importance and effectiveness of the formation of* ϕ_b *to confine and heat up tandem-mirror central electrons*. On the other hand, the similar effects of ϕ_c on ions have already been reported [1].

In this context, we investigate to find the general formula for the relation between T_e and ϕ_b on the basis of the combination of the energy-balance equation with Pastukhov's energy confinement time [6] in place of the two empirical scalings of T_e with ϕ_b employed in Fig. 1. In a quasi-steady state, one can obtain the following equation when equalizing τ_E from the Pastukhov theory [6] with τ_E in the energy-balance equation (1) under the assumption of the predominant loss in the axial direction compared to losses including the radial direction.

$$f(x)=\frac{2.01\cdot10^4 n_c^{\,2}\ln\Lambda}{T_e^{1/2}\left[\frac{V_h}{V_{BB}}P_{hb}+P_{wb}\right]},\ or\quad x=\frac{\phi_b}{T_e}=f^{-1}\left[\frac{2.01\cdot10^4 n_c^{\,2}\ln\Lambda}{T_e^{1/2}\left[\frac{V_h}{V_{BB}}P_{hb}+P_{wb}\right]}\right], \qquad (3)$$

where the units of T_e, ϕ_b, n_c, P_{hb} and P_{wb} are in keV, kV, 10^{18} m^{-3}, W·m^{-3}, and W·m^{-3}, respectively, and x is defined as ϕ_b/T_e.

From Eq. (3), one can reproduce the above-mentioned two empirical scalings of T_e with ϕ_b. A finding of a good approximation of $f^{-1}(x)\approx0.04+0.97\ln[f(x)]$ along with the substitution of standard formula for P_{hb} and P_{wb} allows us to derive the above-employed empirical formula of $\phi_b \approx 1.33+4.39(T_e-0.33)$ with the Taylor expansion in the above-described parameter regime of the high-potential mode [Fig. 1(a)], for instance.

5. Scaling Law of Potential Formation with ECH Powers

Summarized data on ϕ_c [kV] (or ϕ_b due to Fig. 1) as a function of *externally controllable* plug ECH powers, P_{ECH} [kW], and n_c [10^{18} m^{-3}] are plotted in Fig. 3 along with the solid curves from *the empirical scaling* of $\phi_c=1.73\times10^{-4}\,P_{ECH}^{\,1.73}\exp(-0.33n_c)$. According to the scaling, further favorable increase in confining potentials is anticipated with installing more powerful ECH sources beyond the power limit in Fig. 3. Experiments beyond 200-kW ECH injection into tandem-mirror plasmas have never been tried all through the tandem-mirror history.

An assumption of the validity of extrapolation of the experimentally verified scaling theories and relations allows us to make a scenario in search of innovative tandem fusion researches.

The combination of physically well interpreted scalings of (i) the strong ECH relation extended over both high-potential and hot-ion modes with the key parameters of ϕ_c, ϕ_b, T_e, and n_p/n_c [5] [*i.e.*, $\phi_c=c_1 f_1(T_e, \phi_b, n_p/n_c)$ using the functional dependence of f with a constant c (Sections 3 and 4)], (ii) the scaling of the effect of ϕ_b on T_e [*i.e.*, $T_e = c_2 f_2(\phi_b)$], having the consistency with the electron energy-balance equation (Section 4), (iii) the empirical ECH power scaling for the formation of ϕ_c (or ϕ_b with n_p/n_c via Fig. 1) [*i.e.*, ϕ_c or $\phi_b=c_3 f_3(P_{ECH}$, and n_c or n_p/n_c), on the basis of an electron feeding effect due to P_{ECH} from plug to barrier regions (Section 5)], and (iv) the Pastukhov relation with ϕ_c, ϕ_b, T_i, T_e and n_c [6]. A substitution of the scaling (ii) of T_e into f_1 in (i), and then the use of the scaling (iii) of ϕ_b in (i) provides *the scaling of* ϕ_c *with* P_{ECH} in Fig. 4 [*i.e.*, $\phi_c=c_4 f_4(P_{ECH}, n_p/n_c)$]. (v) Consequently, *under the assumption of the validity of the extension of the above-summarized scaling bases* having the theoretical interpretations, the formation of Pastukhov's predicted ϕ_c of 30 kV for confining Q=1 plasmas (see Refs. [1 and 6]) is scaled to require 5-MW P_{ECH} with n_p/n_c=0.3, T_i=16 keV, and $n_c\tau_E = 2.5\times10^{19}$ m^{-3}s with a direct-converter efficiency of 0.8, for instance. For challenging these future possibilities, step-to-step works for verifying the validity of an order-of magnitude extension should be made on the basis of these theoretically supported scalings and established knowledge in the individually realized parameter regime for the high-potential and hot-ion modes.

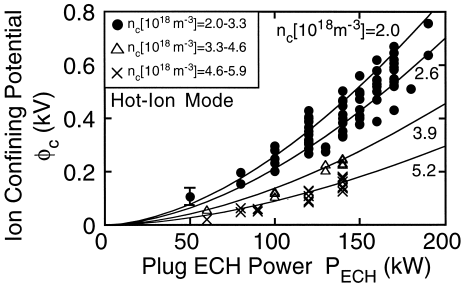

FIG. 3. The scaling of ϕ_c with P_{ECH} as a function of n_c (the hot-ion mode).

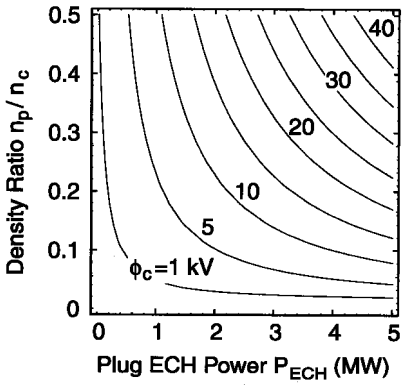

FIG. 4. The scaling of ϕ_c in relation to P_{ECH} and n_p/n_c.

6. Summary

The scalings in the main tandem-mirror operations from 1979 to 2000 on GAMMA 10, characterized in terms of the high-potential mode having kV-order plasma-confining potentials, and the hot-ion mode yielding fusion neutrons from bulk ions with T_i=10-20 keV, are consolidated and generalized. In particular, the novel proposal of the combination of Cohen's strong ECH and Pastukhov's potential-confinement theories provides the generalized physics interpretations covering over both modes upon the observed scalings; that is, the scalings of potential formation and the associated potential effects on improvements in plasma parameters (*i.e.*, T_e, T_i [1], n_c, τ_E due to the formation of ϕ_b and ϕ_c with P_{ECH}) for exploring the future upgraded modes under the extension of these theoretically supported scaling bases.

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