INTERPRETATION OF TRANSPORT BARRIERS AND OF SUBNEOCLASSICAL TRANSPORT IN THE FRAMEWORK OF THE REVISITED NEOCLASSICAL THEORY

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Abstract

„Subneoclassical“ heat fluxes are predicted in the high collisionality regime by the revisited neoclassical theory¸ which includes the roles of Finite Larmor Radius effects and Inertia, that we published earlier. Unlike conventional neoclassical theory, the revisited theory further provides a non degenerate ambipolarity constraint which defines unambiguously the radial electric field. Together with the parallel momentum equation, the ambipolarity constraint leads, under some conditions, to radial electric field profiles with high negative shear akin to those observed in spontaneous edge transport barriers. The predictions of the theory are outlined, with emphasis laid on the interpretation of experimental results such as magnitude of the jumps, width of the shear layer, local scaling laws. Extension of the theory to triggered transitions and cold pulse propagation studies is suggested.

1. INTRODUCTION

Experiments conducted in the last decade led to the discovery of improved confinement modes, the transitions following the spontaneous [1] or triggered [2] generation of high, negatively sheared radial electric fields: those tend to suppress the turbulence and thus the related anomalous transport [3, 4]. Estimates of the ion energy flux across *transport barriers* also led, in some instances [5], to “subneoclassical" values indicating that the reference theory is there invalidated.

We indicated earlier [6] that the „conventional“ neoclassical theory is incomplete since lacking an equation for the bulk toroidal velocity component $U_\varphi \cong U_{\varphi,i}$ or, alternatively, for the radial electric field E_r. At the order at which they are calculated, the magnetic surface average electron and ion fluxes $\Gamma_{e,r}$ and $\Gamma_{i,r}$ are indeed identical, with the result that the ambipolarity equation $\Gamma_{e,r} - \Gamma_{i,r} \propto J_r = o$ (in the absence of a polarized electrode) is trivially verified. Conventional neoclassical theory postulates the scaling relations $L_\Psi \sim r \sim qR$ but $a_i/L_\Psi << 1$, where L_Ψ is a characteristic radial length-scale of the equilibrium profiles [e.g. $L_{T(N)} = d\ell nT(N)/dr$ or $L_{U\varphi\,(U\theta)}$], r the minor radius, qR the connection length and $a_i = c_i/\Omega_i$ the ion Larmor radius [$c_i = (T_i/m_i)^{1/2}$ is the thermal velocity and $\Omega_i = eB/m_i$ the Larmor frequency of singly charged ions]. This assumption is inappropriate at the plasma edge and near transport barriers where $L_{N(T)}$ and $L_{U\varphi\,(U\theta)}$ can be as small as a few centimetres, whereas r and qR are typically of order 1 and 10 m respectively and a_i is of order 1 mm; hence $a_i/L_\psi \sim 1/20$ whereas $L_\psi/qR \sim 1/500$. Keeping accordingly a_i/L_ψ as a finite albeit small parameter leads to the emergence of Finite Larmor Radius and Inertia effects in the theory, which decouple the ion and electron radial flows. The fundamental scaling parameter which characterizes the revisited neoclassical theory, so far only developed for the high collisionality regime, turns out to be

$$\Lambda_1 \equiv \frac{q^2R^2}{(\chi_{\|,i}/N)}\frac{T_i^{'}}{eBr} = \frac{1}{3.9\,\Omega_i\tau_i}\frac{q^2R^2}{rL_T} \qquad (1)$$

(the parallel heat conduction $\chi_{\|,i}$ and the collision time τ_i are defined in Braginskii [7]). We note that i) the counterpart of (1) in the low collisionality regime $(\nu_i^* \equiv qR\nu_i/\varepsilon^{3/2}c_i < 1)$ would be (replace 3.9 τ_i by qR/ci): $(\Lambda_1)_{\ell.c.} = (a_i)_p/L_T$, where $(a_i)_p$ is the Larmor radius in the poloidal magnetic field; that, or a closely related parameter is introduced to describe the squeezing of banana orbits [8];

ii) Λ_1 is of order unity in the framework of the revisited theory if, as assumed in [6], $L_T/r \sim r/qR \sim \varepsilon$ and $\hat{\nu}_i \equiv qR\nu_i / c_i \sim \varepsilon^{-1}$ (the high collisionality regime is characterized by $\hat{\nu}_i \equiv \varepsilon^{3/2}\nu_i^* > 1$), whereas Λ_1 is vanishing ($\sim \nu_i / \Omega_i << 1$) in the framework of the conventional theory;

iii) Λ_1 is the ratio of the parallel ion heat conduction and diamagnetic times scales.

A rigorous and almost immediate consequence of Braginskii's two fluid equations iterated with the new expansion scheme is the „subneoclassical" heat flux

$$q_{i,r} = -[1 + 1.6 q^2 (1 + Q^2 / S^2)^{-1}] \chi_{\perp,i} T_i' \tag{2}$$

where $\chi_{\perp,i}$ is defined in [7], Q and S in [6], Q/S = O ($8\Lambda_1$), and q is the safety factor. The Pfirsch-Schlüter heat flux $- 1.6\, q^2\, \chi_{\perp,i}\, T_i'$, a consequence of the ion poloidal asymmetries inherent to the (here axisymmetric) toroidal geometry, is thus reduced by the factor $1 + Q^2/S^2$. The interpretation is that the diamagnetic rotation combines to the parallel heat flux to reduce poloidal inhomogeneities; this is equivalent to increasing B_θ, or reducing q. Other consequences of the new scaling are:

(a) a modified ambipolar flux including new, mostly pinch terms;

(b) a modified parallel momentum equation (actually better regarded as an equation for $U_{\theta,i}$);

(c) a non degenerate ambipolarity constraint $\Gamma_{i,r} - \Gamma_{e,r} = O(\Lambda_1 \Gamma_{i,r}) = o$, the consequences of which will be discussed in the next Sections.

It should be noted that electrostatic drift wave turbulence also yields $\Gamma_{e,r} \equiv \Gamma_{i,r}$ in leading order. Keeping here the parameter a_i/L_ψ finite, albeit small, would presumably lead to a different ambipolarity constraint (A self consistent theory is still awaiting development). As experimental work on the transition from low (L) to high (H) confinement has revealed that the radial electric field shear increases before fluctuations suppression [1] and that the plasma is much less turbulent in the H mode than in the L mode (raising the obvious question "How does the plasma maintain the more quiescent H-mode state when the driving terms from turbulence ... that existed in L-mode have disappeared?" [9]), we have chosen to concentrate on the possible *neoclassical origin* of the sheared radial electric field profile. As transitions have been observed [9, 10] for values of ν_i^* ranging from 1 to 60, we discard the role of direct orbit losses (the latter could be relevant only if there were a suprathermal ion tail).

2. NEOCLASSICAL THEORY OF ROTATION

Experiment has also revealed that the radial electric field shear increases in L to H transitions prior to any significant change in the ion pressure gradient [1]. We thus *ideally* decouple the evolutions of $U_{\varphi,i}$ and $U_{\theta,i}$ from those of N and T_i and assume, for convenience,

$$T_i = T_{i,s} (Kx + 1)^p, \quad N_i = N_{i,s} (Kx + 1)^{p/\eta i}, \tag{3}$$

where x is measured from the separatrix: x < o in the confined plasma; K < o.

The limiting case where $\Lambda_1 \to o$ is amenable to a simplified but suggestive analytical solution. In that instance, $U_{\theta,i} = k\, \upsilon_{T,i}$ as usual [11], where $\upsilon_{T,i} = T_i' / eB_\varphi$ and k = - 2.1; the ambipolarity equation of the revisited neoclassical theory [6] then reduces to

$$[F\partial_t + (\hat{u}_1 + \hat{u}_2) - h + L_T \partial_x] h = \hat{u}_1 \hat{u}_2 \tag{4}$$

where $h = U_{\varphi,i}/\nu_{T,i}$, $L_T = (Kx + 1)/Kp$ (< o), $F = 0.8\, \chi_{\|,i} / N q^2 \nu_{T,i}^2$, and

$$\hat{u}_1 + \hat{u}_2 = F\nu_{cx} - [-2k + 1.5 + 2\eta_i^{-1} + (2 - z)p^{-1}] \tag{5a}$$

$$\hat{u}_1 \hat{u}_2 = F(B_\theta / B_\varphi)\, \dot{m}_\varphi / \nu_{T,i} - (3 + 6\eta_i^{-1} - 5k)[0.5 + 2\eta_i^{-1} - (2 - z)p^{-1}]. \tag{5b}$$

Friction of the flow with charge exchange neutrals and acceleration $(\dot{m}_\varphi)$ by a beam are taken into account; $z \equiv \partial \ell n\, Z_{eff,i} / \partial x$, $Z_{eff,i}$ entering via main ions collisions with impurities.

Equation (4) can be solved exactly [12] if $\hat{u}_1$ and $\hat{u}_2$ are considered as constants (which, as Ansatz (3), is no major restriction, L-H transition being almost universally observed); this solution involves an arbitrary function of the „retarded time"

$$t_{ret} = t - \int_o^x F(x')\, dx' / L_T\, (x') \tag{6}$$

which must be chosen as to fulfil the boundary condition at the last closed magnetic surface [(6) indeed corresponds to an inwardly propagating signal]. $\hat{u}_1 + \hat{u}_2$ being generally a large number (in absolute value), e.g. O(- 10) if $\eta_i = 1$, p = 1, z = o and $F\nu_{cx} < 10$, the spatial dependence of F(x) and $L_T(x)$ can however be neglected in a first analysis; the approximate regular solution of Eq. (4) is then

$$h = 0.5\left\{(\hat{u}_1 - \hat{u}_2)\ \tanh\ [0.5(\hat{u}_1 - \hat{u}_2)\xi] + (\hat{u}_1 + \hat{u}_2)\ \right\} \tag{7}$$

where $\xi = (x - x_o)/(- L_{To})$ and we have assumed $\hat{u}_1$ and $\hat{u}_2$ to be real (which sets a constraint on the constituting parameters). $x_o(t_{ret})$ is the position of the inflexion point of the exact solution. If $U_{\varphi,i}\cdot(x = o) = o$ is the (stationary) boundary condition, then

$$(Kx_o + 1)^{p(\hat{u}_1 - u_2)} = -(\hat{u}_2 / \hat{u}_1)\ . \tag{8}$$

3. PREDICTIONS OF THE SIMPLIFIED THEORY

Equation (8) has a solution iff $\hat{u}_1\hat{u}_2 < o$ [It can be shown that the exact solution of Eq. (4) diverges if $\hat{u}_1\hat{u}_2 > o$, which sets another limit on the parameter range of tokamak discharges]. If $\hat{u}_1\hat{u}_2 < o$, then x_o is negative corresponding to a position in the confined plasma iff $\hat{u}_1 + \hat{u}_2 < o$ or, cf. Eq. (5a):

$$\nu_{cx} < (\sim 3)\, q^2\, (a_i / L_T)^2\, NT_i / \eta_{o,i}\ , \tag{9}$$

where $\eta_{o,i} = m_i\, \chi_{\parallel,i} / 4$. Inequality (9) implies that high confinement sets in when rotation spin up, owing to FLR and Inertia effects, overcomes friction with charge exchange neutrals. Assuming that the theory could be extrapolated to low collisionality plasmas ($\tau_i \rightarrow qR/c_i$), (9) would yield

$$(T_i)_{thres} \propto N_o^{0.8}\ (RL_T / r)^{1.6}\, m_i^{-0.4}\ B^{0.8}\, I^{0.8} \tag{10}$$

for the minimum edge temperature required for the transition. (10) compares well with a recent observation by the ASDEX-Upgrade group: $(T_e)_{thres} \propto N_{e,edge}^{-0.3}\ B^{0.8}\, I^{0.5}$ [13]; moreover, both (10) and (9) show adverse neutral density and favourable isotopic mass variations, as reported [14, 15].

Other important results are:

(1) Electric fields $(E_r)_+$ = 30 kV m^{-1} and $(E_r)_-$ = - 30 kV m^{-1} are predicted respectively inwards and outwards from the inflexion point x_o, in agreement with reported values [16], if T_i' = - 5 kV m^{-1} and, in accordance with (5a) and (5b), $\hat{u}_1 = -10, \hat{u}_2 = +2$;

(2) The width of the shear layer, defined as the distance between the points where tanh $[0.5(\hat{u}_1 - \hat{u}_2)\xi] = \pm 0.5$, is $2L_T / (\hat{u}_1 - \hat{u}_2) \sim 0.2\ |L_T|$;

(3) The negative E_r shear layer lies close to the LCMS: $x_o / L_{To} \cong (\hat{u}_2 - \hat{u}_1)^{-1}\ \ell n(-\hat{u}_1 / \hat{u}_2)$;

(4) The critical neutral density, defined by the condition $\hat{u}_1 + \hat{u}_2 = o$, is in the range $N_o \sim (1-5)\, x\, 10^{-4}\, Z_{eff,i}\, N_i$, as also found on ASDEX-Upgrade [12];

(5) The neoclassical time-scale associated with the toroidal rotation is $\tau_{U\varphi} \sim \left|\hat{u}_1 + \hat{u}_2\right|^{-1} F$, i.e. $\tau_{U\varphi} \sim 10\ \left|\hat{u}_1 + \hat{u}_2\right|^{-1} (\tau_E)_{neo}$ where $(\tau_E)_{neo}$ is the neoclassical heat transport time-scale if $\hat{u}_1 + \hat{u}_2 << o$, but $\tau_{U\varphi} \sim \nu_{cx}^{-1}$ if $\hat{u}_1 + \hat{u}_2 >> 0$; the former result is in contrast to earlier predictions [17]

4. DISCUSSION AND PROSPECTS

The revisited neoclassical theory of plasmas provides a satisfactory description of many observations related to the L to H transition. The limit considered in Section 2 for analytical simplification purposes may however not be fully adequate as it breaks down when (i) $\left|\Lambda_1\right| \sim 0.125$ and (ii) $\left|\Lambda_1\right| \sim \left|\hat{u}_1 + \hat{u}_2\right|^{-1}$. The poloidal velocity does no longer follow the simple neoclassical law $U_{\theta,i} = k\,v_{T,i}$ under those conditions and new terms appear in Eq. (4). Consequences are: the contribution from $U_{\theta,i}$ to the modification of E_r from L to H phases is increased, as experiments suggest (but, it should be noted, the velocities are mostly obtained from the Doppler shift of impurity lines); moreover, the theory might now be able to explain the results of polarization experiments [2] (the general ambipolarity equation [6] includes an integration constant related to a radial current).

A final noteworthy point is that turbulence and hence anomalous heat and particle transport reduction by a propagating E_r (t_{ret}) profile could possibly explain the cold pulse experiment [18]. This type of behaviour — or rather its opposite (turbulence excitation) — was observed earlier in shock wave experiments [19].

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