NONLINEAR FULL TORUS CALCULATIONS OF RESISTIVE PRESSURE-GRADIENT-DRIVEN TURBULENCE AND ION-TEMPERATURE-GRADIENT-DRIVEN TURBULENCE IN TOROIDAL GEOMETRY

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Abstract

Resistive pressure-gradient-driven turbulence (RPGDT) and ion-temperature-gradient-driven turbulence (ITGDT) seem to play a critical role in explaining the measured transport in tokamaks. The former at the plasma edge [1], and the latter at the plasma core [2]. However, numerical calculations based on these types of instabilities have been limited to flux tube geometry or limited by numerical resolution. Here, we present results for three-dimensional calculations in the full torus geometry.

1. RESISTIVE PRESSURE-GRADIENT-DRIVEN TURBULENCE

Since the decay index of the poloidal and toroidal mode number spectrum of RPGDT is very small, the spectrum is broad and requires the inclusion of many Fourier components to achieve the desired high resolution. To this end, we have simplified the dynamical model to only include time evolution equations for the electrostatic potential, and the pressure. Viscosity and perpendicular transport are included in the equations to provide the energy sink needed to get steady-state turbulence.

Flux coordinates with straight field lines have been chosen to describe the general toroidal magnetic geometry [3]. The dynamical model equations are solved implicitly in time for the linear part while they are solved explicitly in time for the nonlinear terms. A hybrid representation based on finite differences in radius and Fourier mode expansion in the poloidal and toroidal angles is used. With the present computer capabilities, we can resolve adequately the spectrum for this system of equations in a full torus geometry using this representation.

A realistic turbulence calculation would require the inclusion of a source term in the average pressure evolution equation. This source accounts for heating and fueling of the plasma and leads to profiles that are consistent with the fluctuation-induced transport. However, its inclusion requires carrying out the calculation over several confinement times, which is too long for a 3-D nonlinear calculation in toroidal geometry. Hence, we do not include such a source, but we consider two different evolution scenarios: (1) The average pressure evolves nonlinearly in a consistent way with the evolution of the fluctuations. In this case, the average pressure profile is modified by the quasilinear effects, but maintains a nonzero gradient. (2) The average pressure profile is maintained fixed during the evolution. In the first case, two effects are controlling the saturation of the turbulence: the change in the averaged gradients and the dissipation in the high n range. In the second case, the nonlinear saturation is only caused by the transfer of energy to damped high-n modes, where it is dissipated.

When the average pressure profile is maintained fixed, the nonlinear evolution always reaches a saturated state of fully developed turbulence. However, when the average pressure gradient evolves, there are two possible steady state solutions. If the initial pressure gradient is low (for instance when $\beta_0 = 0.01$ and 0.021), the final state evolves to be close to marginal stability and the turbulence at saturation is dominated by a single n-mode. The n-value of this single mode is close to the mode with the largest linear growth rate. The fast growth of the nonlinear dominant mode modifies the average pressure profile in such a way that all the modes become nonlinearly stable. Because the change of the average pressure profile is slight, this stabilization can occur only for low pressure gradients, that is to say, near the linear stability threshold. When the initial pressure gradient is higher, a saturated state of fully developed turbulence is reached.

For a case with the average pressure gradient held fixed and $\beta_0 = 0.087$, we have compared three calculations with 488, 706, and 754 Fourier components. In these cases, the diffusivities are chosen to stabilize all modes with $n > 25$, 35, and 45 respectively. The spectra are flat, and the convergence is good for the cases with 706 and 754 modes included in the calculation. The spectrum at saturation for the best converged case is shown in Fig. 1. The value of the different components is averaged in time during the stationary phase and in radius between 0.75 and 0.85. A power-law fit to the fluctuation spectrum gives $\overline{\Phi^2_{mn}} \sim n^{-5/3}$ for wave numbers in the range between n = 10 and n = 45. Here, the upper bar indicates time average.

We have characterized the spatial-temporal structure of the turbulence by calculating time and radial correlation functions at different radial and poloidal positions. The correlation function is calculated by averaging the coherence on the toroidal angle. For the high-β cases, there are significant differences between the linear and nonlinear phase in the numerical evolution. In the linear phase of the calculation when a single mode dominates, the radial correlation length is of the order of the profile scale length. This is a consequence of the ballooning structure of the mode. However, in the nonlinear phase, the radial correlation length is reduced and turns out to be of the order of the width of the individual poloidal components of the linear eigenfunction. This result clearly indicates a reduction of the characteristic radial scale of the fluctuations in the nonlinear phase. For low β-values, when the spectrum is dominated by a single mode, the radial dependence of the coherence is similar to that of a linear eigenfunction.

In the linear phase, the dependence of the coherence with the poloidal angle is very strong and it is weak in the nonlinear phase. Therefore, the spatial structure of the turbulence is fairly homogeneous and very different from the linear eigenmode structure. In spite of the radial correlation length of the fluctuations being nearly constant on a flux surface, both r.m.s. fluctuation levels and fluctuation induced fluxes have strong poloidal asymmetries in the nonlinear phase and the fluctuation level varies along the poloidal direction.

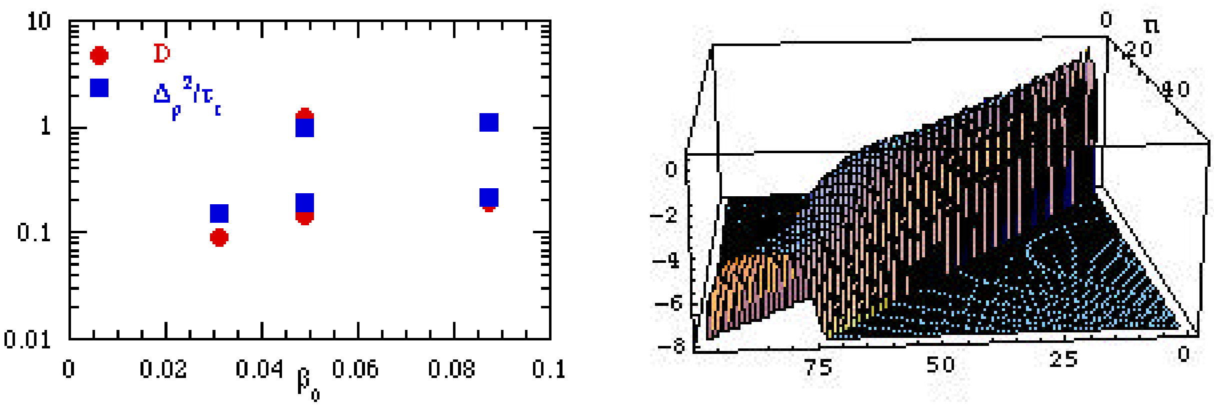

FIG. 1. Spectrum of logΦ^2_{mn} *at saturation for* β_0*=0.087 (left), and comparison of nonlinear diffusion coefficient D and mixing length estimate vs* β *at* ρ*/a=0.8 (right).*

The calculated values of the diffusion coefficient and the fluctuations levels at saturation agree well with the mixing length estimates based on the nonlinear correlation length and decorrelation time. This is shown in Fig. 1, where we compare the numerically obtained diffusion coefficient $D = \langle \tilde{V}^\rho \tilde{p} \rangle / |d\langle p \rangle / d\rho|$ with the mixing length estimate Δ_ρ^2 / τ_c as a function of β for a fixed radial position. Here, the flux, correlation length, and correlation time are averaged in the poloidal angle. We do not include in the last comparison the cases for which the saturated state is dominated by a single n component because in those cases the mixing-length approach does not have any relevance to the saturation dynamics.

2. ION-TEMPERATURE-GRADIENT-DRIVEN TURBULENCE

Ion temperature gradient driven turbulence (ITGDT) [2] is generally believed to cause the experimentally observed anomalous loss of particles and heat at the core of tokamaks. Significant progress has been made in the theoretical understanding of ITGDT over the past few years, in large part due to the development of toroidal gyrofluid computational models [4]. These gyrofluid models have mostly been limited to a local flux-tube description of the plasma and to circular geometry for computational tractability reasons.

We report here on progress towards nonlinear toroidal gyrofluid models covering the full plasma cross section. To keep these full cross section calculations at a computationally manageable level, we only include time evolution equations for the perturbed ion density (vorticity) and perturbed ion parallel velocity, and a perturbed ion temperature equation with a simple parallel linear Landau closure [5]. Moreover, the electrons are treated as adiabatic and the electrostatic approximation is used throughout.

The same representation of the magnetic geometry and the same numerical scheme as for RPGDT have been adopted to solve the Landau fluid system of equations for ITGDT. We note that using straight field lines flux coordinates allows the study of ITGDT for shaped, high plasma beta equilibria.

Full cross section nonlinear toroidal Landau fluid calculations of ITGDT have been performed for eventual comparison with flux-tube models. For illustrative purposes, a low beta equilibrium in circular geometry with a monotonically increasing q profile has been chosen. These calculations have been carried out with 400 radial grid points (covering 100 ion Larmor radii), 1120 poloidal (m) and toroidal (n) modes, with 51 distinct toroidal (n) harmonics. Only through massively parallel implementation of the model on the 512 nodes CRAY T3E at the United States Department of Energy (USDOE)'s National Energy Research Scientific Computer Center (NERSC) has this high degree of high resolution been possible.

Linear results from these calculations are shown in Fig. 2 where each poloidal harmonic forming a typical potential eigenfunction with n = 10 as a function of radius and contours of its projection in the poloidal plane are displayed in panels a and b respectively. The ballooning structure typical of these toroidal eigenmodes driven unstable by a combination of ion temperature gradient and magnetic curvature is apparent in panel b, along with the global radial character of these low-n toroidal eigenmodes. We also note that modes with n > 36 are stable largely because of ion Landau damping.

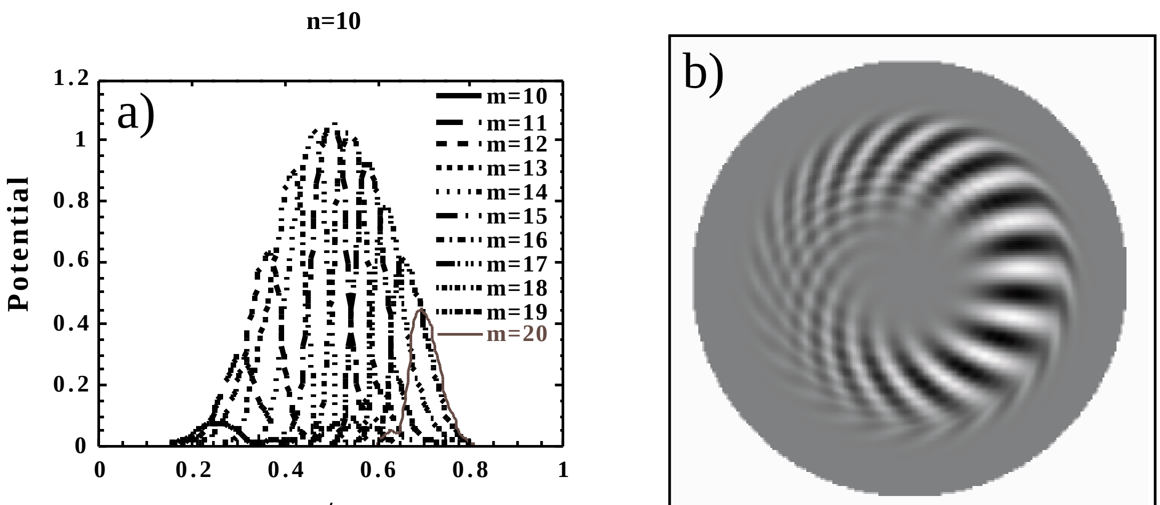

FIG. 2. Linear n=10 toroidal eigenmode

These calculations have been evolved nonlinearly in time to a near steady-state. Salient features of the nonlinear evolution include generation of large poloidal sheared flows by the fluctuations themselves through Reynolds stress and quasilinear modification of the temperature profile by the growing fluctuations. The strong effect that the self-generated sheared poloidal flow has on the spatial structure of the fluctuations is illustrated in Fig. 3 where contours of the potential for calculations without and with self-generated poloidal flow are shown in panels a and b respectively.

It is clear from Fig. 3 that the extended radial structures which dominate the potential fluctuations without flow (panel a) have been broken up by the self-generated poloidal flow (panel b). With flow, there is also a region where the fluctuations have been completely suppressed. It can be further inferred from panel b that the poloidal sheared flow exhibits a structure which is rather global in character. This is confirmed by direct examination of the radial structure of the poloidal sheared flow (not shown). Moreover, it is apparent from Fig. 3 that the poloidal and toroidal mode numbers associated with the near steady-state structures are also lower with flow than without. This is again confirmed by examination of the poloidal and toroidal mode number spectra time-averaged over the near steady-state phase (not shown) which exhibit higher amplitude tails at high *m* and high *n* indicating energy transfers to smaller scale structures through nonlinear mode couplings in the absence of self-generated poloidal sheared flow. Somewhat paradoxically, the ion thermal diffusivities obtained in the calculations appear to be comparable in near steady-state, except in the region where the fluctuations have been completely suppressed by the self-generated poloidal sheared flow.

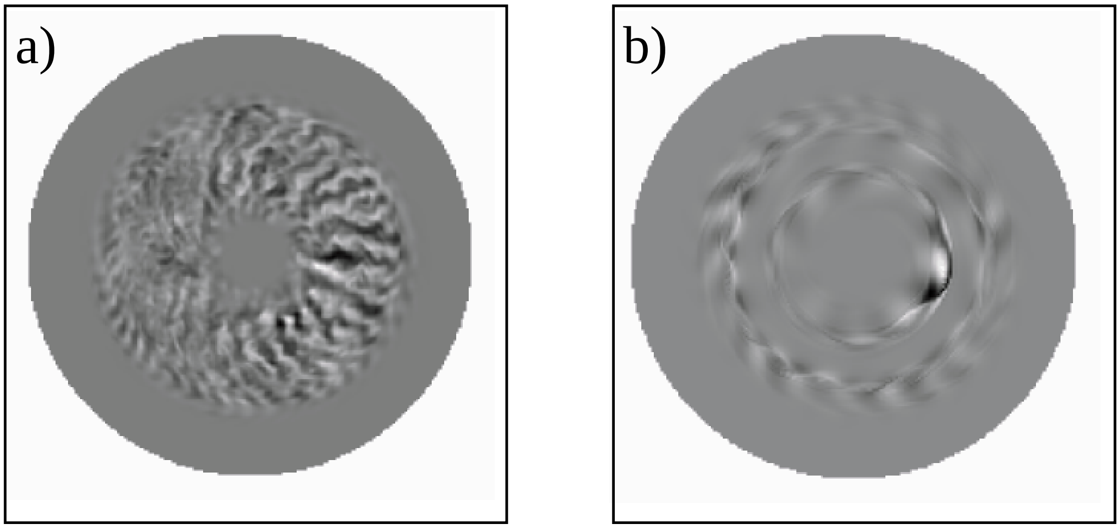

FIG. 3. Near steady-state contours of potential fluctuations without (a) and with self-generated poloidal sheared flow (b).

The preliminary results presented herein indicate that nonlinear, global, full plasma cross section, full torus, toroidal, Landau fluid calculations of ion temperature gradient driven turbulence are feasible on present massively parallel computers. The results also emphasize the profound influence self-generated poloidal sheared flows can have on the structure of the fluctuations.

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