Nonlinear Dynamics of the Reversed-Field Pinch: Torques, Dynamo, and Reconnection

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Abstract: The magnetic field configuration of the Reversed-Field Pinch (RFP) typically exhibits resistive tearing modes of poloidal mode number m = 1 resonant in the plasma core and m = 0 resonant in the plasma edge. In the Madison Symmetric Torus (MST) RFP, these fluctuations cause electromagnetic torques which alter the flow profile, and magnetic reconnection and dynamo effects which alter the magnetic configuration and current density profile. Described in this paper are three key physics results: 1) The discovery of *internal* electromagnetic torques between two core modes, a three-wave interaction requiring the mediation of the m = 0, n = 1 mode at the plasma edge. 2) Direct measurements of the $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{B}} \rangle_{\parallel}$ dynamo at the plasma edge confirm that it balances Ohm's law and is primarily driven by the m = 0 mode. 3) Measurements across the reconnection layer at the q = 0 resonant surface demonstrate the dominance of m = 0 current density fluctuations in the vicinity of this resonant surface and show a phase flip of the radial plasma flow velocity fluctuations across the resonant surface.

1. Introduction

The magnetic field configuration of the Reversed-Field Pinch (RFP) typically exhibits resistive tearing modes of poloidal mode number m = 1 resonant in the plasma core and m = 0 resonant in the plasma edge. In the Madison Symmetric Torus (MST) RFP, these fluctuations cause electromagnetic torques which alter the flow profile, and magnetic reconnection and dynamo effects which alter the magnetic configuration and current density profile. Described in this paper are three key physics results: 1) We have confirmed the existence of *internal* electromagnetic torques between two core modes, a three-wave interaction requiring the mediation of the m = 0, n = 1 mode at the plasma edge. 2) Direct measurements of the $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{B}} \rangle_{\parallel}$ dynamo at the plasma edge confirm that it balances Ohm's law and is primarily driven by the m = 0 mode. We conclude that although the dynamo is a global effect, it arises from a superposition of relatively localized reconnection events. 3) Measurements across the reconnection layer at the q = 0 resonant surface demonstrate the dominance of m = 0 current density fluctuations in the vicinity of this resonant surface and show a phase flip of the radial plasma flow velocity fluctuations across the resonant surface. This is the flow pattern expected to be associated with reconnection, but the "current sheet" is wider than expected, similar to the calculated width of the island at the m = 0 surface.

2. Internal electromagnetic torques

The dynamics of locked modes and the RFP dynamo involve processes that are inherently nonlinear, requiring non-zero products of two or more fluctuating quantities. For example, the nonlinear internal torque (which redistributes the flow profile) is due to the interaction between a mode and the resonant current perturbation produced by a pair of other modes: $T_{\mathrm{em}}^{\mathrm{NL}} = \left| \mathbf{R} \times \tilde{\mathbf{j}}_{\mathbf{k}}^{\mathrm{NL}} \times \tilde{\mathbf{B}}_{\mathbf{k}} \right|$, where $T_{\mathrm{em}}^{\mathrm{NL}}$ is the nonlinear electromagnetic torque, $\mathbf{k}$ is the wavevector of the mode, $\tilde{\mathbf{B}}_{\mathbf{k}}$ is the mode amplitude, and $\tilde{\mathbf{j}}_{\mathbf{k}}^{\mathrm{NL}}$ is the nonlinearly produced current density perturbation at the mode's rational surface. In MST, we expect the largest torque on the (m = 1, n = 6) mode, involving adjacent m = 1 modes and the usually stationary (0,1) mode and satisfying a wavevector sum rule [1]:

$$T_{(1,6)}^{\mathrm{NL}} \approx C_{(1,7),(0,1)}^{(1,6)} \tilde{B}_{(1,6)} \tilde{B}_{(1,7)} \tilde{B}_{(0,1)} \sin\left(\delta_{(1,7)} - \delta_{(1,6)} - \delta_{(0,1)}\right) \\ - C_{(1,5),(0,1)}^{(1,6)} \tilde{B}_{(1,6)} \tilde{B}_{(1,5)} \tilde{B}_{(0,1)} \sin\left(\delta_{(1,7)} - \delta_{(1,5)} - \delta_{(0,1)}\right), \quad (1)$$

where $C_{(m',n'),(m'',n'')}^{(m,n)}$ are the nonlinear coupling coefficients, $\tilde{B}_{(m,n)}$ and $\delta_{(m,n)}$ are the amplitude and time dependent phase of the (m,n) mode respectively, and the appropriate surface average is taken. Since the most dramatic changes in mode rotation take place during a sawtooth crash in MST (Fig. 1), coincident with rapid growth of the mode amplitudes, we expect the nonlinear torque to be largest during the crash. Figure 2 contains a plot of the torque on the (1,6) mode, ensembled over a large number of sawtooth crashes. Note that the rapid deceleration of the (1,6) mode just before the sawtooth crash is coincident with a large nonlinear torque [2]. In a related experiment, the q = 0 surface was removed from the plasma by operating non-reversed discharges. Sawteeth still occurred, illustrated by the peaks in mode amplitude in Fig. 3b. But, by removing the m = 0 channel (Fig. 3c), nonlinear coupling was diminished and the rapid deceleration of the (1,6) mode at the crash was eliminated (compare Fig. 3a to Fig. 1c). An active experiment has also been done in which a stationary external magnetic perturbation resonant only with the (1,6) mode was applied; all of the (1,n) modes were locked by this perturbation, contrary to the prediction of linear theory that a perturbation will affect only modes resonant with it.

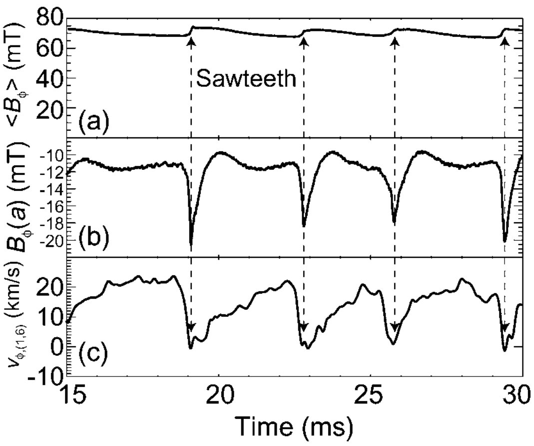

Fig. 1. Dynamics during sawtooth crashes in a typical MST discharge. a) Volume averaged toroidal field, b) toroidal field at plasma edge, and c) toroidal phase velocity of (1,6) mode.

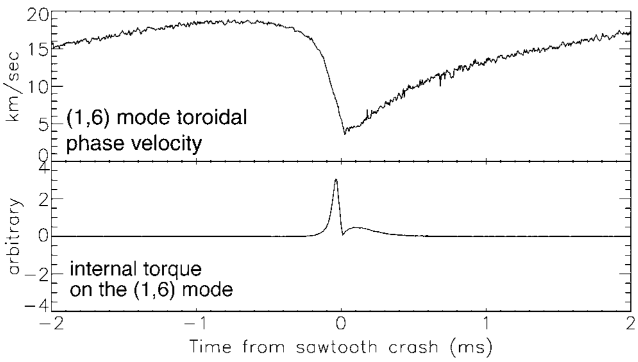

Fig. 2. Rapid deceleration of the (1,6) *mode at the sawtooth crash and the nonlinear torque likely responsible.*

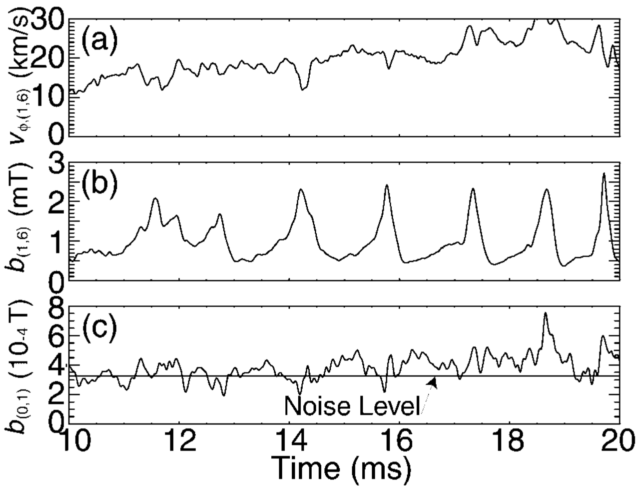

Fig. 3. An MST discharge with no m = 0 *resonance. a) (1,6) mode toroidal phase velocity, b) (1,6) mode amplitude, and c) (0,1) mode amplitude.*

3. RFP Dynamo

In addition to mediating internal torques in MST, the m = 0 mode also drives the MHD dynamo in the edge [3]. This dynamo is the correlated cross-product of two fluctuating quantities, the plasma flow velocity and magnetic field. We have made spatially resolved direct measurements of these quantities in the edge of MST, using an innovative spectroscopic probe to measure the impurity flow fluctuations [4]. As is the case for the nonlinear torque, dynamo activity is largest at the sawtooth crash. Individually, both $\tilde{v}$ and $\tilde{B}$ rise dramatically at the crash (Fig. 4), but more importantly the fluctuation-induced dynamo $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{B}} \rangle_{\parallel}$ rises and provides a time-dependent balance of Ohm's law, $E_{\parallel} + \langle \tilde{\mathbf{v}} \times \tilde{\mathbf{B}} \rangle_{\parallel} = \eta j_{\parallel}$, where η is the parallel resistivity and $E_{\parallel}$ and $j_{\parallel}$ are parallel components of the electric field and current (Fig. 5). The current density profile was obtained using an insertable Rogowski coil [5]. The plasma resistivity was not directly measured, since measurements of local effective ion charge state Z_{eff} were not available. However, at the low electron temperature (≤ 50 eV as measured by Langmuir probes) of the edge plasma in MST, the probability of stripping impurity ions to high charge states is small; hence, a Z_{eff} as high as 2 is unlikely. For the present comparison, a Z_{eff} of 1.5 has been assumed and η is calculated from Spitzer's formula. Since the contribution of the current term in Ohm's law is modest, the uncertainty in the estimated resistivity does not affect the conclusion that the $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{B}} \rangle_{\parallel}$ dynamo balances Ohm's law in the edge of MST.

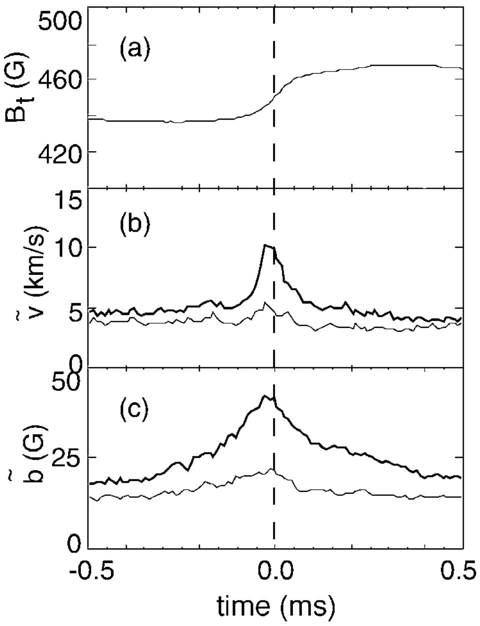

Fig. 4. The dynamo event cycle in MST. Shown are (a) the average toroidal magnetic field flux in the plasma, b) the amplitude of the toroidal (heavy line) and radial (light line) velocity fluctuations, and (c) the amplitude of the toroidal (heavy line) and radial (light line) magnetic field fluctuations.

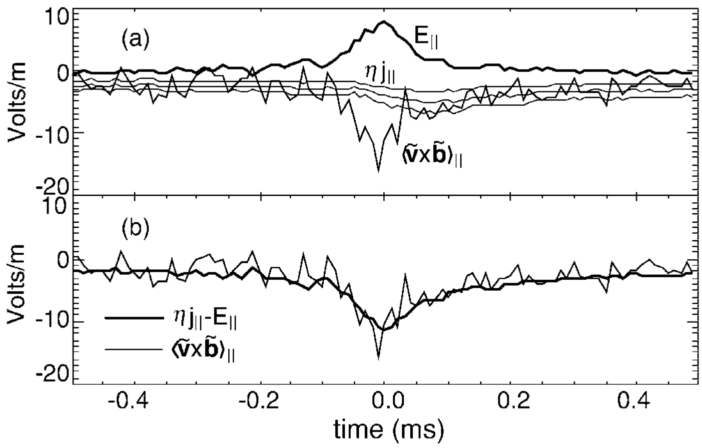

Fig. 5. Ohm's law balance at r/a *= 0.92 during a dynamo event cycle. a) The parallel electric field, fluctuation-induced dynamo, and* $\eta j_{\parallel}$ *with* Z_{eff} *= 1, 1.5, and 2. b) The fluctuation-induced dynamo balances Ohm's law with* Z_{eff} *= 1.5.*

However, at r/a < 0.85 (probe insertion deeper than the q = 0 reversal surface), the dynamo term virtually disappears. This is true despite the fact that the magnitudes of velocity and magnetic fluctuations individually remain large. Separation of the dynamo product into its two constituent terms $\tilde{v}_r\tilde{B}_t$ and $\tilde{v}_t\tilde{B}_r$ reveals that at large radii both terms contribute to the dynamo. Deeper than the reversal surface, however, the $\tilde{v}_r\tilde{B}_t$ component changes sign and becomes an anti-dynamo (opposite in direction to the current), competing

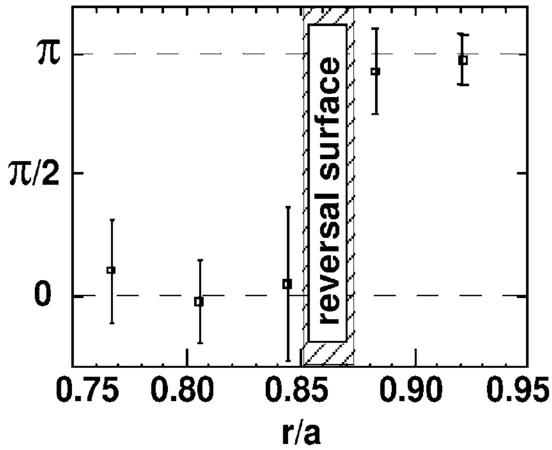

Fig. 6. The phase flip of the radial velocity fluctuations across the q *= 0 surface.*

with $\tilde{v}_t\tilde{B}_r$ and effectively canceling the net dynamo effect. Correlations of each fluctuating quantity with a reference signal from magnetic coils at the edge of MST show that $\tilde{v}_t$, $\tilde{B}_r$, and $\tilde{B}_t$ maintain their phase with radius, while $\tilde{v}_r$ flips phase by π radians at the reversal surface, producing the change in sign of the $\tilde{v}_r\tilde{B}_t$ contribution (Fig. 6). Linear MHD theory predicts this phase change in $\tilde{v}_r$ across the q = 0 resonant surface as a hallmark of a magnetic reconnection layer.

Spectral analysis confirms that the correlated velocity fluctuations are predominantly n = 1; given the q-profile of MST, these are likely resonant m = 0 modes. These results are consistent with nonlinear MHD computation, which predicts that the m = 0 component of the fluctuation-induced dynamo should be dominant at the extreme edge of MST. Since earlier measurements showed the core dynamo to be m = 1 and localized to those resonant surfaces [6], we conclude that although the dynamo is a global effect, it arises from a superposition of relatively localized reconnection events.

4. Reconnection

Magnetic reconnection – the breaking and reconfiguration of magnetic field lines in a plasma – occurs in numerous natural settings and in many magnetically confined plasmas in the laboratory. Magnetic reconnection is expected to be associated with a large current (a "current sheet" that is a singularity in ideal magnetohydrodynamics) at the location of the reconnection. Using insertible Rogowski probes, we have measured the characterisitics of the current sheet associated with spontaneous reconnection around the q = 0 surface in MST [7]. The current density fluctuations near the q = 0 surface are dominantly m = 0 (Fig. 7). However, the current layer is not confined to a resistive tearing layer, as calculated from linear MHD theory. Inclusion of additional effects in Ohm's law predicts different scale lengths for the layer width. The electron inertial effect in Ohm's law yields a scale size of the electron skin depth (c/ω_{pe}), and the electron pressure gradient in combination with ion inertia leads to a scale size of the ion acoustic gyroradius

$$\rho_s = \frac{\left(\left(T_e + T_i\right)m_i\right)^{\frac{1}{2}}}{eB_0} \qquad (2)$$

The radial extent of the measured current layer is larger than both c/ω_{pe} and ρ_s. Moreover, we do not detect current structures at these scales. Rather, the layer width is of the order of the calculated width of the magnetic island

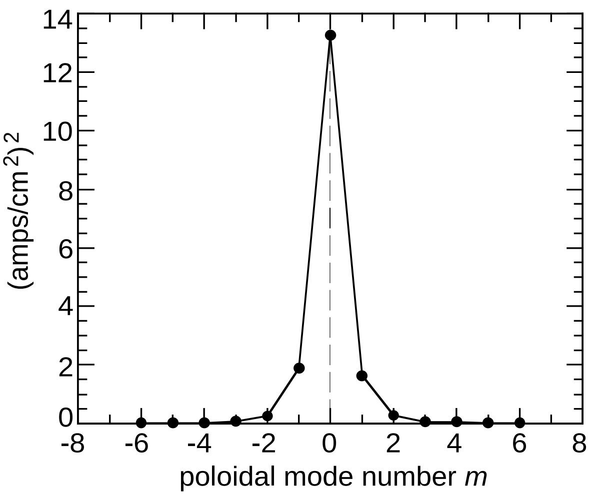

Fig. 7. Current density fluctuations are m *= 0 at* q *= 0.*

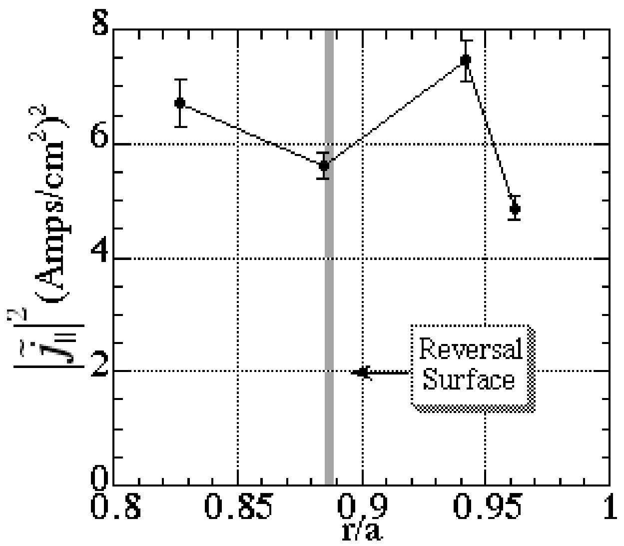

*Fig. 8. Edge resonant (*m = *0) parallel current density fluctuation power during sawtooth crashes, illustrating the breadth of the reconnection layer.*

associated with the reconnection (Fig. 8). Hence, parallel motion of particles along the magnetic field (corresponding to radial transport of parallel current) may account for the measured layer width. The measured radial charge transport from the magnetic fluctuations associated with reconnection is generally small. This is in agreement, in part, with theoretical expectation for the fluctuations of the tearing modes, which are the cause of reconnection.

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