Flow shear stabilization of hybrid electron-ion drift mode in tokamaks

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ABSTRACT

In this paper, a model of sheared flow stabilization on hybrid electron-ion drift mode is proposed. At first, in the presence of dissipative trapped electrons, there exists an intrinsic oscillation mode in tokamak plasmas, namely hybrid dissipative trapped electron-ion temperature gradient mode (hereafter, called as hybrid electron-ion drift mode). This conclusion is in agreement with the observations in the simulated tokamak experiment on the CLM [1,2]. Then, it is found that the coupling between the sheared flows and dissipative trapped electrons is proposed as the stabilization mechanism of both toroidal sheared flow and poloidal sheared flow on the hybrid electron-ion drift mode, that is, similar to the stabilizing effect of poloidal sheared flow on edge plasmas in tokamaks, in the presence of both dissipative trapped electrons and toroidal sheared flow, large toroidal sheared flow is always a strong stabilizing effect on the hybrid electron-ion drift mode in internal transport barrier location, too. This result is consistent with the experimental observations in JT-60U[3,4].

1. INTRODUCTION

In the last decade, sheared flow stabilization model in fusion research has successfully explained the formation of transport barriers in magnetically confined devices. This model was originally developed to explain the formation of the edge transport barrier in tokamaks for L-H transition [5-7]. Recently, this concept has been applied to study the internal transport barrier (ITB) formed at the plasma core with large toroidal sheared flow and converse (or weak) magnetic shear [3,8]. The edge poloidal sheared flow is always a strong stabilizing effects on the instabilities in tokamaks. As pointed out by Burrell [9], $\mathbf{E}\times\mathbf{B}$ flow shear stabilization model is of considerable physical significance: it is not often that a system self-organizes to a higher energy state with reduced turbulence and transport when an additional source of free energy is applied to it. Oppositely, we notice that, as show in Ref. 10, if toroidal flow shear is large enough, then the pure Kelvin-Helmhotz instability is excited. It is necessary to study the $\mathbf{E}\times\mathbf{B}$ flow shear effects further. In recent experiment on CLM [1], the transition from the slab to the toroidal branch of the ITG mode was studied by increasing the curvature drive provided with the mirror ratio and trapped electron fraction, it means that there may be a hybrid dissipative trapped electron ITG mode. Koide et al. [3] observed the spontaneous formation of internal and edge transport barriers in JT-60U high-β_P discharges. A large toroidal velocity shear or jump was observed across the $q=3$ surface (the internal transport barrier location) as if momentum transfer across the internal transport barrier was significantly reduced.

2. HYBRID ELECTRON-ION DRIFT MODE

Motivated by the experiment in the CLM, we adopt a simple radial inhomogeneous sheared slab configuration. The magnetic field is given by $\mathbf{B}=B_0\left[\hat{z}+(x/L_s)\hat{y}\right]$, where $L_s=\left(B_y^{'}(x)/B_0\right)^{-1}$ is magnetic shear length. Quasi-neutrality with non-adiabatic electron response, namely, $\tilde{n}_e=(1-i\delta_e)\tilde{\phi}$, where $\delta_e(k,\omega)\cong\left[(\omega_{*e}-\omega)/\nu_{eff}\right](2\varepsilon)^{1/2}$ [11] and electrostatic dynamics are assumed. Obviously, the non-adiabatic electron response will increase with the increasing of the trapped electron population fraction $f_{tr}\equiv\sqrt{2\varepsilon}$ and with the decreasing of the effective collision frequency $\nu_{eff}=\nu_{ei}/\varepsilon$. A fluid description of ion temperature gradient-driven turbulence consists of continuity equation for the ion density $\tilde{n}_i$, motion equation of parallel ion velocity $\tilde{V}_{\parallel i}$ and evolution equation for ion pressure P_i:

$$\frac{\partial\tilde{n}_i}{\partial t}+\nabla\cdot\left[n_i(\mathbf{V}_E+\mathbf{V}_{di})\right]+n_i\nabla\cdot\mathbf{V}_{pi}+\nabla_\parallel\left(n_i\tilde{V}_{\parallel i}\right)=0, \qquad (1)$$

$$m_i n_i\left(\frac{\partial\tilde{V}_{\parallel i}}{\partial t}+\mathbf{V}_E\cdot\nabla\tilde{V}_{\parallel i}\right)=-en_i\nabla_\parallel\Phi-\nabla_\parallel\tilde{P}_i, \qquad (2)$$

$$\frac{\partial P_i}{\partial t} + \mathbf{V}_E \cdot \nabla P_i + \Gamma P_i \nabla_{\|} \tilde{V}_{\| i} = 0, \tag{3}$$

where $\mathbf{V}_E = (c/B)\hat{b} \times \nabla\Phi$ and $\mathbf{V}_{di} = (c/eBn_i)\hat{b} \times \nabla P_i$ are, respectively, electric field drift velocity ($\mathbf{E}\times\mathbf{B}$ drift) and ion diamagnetic drift velocity, which describe the perpendicular ion dynamics to the first order in $O(\omega/\omega_{ci})$, ω_{ci} is the ion cyclotron frequency. In the next order, the generalized ion polarization drift is given by $\mathbf{V}_{pi} = -\frac{c^2 m_i}{eB^2}\left(\frac{\partial}{\partial t} + \frac{c}{B}\hat{b}\times\nabla\Phi\cdot\nabla + \frac{c}{eBn_i}\hat{b}\times\nabla P_i\cdot\nabla\right)\nabla_{\perp}\Phi$. Finally, we obtain a eigenmode equation for ITG mode including non-adiabatic electron response

$$\frac{d^2}{dx^2}\tilde{\phi} + \left[-b_s + \frac{1-\Omega(1-i\delta_e)}{\Omega + (1+\eta_i)/\tau} + \frac{x^2}{x_s^2}\right]\tilde{\phi} = 0. \tag{4}$$

This is a Weber equation, where $b_s = k_y^2$, $x_s = \Omega L_s/L_n$, Γ-term is negligible when L_n/L_s <<1. The solution of Eq.(16) can be obtained in terms of Hermite polynomials

$$\tilde{\phi} = \exp\left(i x^2/2x_s\right) H_n\left(\sqrt{i x^2/x_s}\right), \tag{5}$$

where $n = 0,1,2,\cdots$ is the radial eigenmode number. H_n is Hermite polynomials, Δ is the radial mode width and is determined by $\Delta^{-2} = -\mathrm{Im}[(2x_s)^{-1}]$. The eigenmode frequency Ω is given by the solution of the linear dispersion relation determined by the WKB eigenvalue condition for the Weber equation

$$\Omega^2(1+b_s) + \Omega\left[b_s\frac{1+\eta_i}{\tau} - 1 + i(2n+1)\frac{L_n}{L_s}\right] + i(2n+1)\frac{1+\eta_i}{\tau}\frac{L_n}{L_s} = i\frac{\omega_{*e}}{\nu_{eff}} f_{tr}(1-\Omega)\Omega^2 . \tag{6}$$

This is a cubic equation for normalized frequency Ω, the left-hand side is the standard dispersion relation for the ITG mode, and the right-hand side represents the modification due to dissipative trapped electrons. it means that in the absence of the nonadiabatic electron response, that is, $\delta_e = 0$, the cubic Eq. (6) reduces to usual dispersion relation for the ion temperature gradient mode

$$\Omega^2(1+b_s) + \Omega\left[b_s\frac{1+\eta_i}{\tau} - 1 + i(2n+1)\frac{L_n}{L_s}\right] + i(2n+1)\frac{1+\eta_i}{\tau}\frac{L_n}{L_s} = 0 . \tag{7}$$

Eq.(7) describes a pair of modes, only one is unstable, that is, ITG mode. Besides the trapped electron-modified ITG mode, Eq.(6) includes another unstable root yet, that is, a hybrid dissipative trapped electron-ion temperature gradient. In the case L_n/L_s <<1, Eq.(6) gives:

$$\gamma \cong (2n+1)\frac{1+\eta_i}{\tau}\frac{L_n}{L_s}\frac{1}{1-b_s K}\left[1 + (2n+1)\frac{1+\eta_i}{\tau}\frac{L_n}{L_s}\frac{\delta_e}{(1-b_s K)^2}\right]. \tag{8}$$

This implied that there exists an intrinsic oscillation mode in tokamak plasmas, that is, a hybrid dissipative trapped electron ion temperature gradient (ITG) mode. When trapped electron fraction is sufficient high and the trapped electrons are dissipated strongly, the mode is determined by the dissipative trapped electron dynamics and propagated in electron diamagnetic direction, namely, the mode appears to be a hybrid dissipative trapped electron ion temperature gradient (ITG) mode. Analytical result can reduce to usual predictions of the ion temperature gradient-driven instability in the absence of the dissipative trapped electron response. Numerical calculation indicates that in the absence of the dissipative trapped, the hybrid dissipative trapped electron ITG mode reduces to the usual ITG mode, and that when the electron nonadiabatic electron response is sufficient strong, the hybrid electron-ion drift mode propagates in electron diamagnetic direction, see Fig. 1. Analytical and numerical results are agreement with the experimental observations in CLM [1,2].

3. SHEARED FLOWS STABILIZATION ON HYBRID ELECTRON-ION DRIFT MODE

On the basis of the hybrid electron-ion drift mode above, we further consider the effects of $\mathbf{E}\times\mathbf{B}$ sheared flows on this hybrid electron-ion drift mode. The equilibrium flow velocity has form

$$\mathbf{V}_0(x) = \mathbf{V}_0(0) + (x/L_y)V_{0y}\hat{y} + (z/L_{0z})V_{0z}\hat{z}, \tag{9}$$

where $L_y = (d\ln V_{0y}/dx)^{-1}$ and $L_z = (d\ln V_{0z}/dx)^{-1}$ the scale length of poloidal velocity and toroidal velocity, respectively. The definition of the convective derivative in Eqs. (1-3) contains the poloidal and toroidal equilibrium flow velocities, that is,

$$\omega - \mathbf{V}_0(x)\cdot\mathbf{k} = \omega - \mathbf{V}_0(0)\cdot\mathbf{k} - (x/L_y)V_{0y}k_y - (x/L_z)V_{0z}k_z \cong \omega - (x/L_y)V_{0y}k_y \,, \quad (10)$$

According to the drift ordering $k_z = (x/L_z)k_y << k_y$, we ignored all terms of order higher than the term $O(x^2/L^2)$. Including both dissipative trapped electrons and sheared flows, Eqs. (1-3) become

$$[1-\hat{\Omega}(1-i\delta_e)]\tilde{\phi} + (\hat{\Omega}+K)\nabla_\perp^2\tilde{\phi} + sK\tilde{\phi} = 0\,, \quad (11)$$

$$\hat{\Omega}\tilde{V}_\parallel - sK\tilde{p} = sK\tilde{\phi} - J_z\tilde{\phi}\,, \quad (12)$$

$$(\Gamma/\tau)ax\tilde{V}_\parallel - \tilde{\Omega}\tilde{p} = -K\tilde{\phi}\,, \quad (13)$$

where $\hat{\Omega} = \Omega - J_z x$, $J_y = (V_{0y}/c_s)(L_n/L_y)$ and $J_z = (V_{0z}/c_s)(L_n/L_z)$ are, respectively, poloidal and toroidal sheared flow parameters, which describe the synergistic effect of flow shear and flow magnitude. Finally, we obtain a single eigenmode equation for $\tilde{\phi}$

$$\frac{d^2\tilde{\phi}}{d^2x} + \left[-b_s + \frac{1-\hat{\Omega}(1-i\delta_e)}{\hat{\Omega}+K} - \frac{J_z s x\hat{\Omega}}{(\hat{\Omega}+K)(\hat{\Omega}^2-\Gamma s^2x^2/\tau)} + \frac{s^2x^2}{\hat{\Omega}^2-\Gamma s^2x^2/\tau}\right]\tilde{\phi} = 0\,. \quad (14)$$

Similar to the treatment above, we can obtain the growth rate. First, the dissipative trapped electrons play a key role for stabilizing effect of the toroidal sheared flow on the hybrid electron-ion drift mode. If the non-adiabatic electrons are absent in the system, the solution of Eq.(14) reduced to the pure toroidal sheared flow-enhanced ITG mode which shows that pure toroidal sheared flow is a destabilizing effect on ITG mode, that is, the pure toroidal sheared flow, acting as a dominant free energy source, drives a sheared flow-enhanced purely Kelvin-Helmhotz instability, as show in Ref. 10:

$$\gamma = \frac{sK}{1-b_sK-J_z^2/4K}\,. \quad (15)$$

In the absence of sheared flows, that is, $J_y = 0$, $J_z = 0$, the results reduced to the hybrid dissipative trapped electron-ion temperature gradient mode, as show above:

$$\gamma \cong \frac{sK}{1-b_sK}\left[1-\frac{sK}{(1-b_sK)^2}|\delta_e|\right]. \quad (16)$$

In the presence of both dissipative trapped electrons and toroidal sheared flow, we obtain linear growth rate for hybrid electron-ion drift mode approximately

$$\gamma \cong \frac{sK}{1-b_sK-J_z^2/4K}\left[1-\frac{sK}{(1-b_sK-J_z^2/4K)^2}|\delta_e|\right]. \quad (17)$$

We know it is not often that a system self-organizes to a higher energy state with reduced turbulence and transport when an additional source of free energy is applied to it. In the presence of both dissipative trapped electrons and toroidal sheared flow, it is obvious that strong coupling between the dissipative trapped electrons and the toroidal sheared flow [the second term of Eq.(17)] results in strong stabilizing effect of the toroidal sheared flow on the hybrid electron-ion drift type mode. That is, for small toroidal velocity shear (or weak velocity shear), the linear growth rate increases with the sheared flow parameter J_z; but for sufficiently large value of parameter J_z, the linear growth rate will decreases with the increase of parameter J_z. In particular, within the ITB region, the toroidal rotation velocity, especially its shear is so large that the toroidal sheared flow parameter J_z is large enough to suppress the hybrid electron-ion drift mode. In other words, the coupling between the sheared flows and non-adiabatic electrons is proposed as the stabilization mechanism of toroidal sheared flow on the hybrid electron-ion drift type mode. Numerical calculation shows same result, see Fig. 2. It is consistent with the experimental observation in the ITB region where further confinement improvement is always associated with large toroidal sheared flow [3,4]. Toroidal sheared flow stabilization appears to offer favorable prospects for high confinement operation for fusion reactor.

Similarly, in the presence of both dissipative trapped electrons and poloidal sheared flow, we have

$$\gamma \cong \frac{sK}{1-b_sK-J_y^2/4K}\left[1-\frac{sK}{(1-b_sK-J_y^2/4K)^2}|\delta_e|\right]. \quad (18)$$

It also showed that for small poloidal velocity shear parameter J_y (or weak shear), linear growth rate increases with shear parameter J_y; but for sufficiently large value of shear parameter J_y, the linear growth rate will decreases with increasing shear parameter J_y. We have expanded the potential function with a polynomial in x. The terms which vary as x^3 or higher tend to destroy the quadratic structure at large value of x, Physically, these terms represent the effects of higher shear damping when the velocity shear causes the mode to deviate too far from the mode rational surface.

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REFERENCES

[1] J. Chen and A. K. Sen, Phys. Rev. Lett. **72**, 3997 (1994).
[2] L. Bai, X. M. Qiu, L. Huang, and X. M. Song, Phys. Plasmas **3**, 3004 (1996).
[3] Y. Koide *et al*., Phys. Rev. Lett. **72**, 3662 (1994).
[4] L.Bai, A.Fukuyama and M.Uchida, Phys Plasmas **5**, 989 (1998); Plasma Phys. Control. Fusion **40**, 785 (1998).
[5] F. Wagner *et al*., Phys. Rev. Lett. **49**, 1408 (1982).
[6] S.-I.Itoh and K. Itoh, Phys. Rev. Lett. **60**, 2276 (1988).
[7] X. M. Qiu, L. Bai, and L. B. Ran, Chin. Phys. Lett. **9**, 527 (1992); *Plasma Physics and Controlled Nuclear Fusion Research*, 1992 (IAEA, Vienna, 1993) Vol.**2**, p.529.
[8] B. W. Rice *et al*., Phys. Rev. Lett. **79**, 2697 (1997).
[9] K. H. Burrell, Phys. Plasmas **4**, 1499 (1997).
[10] P. J. Catto, M. N. Rosenbluth, and C. S. Liu, Phys. Fluids **16**, 1719 (1973).
[11] P. L. Similon and P. H. Diamond, Phys.Fluids **27**, 926 (1984).

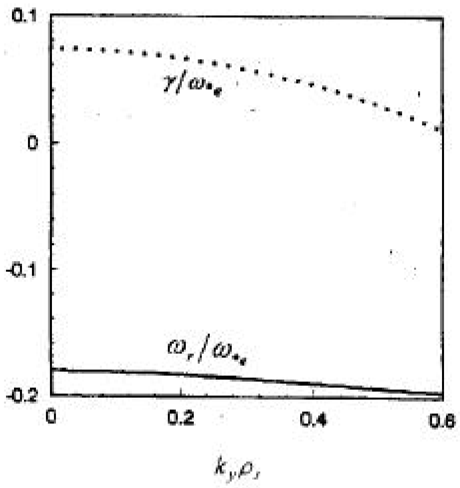

Fig. 1. Normalized mode growth rate γ/ω_{*e} (dotted line) and real frequency ω_r/ω_{*e} (solid line) of the dissipative trapped electron-modified η_i -mode with wave number $k_y\rho_s$ for $L_n/L_s = 0.1$, $\eta_i = 2$, $T_e/T_i = 1$, $n = 0$.

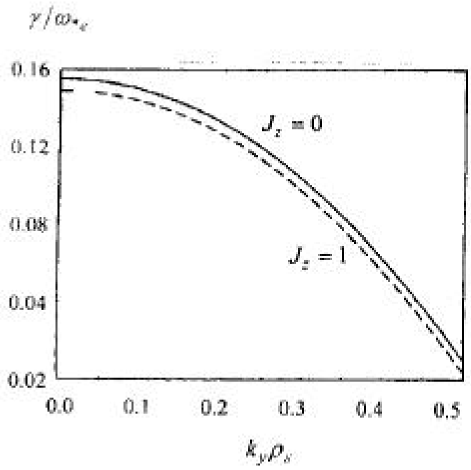

Fig. 2. Noemalized mode growth rates γ/ω_{*e} of $J_z = 0$ hybrid electron-ion drift mode vs $k_y\rho_s$ with (solid line) and $J_z = 1$ (dashed line).