SELF-CONSISTENT QUASILINEAR FOKKER-PLANCK - MAXWELL MODELLING OF ION CYCLOTRON RESONANCE HEATING IN TOKAMAK PLASMAS

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Abstract

Models of the radiofrequency (rf) response and of the rf quasilinear diffusion operator of tokamak plasmas have been derived with identical approximations; they are under implementation in a full-wave code and a bounce-averaged quasilinear Fokker-Planck (QLFP) code, respectively. Combined iterative use of the codes will allow self-consistent simulations of ICRH in general tokamak geometry, with due account taken of the mutual influence between the distribution function of the heated species, which is strongly nonmaxwellian under intense ICRH, and the rf wave pattern. This first application focuses on the fundamental cyclotron interaction (i.e. typically on minority heating scenarios), transport across magnetic surfaces and finite Larmor radius effects currently being neglected. Theory and its numerical implementation are based on weak Galerkin formulations of Maxwell's equations and of the QLFP equation. Two particularly attractive benefits result from this approach: i) the power depositions associated with each equation are automatically consistent with each other; ii) elementary building blocks common to the rf response and the QL operator are evaluated only once, yielding drastic computer time savings.

1. INTRODUCTION

Quasilinear theory relies on an analysis of rf heating on two well-distinct timescales: the rf wave period itself, and the much larger characteristic time of the evolution associated with net heating and collisional relaxation. Owing to the complexity of the problem in toroidal geometry, these two sub-problems have been addressed independently in the past:

i) Solution of the wave propagation and absorption problem, i.e. of Maxwell's equations in the plasma described by a suitable equivalent dielectric, and subject to appropriate boundary conditions. A realistic description of the interactions between particles and high frequency waves requires a kinetic treatment, provided by the linearized Vlasov equation. The full-wave problem is advantageously based on a weak Galerkin [1] formulation of the Maxwell-Vlasov system, namely (e.g. [2])

$$\frac{i}{2}\int_V \left[\frac{1}{\omega\mu_0}\nabla\times\mathbf{F}^*.\nabla\times\mathbf{E}-\omega\varepsilon_0\,\mathbf{F}^*.\mathbf{E}\right]\mathrm{d}r^3+\sum_\beta W_{\mathbf{FE}\beta}=-\frac{1}{2}\int_V \mathbf{F}^*.\mathbf{j}_s\,\mathrm{d}r^3 \tag{1}$$

where V is the tokamak chamber, $\mathbf{E}$ the rf electric field, and $\mathbf{j}_S$ the rf antenna current density. This equation is required to hold for arbitrary but sufficiently well-behaved vector fields $\mathbf{F}$, the case $\mathbf{F} = \mathbf{E}$ giving Poynting's theorem. The global rf dielectric response of particle species β is defined as [2]

$$W_{\mathbf{FE}\beta}=\frac{1}{2}\int \mathbf{F}^*.\mathbf{j}_\beta\,\mathrm{d}r^3=\frac{q_\beta}{2}\int \mathrm{d}r^3\int \mathrm{d}v^3\, f_\beta\,\mathbf{F}^*.\mathbf{v} \tag{2}$$

where $\mathbf{j}_\beta$ is the rf current density and f_β the perturbed distribution function of the species; both quantities are linear integro-differential expressions of the rf electric field $\mathbf{E}$ and the distribution function $F_{0\beta}$. Solving the linear wave problem (1) requires one to know $F_{0\beta}$ - a constant on the rf timescale - *a priori*, in order to evaluate the dielectric response (2). Until the present paper, the numerical modelling of wave propagation in tokamak geometry has been restricted to Maxwellian equilibria; we now allow $F_{0\beta}$ to be an arbitrary function of the constants of the unperturbed particle motion: $F_{0\beta}(\rho,v,x)$ where ρ is a radial coordinate, v the velocity, $x = 2\mu B_a/(mv^2)$ = (magnetic moment / kinetic energy × reference induction[1]). Radial drift motion of the guiding centres and finite Larmor

[1]For instance, the choice $B_a = B_m(\rho)$ (minimum value of the induction on the current magnetic surface) made in

radius effects are included in the theory (see [2], where the toroidal invariant P replaces the radial coordinate in the presence of drifts), but are not addressed in the present contribution. Expansion of **E** and **F** in toroidal and poloidal Fourier modes allows an accurate description of wave dispersion along the equilibrium magnetic field in the presence of rotational transform. Axisymmetry yields independent toroidal modes ($\exp[in\phi]$), but the poloidal modes of **E** ($\exp[im_1\theta]$) and **F** ($\exp[im_2\theta]$) are strongly coupled. Truncation of these Fourier series and a finite element representation of the radial variations complete the discretization, and yield a linear algebraic system. Its solution gives the rf field pattern, and thence the power deposition to the various plasma species.

ii) Evolution of the equilibrium under the influence of heating and Coulomb relaxation: on this timescale, $F_{0\beta} = F_{0\beta}(\rho,v,x;t)$ is now a function of the time and satisfies the bounce-averaged quasilinear Fokker-Planck (QLFP) equation (see e.g. [3]). In view of numerical applications, it is also interesting to write this equation in Galerkin form [3, 4], obtained by multiplication of the QLFP by a sufficiently smooth test function G and integration over the constants of the motion (com) ρ, v, x defined above: this yields

$$\frac{\partial}{\partial t}\int_{com} G\, F_{0\beta} = \tilde{\mathcal{Q}}(G, F_{0\beta}) + \int_{com} G\,\{(\mathcal{C} - \mathcal{L})F_{0\beta} + S\} \qquad (3)$$

in which $\mathcal{C}$ is the Coulomb collision operator, $\mathcal{L}$ accounts for the particle losses and S for the sources (see [3-4] for explicit expressions of these coefficients); in the present paper we focus on the contribution $\tilde{\mathcal{Q}}$ of the rf wave diffusion operator $\mathcal{D}$

$$\tilde{\mathcal{Q}}(G, F_{0\beta}) = \int_{com} G\,\mathcal{D}\,F_{0\beta} = -\int_{com}\sum_{p}\mathcal{B}_p\, h_p(G)\, h_p(F_{0\beta}) \qquad (4)$$

where p is the cyclotron harmonic index and $\mathcal{B}_p$ (which depends quadratically on the rf field) accounts for the resonant wave-particle interactions. A compact notation has been introduced for the triple integral over the constants of the motion:

$$\int_{com} \equiv \sum_{\sigma=\pm1}\int d\rho \int dv \int dx\, J' \quad , \qquad J' = (a\, v^3\, d\Psi / d\rho) / (4\pi\, \omega_b\, B_a) \qquad (5)$$

where σ is the sign of $v_{//}$ and J' is the Jacobian of the transformation from the particle variables (**r**,**v**) to the constants of the motion and angle variables (ρ,v,x,φ,ϕ,$\omega_b t$). Ψ is the poloidal flux function, ω_b the bounce frequency, *a* = 1 for a passing particle and 1/2 for a trapped particle. φ is the gyrophase, ϕ the toroidal angle of the guiding centre (gc), and t is the time parametrizing the gc motion. Last but not least, the operators h_p are given by

$$h_p(\quad) = \{-\frac{1}{v}\frac{\partial}{\partial v} + \frac{2}{v^2}\left(x - p\,\frac{\omega_{ca}}{\omega}\right)\frac{\partial}{\partial x}\} \qquad (6)$$

Consistently with the rf response (2) used in the wave equation, radial transport is not included in the QLFP yet (this is planned for future work). Given the rf field pattern, equation (3) is solved on a set of magnetic surfaces. This yields $F_{0\beta}$, thence the detailed power deposition (per guiding centre orbit) and the driven equilibrium current.

2. METHOD OF SOLUTION

The purpose of the present work is to treat the two equations (1) and (3) as genuinely coupled, for the first time in practical applications. The rf response (2) and the QL term (5) have been derived with the same simplifying assumptions, with the very important consequence that the two associated power absorptions are automatically identical [3, 6]: the total deposition of rf power to species β is given by either of the two expressions respectively obtained from equations (2) and (3):

$$P_\beta = \mathrm{Re}\, W_{\mathbf{EE}\beta} \equiv \tilde{\mathcal{Q}}(m_\beta v^2 / 2, F_0) \qquad (7),$$

Moreover, our formulation puts forward elementary building blocks common to the global rf response and the QL operator; these coefficients are independent of the rf field and the distribution function. Their sharing between the two codes allows important savings of computer time. The level

reference [4] yields an invariant x between 0 and 1.

of sophistication of the coupled model is determined by the amount of physics included in these terms, see further. RF heating and current drive are modelled self-consistently in tokamak geometry, by iterative use of the CYRANO full-wave code [7] and a QLFP code [3, 4], see Fig. 1. (Another approach has been taken by Hellsten *et al.* [8], who solve the QLFP with a Monte Carlo method.)

Wave code:	***QLFP code:***
Evaluate rf response (2)	Particle distribution $F_{0\beta}$
Solve Maxwell's eqs. (1)	One time step of Fokker-Planck eq. (3)
rf field **E**	Evaluate rf diffusion term (4)

FIG. 1. Schematic Maxwell / Fokker-Planck iteration.

3. FIRST APPLICATION OF THE THEORY

We wish to demonstrate the feasibility of our method on a simplified situation where the fundamental cyclotron interaction is the dominant process. This approximation is suitable for ICRH minority heating scenarios provided (i) the minority species is sufficiently dilute to allow linearization of the collision term in the QLFP; (ii) the electron density is sufficiently high to avoid a too energetic minority tail which would require finite Larmor radius corrections. In this context, we assume a single nonmaxwellian species and consider a single toroidal Fourier mode n, e.g. the dominant mode of a phased rf antenna array with high directivity. We account for the Landau interaction (i.e. the p=0 terms) as discussed in [2], but neglect transit time magnetic pumping.

3.1. Plasma contribution to the wave equation

The global rf response, equation (2), reduces to[2]

$$W_{\mathbf{FE}\beta} = \int d\rho \sum_{\substack{m_1, m_2 \\ = -\infty}}^{+\infty} \sum_{p=-1}^{+1} F^*_{L,m_2} \mathcal{M}^p_{21} E_{L,m_1} \qquad (L = +, -, // \text{ for resp. } p = 1, -1, 0) \qquad (8)$$

The matrix elements between poloidal modes m_2 and m_1 of test function and rf field components are

$$\mathcal{M}^p_{21}(\rho) = \frac{d\Psi}{d\rho} \sum_{\sigma=\pm 1} \int dv \;\; \Xi \int dx \;\; [v^2 h_p(F_0)\, D^p_{21} + F_0\, K^p_{21} - b(F_0)\, J^p_{21}] \qquad (9),$$

$$\Xi = (a\pi q^2_\beta v^3) / (2 m_\beta B_a), \qquad\qquad b(F_0) = 3F_0 + 2x\, \partial F_0 / \partial x \qquad (10)$$

In the present study, F_0 is supplied by the QLFP to the wave code on a finite element mesh, and expression (9) is integrated numerically over x and v. (Note that in a Maxwellian plasma, these matrix elements are poloidal Fourier transforms of the Fried-Conte dispersion function.) The matrix elements depend on two kinds of elementary contributions:

i) Resonant terms[3]:

$$D^p_{21} \equiv i \int_0^{\tau_b} \frac{e^{i(m_1 - m_2)\theta(t)}}{\dot\gamma(t)} \; I_p(t)\, dt \equiv (Res + i\, Pv)\, D^p_{21} \qquad (11),$$

where the time integral is over one poloidal revolution of the guiding centre orbit;

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[2]The notation for field polarized components in the present paper is as in [2].

[3]N.B.: Correspondence with [5, equation (8)]: $I^p_{21} = v^2\; \Xi\; D^p_{21}$. As a general remark, the evolution of our notation results from a progressive merging of two previously independent codes.

$$\dot{\gamma} = \omega - p\,\omega_c - n\,\dot{\phi} - \overline{m}\,\dot{\theta}, \qquad \overline{m} = (m_1 + m_2)/2 \tag{12}$$

$$l_{\pm 1} = (v \quad / v)^2 / 2, \qquad l_0 = (v_{//} / v)^2 \tag{13}$$

The D^p_{21} consist of a residue term and a principal value contribution, respectively associated with collisionless wave damping and the propagation characteristics of the plasma;

$$\mathit{Res}\ D^p_{21} = \pi \sum_{\text{res.}} \frac{e^{i(m_1 - m_2)\theta(t)}}{|\dot{\gamma}(t)|}\, l_p(t) \Bigg|_{t=t_{res}}, \qquad \mathit{Pv}\ D^p_{21} = ⨍_0^{\tau_b} \frac{e^{i(m_1 - m_2)\theta(t)}}{\dot{\gamma}(t)}\, l_p(t)\, dt \tag{14}$$

The summation is over the eventual resonances $\dot{\gamma}(t_{res}) = 0$ occurring along the guiding centre orbit, and $⨍$ indicates a Cauchy principal value.

ii) Nonresonant, purely reactive terms[4]:

$$K^p_{21} \equiv \frac{i}{\omega} \int_0^{\tau_b} e^{i(m_1 - m_2)\theta(t)}\, dt\ , \qquad J^p_{21} \equiv \frac{i}{\omega} \int_0^{\tau_b} e^{i(m_1 - m_2)\theta(t)}\, l_p(t)\, dt \tag{15}$$

The residue term in (14) is evaluated analytically; the principal value and the integrals of equation (15) are computed semi-analytically in general tokamak geometry.

3.2. Plasma contribution to the Fokker-Planck equation

The quasilinear term of the Fokker-Planck equation is given by equation (4) with the cyclotron harmonic series truncated to -1 ≤ p ≤ 1. The coefficients therein are given by

$$\mathbf{B}_p = \frac{1}{2\tau_b} (q_\beta v / m_\beta)^2 \sum_{m_1, m_2} \mathrm{Re}\left\{ E^*_{L, m_2}\ E_{L, m_1}\ \mathit{Res}\, D^p_{21} \right\} \tag{16}$$

As mentioned in Section 2, the resonant term $\mathit{Res}\, D^p_{21}$ is an elementary building block common to both equations. The $\mathbf{B}_p$ are directly supplied by the wave solver to the QLFP. The power deposition identity, equation (7), is readily checked on equations (8) and (16).

4. CONCLUSIONS

We have presented a self-consistent approach to the problems of wave propagation and quasilinear diffusion in tokamak geometry, and given expressions suitable for the description of fundamental cyclotron interactions. These results are under implementation in two coupled numerical codes, organized according to the scheme of Fig. 1. Their initial exploitation will address the simulation of selected minority heating scenarios.

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