Resistive edge modes, A scenario for the L-H transition due to heat flux

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Abstract

A scenario for the L to H mode transition in tokamaks due to heat flux is presented. The mechanism is stabilization of a strongly ballooning resistive edge mode by the ion diamagnetic drift. We find that this stabilization may occure before that of poloidal rotation as a function of increasing ion temperature gradient at the edge. We have also found a new type of toroidal condensation instability which is found to dominate transport in the H-mode edge of tokamak plasmas when the temperature gradient is steeper than the density gradient. A particle pinch develops at the edge in agreement with experimental observations and the transport can be dominated by the ion or electron channel.

Introduction

It is evident that the leading candidates for core transport, i.e. η_i and trapped electron modes are not able to produce the increase in transport coefficients with radius observed in, L modes in the edge region. In H mode[1], on the otherhand, the observed particle pinch[2] has been difficult to explain. It is thus essential to look for other transport mechanisms that may dominate at the edge.In the present work we focus our interest on MHD type modes in the collision dominated edge. The presence of an electrostatic resistive ballooning mode with an ideal MHD growthrate was pointed out in Ref. 3. Such a mode occurs when the resistivity is strong enough to prevent electron motion along the field lines.

In the present work we focus on effects of temperature gradients and perturbations. We expect temperature gradients to be particularly important in transitions from L to H modes since such transitions occur for strong plasma heating. We then expect a situation with $\eta = L_n/L_T > 1$ to develop. A generalization of the new resistive ballooning mode branch with MHD growthrate found in Ref. 3 then leads to an FLR type stabilization at a critical η_i. Since this stabilization is likely to occur for a smaller η_i than the stabilization by rotation, we will, in the following, focus attention on the FLR type effects and, for simplicity, omit effects of rotation. In the local limit this system is 2 dimensional due to collisions. It is very similar to the system derived in Ref.4 where trapping made a part of the electrons two dimensional. In this system a transition to an H-mode was obtained in predictive simulations due to FLR stabilization of the $\eta_i$mode for large η_i[5]. The stabilization of the resistive ballooning mode, however, occurs at much smaller η_i. In H-mode, η_i and η_e increase further at the edge due to the improved confinement. We expect that this would stabilize also the drift type resistive modes[6,7]. In the H-mode barrier we find an MHD type (2d) mode which has the character of a toroidal condensation instability[8]. This mode, which is symmetric in ion and electron dynamics, is not stabilized by large temperature gradients and gives a particle pinch. It has both real frequency and growthrate of the order of the magnetic drift frequency and thus requires a careful treatment of the energy equation. For the real eigenfrequency, the symmetry in ion and electron quantities is shown in a propagation in the electron drift direction when the ion temperature gradient is steeper than the electron temperature gradient and vice versa. The mode can be stabilized by parallel conductivity but this effect is not as strong as for the resistive ballooning mode.

Formulation

We consider a strongly resistive edge region where the parallel conductivity is weak. In this regime the dynamics is essentially only perpendicular to the magnetic field. In H-mode the driving instabilities have small growthrates and, as we will see, of the order of the magnetic drift frequency. We will thus carefully retain magnetic curvature also in the energy equation. We thus use the advanced fluid formulation from Refs. 4 and 5 but add parallel conductivity.

$$\frac{3}{2}n_j\left(\frac{\partial}{\partial t}+\vec{V}_j\cdot\nabla\right)T_j+\rho_j\nabla\cdot\vec{V}_j=-\nabla\cdot\vec{q}_j+\nabla_{\|}(\chi_{\|j}\nabla_{\|}T_j) \tag{1}$$

where $\vec{q}_j=\vec{q}_{*j}=\frac{5}{2}\frac{P_j}{m_j\Omega_j}\left(\hat{e}_{\|}\times\nabla\delta T_j\right)\ \hat{e}_{\|}=\vec{B}_o/B_o,\ \chi_{\|e}=3.16\frac{n_eT_e}{m_e\nu_{ei}},\ \chi_{\|i}=0.$

Eq. (1) defines the truncation of the fluid hierarchy. The momentum equation leads to the usual low frequency fluid drifts

$\vec{V}_E=\frac{c}{B}(e_{\|}\times\nabla\phi),\ \vec{V}_{*j}=\frac{c}{q_jn_jB}(\hat{e}_{\|}\times\nabla P_j),\vec{V}_{P_i}=-\frac{c}{B\ \Omega_i}\left(\frac{\partial}{\partial t}+\vec{V}_i\cdot\nabla\right)\nabla\phi$

We also define the diamagnetic drift frequency $\omega_{*T_j}=\vec{k}\cdot\vec{V}_{*j}$. The diamagnetic drift corresponding to only density gradient is denoted ω_{*j}. We also note the relations $\nabla\cdot\vec{V}_E=\frac{q}{T}\vec{V}_D\cdot\nabla\phi$ and $\nabla\cdot(n\vec{V}_*)=\frac{1}{T}\vec{V}_D\cdot\nabla P$ where $\vec{V}_D$ is the magnetic drift $\vec{V}_D=\frac{T}{m\Omega_c}\left(\hat{e}_{\|}\times\frac{\nabla B}{B}\right)+\frac{T}{m\Omega_j}(\hat{e}_{\|}\times\vec{\kappa})$ where $\vec{\kappa}=(e_{\|}\cdot\nabla)e_{\|}$ is curvature vector.

We consider a collision dominated case where collisions are strong enough to make the parallel electron motion subdominant. This means that electromagnetic effects also should be weak and we will for the low β edge completely ignore them. We will, however, include parallel electron motion as a small term in order to see when it becomes important. We note that the ion dynamics now is the same as for the toroidal η_i mode in Ref.4. When collisions are strong enough to completely eleminate parallel electron motion we obtain the same description for the full electron population as for the trapped electrons in Ref. 4. This means that we in this, local, limit can recover our system by taking the trapped fraction equal to 1 in Ref. 4, 5. In the general case we may write the parallel electron current as

$$j_{\|e}=n\ e\ D_e\ \hat{e}_{\|}\cdot\left(\frac{1}{n}\nabla_{\|}n-\frac{e}{T_e}\nabla_{\|}\phi+1.71\frac{\nabla_{\|}T_e}{T_e}\right) \tag{2}$$

$D_e=T_e/(0.5\nu_{ei}m_e)$ Temperature perturbations enter through curvature terms of the type (7) and through $j_{\|e}$. We introduce $\eta_e=L_{Te}/L_n,\ L_T=-T/dT/dr,\ L_n=-n/\frac{dn}{dr}$ We have ignored $k_{\|}^4D_e^2$ terms since parallel electron motion is assumed to be subdominant. The ion response is analogous to the electron response but without parallel thermal conductivity. Also ion sound effects have been ignored in the following but can easily be added. In order to obtain an eigenvalue equation we make the replacement $k_{\|}^2\rightarrow-\frac{1}{q^2R^2}\frac{\partial^2}{\partial\theta^2}$ The modes we study here are now of an MHD type and the eigenvalue equation is derived from the condition

$$\nabla\cdot\vec{j}=0 \tag{3}$$

We will only consider the strong ballooning regime and will thus not operate on ω_D with $k_{\|}$. We then obtain an eigenvalue equation which is quartic in ω.

So far no ordering has been assumed for η_i, η_e or ϵ_n. In a fluid treatment we must always assume $k_\perp^2\rho_s^2$ to be small for $\tau\sim 1$. The characteristic frequencies are ω_*, $\omega_{*T}=\omega_*(1+\eta)$ and ω_D. The eigenfrequency of course, has to be determined by these frequencies but can reach the ideal MHD-growthrate $\gamma_{MHD}\sim\left(\frac{\omega_*\omega_D}{k_\perp^2\rho_s^2}\right)^{1/2}$.

Edge Ordering

We will now introduce a specific edge ordering. The most general edge ordering is $\epsilon_n\ll 1$ which is almost always fulfilled. This separates the frequencies into "High" frequency of $\sim\omega_*$ and low frequency $\sim\omega_D$. For strong heating ω_{*T} will define the highest drift frequency in the system. When ω_{*iT} becomes comparable to γ_{MHD} the system goes over to H mode for which $\gamma\ll\gamma_{MHD}$. As it turns out the FLR stabilization due to ω_{*iT} will also be effective on resistive drift modes with $\omega\sim\omega_*$ but a mode with $\omega\sim\omega_D$ will remain. This mode corresponds to the trapped-electron-etai mode in the enhanced confinement state of Ref.5 and is mathematically identical if we assume all electrons to be trapped in Ref.5 and take the local limit in the present work.

For the resistive ballooning mode, with growthrate of the order of the interchange frequency we ignore the electron temperature perturbations and gradient. Then using the ordering of ϵ_n we reduce the relation to second order in ω. By making a strong ballooning approximation we can then derive the dispersion relation

$$\omega(\omega - \omega_{*iT} + i\gamma_D) = \frac{\omega_{*e}\omega_{Di}}{k_\theta^2 \rho_s^2}, \tag{4}$$

where

, $= 1 + \eta_i + \tau$, $\gamma_D = \frac{|s|}{k_\theta \rho_s}\sqrt{-i(\omega - \omega_{*iT})(1 - \frac{\omega_{*e}}{\omega})\frac{D_e}{q^2 R^2}}$ and $D_e = \frac{2T_e}{\nu_e m_e}$. In the local limit we can easily include electron temperature gradients by adding $\tau\eta_e$ to , . The expression for the damping due to thermal conduction, γ_D however gets considerably more complicated.
The strong ballooning approximation can easily be fulfilled for edge parameters. The fastest growing mode is obtained at $k_\theta \rho_s = 0.15$. Below this value the conductivity damping (shear damping) is large and above it the FLR stabilization dominates. When we include the electron temperature gradient, complete FLR stabilisation typically occurs at η_i between 3 and 5. A stabilisation due to neoclasical poloidal rotation $v_\theta = \eta_i v_*$ can be estimated at

$$\eta_i \geq k_\theta L_{Ti}\sqrt{\frac{\epsilon_n}{k_\theta^2 \rho_i^2}}, \tag{5}$$

Here we balanced the shearing rate with the growthrate, taking L_{Ti} as the shearing length scale. For $\epsilon_n \simeq k_\theta^2 \rho_i^2$ this threshold is typically around 8. We thus expect the FLR stabilisation to occur before that of sheared rotation in a scenario where the transition is caused by increased heat flow.
We note that a similar transition was previously obtained in a system of eta-i modes and trapped electron modes in predictive transport simulations. The growthrate (local) in that system was

$$\gamma = \omega_{*e}\sqrt{\frac{\epsilon_n}{\Lambda}(\frac{\eta_i}{\tau} + f_t \eta_e) - \frac{k^4 \rho_s^4 \eta_i^2}{4\tau^2 \Lambda^2}} \tag{6}$$

where $\Lambda = 1 - f_t + k^2 \rho_s^2$ and f_t is the fraction of trapped electrons. For $f_t = 0.65$ the transition ocurred at $\eta_i = 15$. For $f_t = 1$ we recover the local limit of our edge mode for large eta. We may thus unify the descriptions of core modes and edge modes by interpreting f_t as the fraction of electrons that for various reasons, i.e. trapping, collisions, induction, do not move along the magnetic field lines. The H-mode transition in the drift wave system gave an H-factor of about 2.5.

In the enhanced confinement state we have eliminated modes with growthrates of the order of the diamagnetic drift frequency. We may, however, still have modes with growthrate of the order of the magnetic drift frequency. This is the reason why we have kept all the magnetic drift terms in our fluid description. Thus, while our general system is of fourth order in ω we can now ignore the two highest orders thus again obtaining a quadratic dispersion relation in the strong ballooning limit. Keeping the conductivity damping term from the analytical solution of the eigenvalue problem we obtain.

$$\omega^2 - \frac{10}{3}\xi \omega_{De}\omega + \frac{5}{3}\omega_{De}^2 \delta = i\gamma_D^2 \tag{7}$$

where

$$\xi = \frac{\eta_i - \eta_e + 0.5(\tau - \frac{1}{\tau})}{\eta_i + \tau\eta_e + 1 + \tau}\ , \delta = \frac{\eta_i + \frac{1}{\tau}\eta_e - \frac{7}{3}(1+\tau)}{\eta_i + \tau\eta_e + 1 + \tau} \tag{8}$$

The expression for γ_D is quite complicated but for the edge ordering we are using it is, in the strong balloning case, considerably smaller than ω_D. Its is thus relevant and interesting to consider the local limit where γ_D is ignored.

In the local limit the solution is symmetric in electron and ion quantities. The mode thus propagates in the electron drift direction when $\eta_i > \eta_e$ and vice versa if $T_e = T_i$. We also note that a necessary condition for instability is:

$$\eta_i + \frac{\eta_e}{\tau} > \frac{7}{3}(1 + \frac{1}{\tau}) \tag{9}$$

In the local limit $j_{||e} \approx 0$ and $\nabla \cdot \vec{V}_{\rho i} \approx 0$. Eq. (3) then reduces to the condition

$$\delta p = 0 \tag{10}$$

The instability is thus of the condensation type. It can be seen as a usual thermal instability where, due to the condition $\eta > 1$, the electron convective temperature perturbation through the divergence of the diamagnetic flux gives the dominant contribution to δn. This relation replaces the Boltzmann equation for electrons in the eta-i mode and thus makes ϕ inversely proportional to η_e in the ion feedback loop and to η_i in the electron feedback loop. This is why η_i and η_e appear in the denominators of ξ and δ and this is why a further increase in η_i and/or η_e does not FLR stabilize this mode. We also note that $\nabla \cdot \vec{q}_*$ gives a fluid resonance for the temperature perturbation. In this way it gives the necessary phase shift for instability (without it one ω factors out). The inclusion of $\nabla \cdot \vec{q}_*$ is thus necessary for the condensation instability.

Transport

We may use the transport coefficients from Ref. 4,5 where we take the trapped fraction, $f_t = 1$. The effective diffusion coefficients are obtained by using quasilinear theory, Ficks law and by balancing the growthrate with the $E \times B$ convective nonlinearity for the saturation level. Using the present ordering and taking $\eta \gg 1$ we have

$$D = \Delta_n \frac{\gamma^3 / < k_x^2 >}{\omega_{*e}^2} \tag{11}$$

$$\Delta_n = \frac{10}{3} \eta_e \frac{\epsilon_n^2}{N} (\xi - 1) \tag{12}$$

N is defined as the magnitude squared of the denominator of the electron density response. It is

$$N = \frac{25}{9} \epsilon_n^4 \left[\frac{20}{3} (\xi - 1)(\xi - \delta) + (1 - \delta)^2 \right] \tag{13}$$

Since N is always positive and since $| \xi | < 1$ for positive η we always have a particle pinch. The thermal fluxes are, however, generally outward.

We believe that the present mode has a high potential for explaining several experimental observations in H-mode in addition to the particle pinch. In particular the thermal diffusion can be carried mainly by the ion or electron channel depending on the relative strength of electron and ion heating. The damping γ_D also supports an improvement in confinement with current and ion mass[2]. We also note that the assumption of large η means that the L to H mode transition is triggered by a strong heat flow. The present results again emphasizes the importance of toroidal effects and the perpendicular dynamics. Similar phenomena may occur in widely different parameter regimes where parallel dynamics can be impeded by very different physics e.g. particle trapping, electromagnetic effects or resistivity.

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