OPTIMIZATION STUDY OF L=1 HELICAL MAGNETIC AXIS SYSTEMS

K.H. SAITO, M. AIZAWA, K.N. SAITO, I. KAWAKAMI, S. SHIINA
Atomic Energy Research Institute,College of Science and Technology,
Nihon University, Tokyo, Japan 101-8308

Abstract

A new approach to the transport improvement in helical magnetic axis stellarator is proposed. First of all, the proposal is presented for L=1 helical axis systems with roughly quasihelical symmetry, then its compatibility with equilibrium and stability would be optimized and controlled. In the optimization, three key requirements are considered; the reduction of effective toroidal curvature ε_T for localized trapped particles, the suppression of indirect magnetic island formation and the magnetic well improvement by some field modifications. The important features is to find the field modifications to satisfy the other two requirements. The reduced ε_T configuration is found to be nearly omnigenous while far from quasihelical, giving rise to the possibility of stellarator design study in a wider parameter domain than quasisymmetry approaches. Up-to-date results show both the negative coil pitch modulation and the superposition of a relatively weak L=−1 field to satisfy the compatibility within the central beta value up to 2% calculated. The compatibility study at higher beta and lower aspect ratio is progressing by optimizing configuration parameters with the combination of some field modifications proposed to improve magnetic well.

1. INTRODUCTION

As reported in previous paper for L=1 systems (major radius R=2.1m)[1], the small N(field period number) system(N=4) satisfies Mercier criterion up to a volume average beta $<\beta>\sim 4.5\%$ due to the formation of a global magnetic well at zero beta by means of both the negative coil pitch modulation($\alpha^* = -0.45$) in coil winding law $\theta = N\varphi + \alpha^* \sin N\varphi$ and the superposition of toroidal magnetic field. On the other hand, the large N system(N=17) with $\alpha^* = -0.2$ has the significantly small loss rate of $\sim 2.5\%$ when $<\beta>=0$ and $\rho_i/R = 3.5 \times 10^{-3}$ for 100keV protons (uniformly distributed in velocity and spatial spaces) started at aspect ratio A=15. The aspect ratio is $R/a = 7.5 \sim 10$ depending on α^*. For comparison, the N=4 system with an optimum bumpy field has a loss rate of $\sim 22\%$ when $<\beta>=0$ and $\rho_i/R = 2 \times 10^{-3}$,$\sim 19\%$ when $<\beta>=2.9\%$ and $\rho_i/R = 2 \times 10^{-3}$, for 1keV protons started at aspect ration A=23. Here ρ_i is ion Larmor radius. In this paper, the optimization study of L=1 system is carried out by optimizing N number and coil aspect ratio so that the equilibrium, stability and transport are compatible at high beta and low aspect ratio. In this study, three key requirements are considered as follows.

2. THREE KEY REQUIREMENTS FOR OPTIMIZATION

It follows from near-axis analysis that L=1 helical fields have roughly quasihelical symmetry for which magnetic field strength B is a function of only a single linear combination of the angles, say $\varsigma \equiv N\varphi - L\theta$: One can think of ς as roughly constant if one were to remain under a given coil. :On axis the pressure gradient vanishes, so B must have non-vanishing gradient in order to balance magnetic curvature in the equation, $\boldsymbol{j} \times \boldsymbol{B} = \kappa B^2 - \nabla_\perp B^2/2 = 0$. Hence, L=1 fields must be present. Thus, if only a single helicity is present, it must be L=1. However, the quasihelical symmetry is disturbed by both the toroidal magnetic Fourier term $B_{n,m} = B_{0,1}$ and the presence of the satellite harmonics of main L=1 helical field $B_{N,1}$, $B_{N,1+i}(i = \pm1, \pm2, \cdots)$, although $i \neq -1$ harmonics are relatively small. These symmetry-breaking terms depend on field modifications to improve magnetic well, and their effects on stability, equilibrium and transport are optimized and controlled if necessary.

2.1. Reduction of effective toroidal curvature

For the transport optimization of stellarator, there are two quasisymmetric approaches, or quasihelical[2] and quasitoroidal[3] symmetrizations. In parallel with quasisymmetric approaches, omnigeneity techniques for directly reducing the particle drifts away from magnetic surfaces have been developed[4]; the bounce action, or zero-order bounce adiabatic invariant J_0 is constant on a magnetic surface. New approach to the transport improvement and the low aspect ratio in helical magnetic axis stellarator as L=1 systems is proposed. We introduce the effective toroidal curvature term ε_T for localized trapped particles[5], defined as $\varepsilon_T \equiv -2\left(B_{0,1}+B_{N,L-1}\right)/B_{0,0}$ in magnetic field Fourier decomposition $B_{n,m}$, where n and m are toroidal and poloidal harmonic number, respectively. For L=1 systems, both $B_{0,1}$ and $B_{N,0}$ are main helical symmetry-breaking terms, depending on field modifications proposed for improving magnetic well. Especially, the negative pitch modulation($\alpha^* < 0$) in coil winding law $\theta = N\varphi + \alpha^* \sin N\varphi$, which forms a local magnetic well in vacuum, reduces ε_T and then leads to a nearly complete collisionless particle confinement in the case of N (field period number)=17, $\alpha^* = -0.2$(major radius $R = 2.1m$, coil aspect ratio R/R_C=7). The smallness of ε_T means a cancellation of two symmetry-breaking terms for localized trapped particles, then an effectively quasihelical symmetrization for them. Really the singular solutions of drift equations with the introduction of ε_T predict the better trapped particle confinement in configuration with the smaller ratio of $|\varepsilon_T/\varepsilon_L|$, where $\varepsilon_L \equiv 2B_{N,1}/B_{0,0}$ is main helical harmonic field. Actually the predictions agree well with the observations by particle orbits calculation. Furthermore, the neoclassical transport losses in the low-collisionality($1/\nu$) regime is the smaller in the smaller ε_T configuration. In order to understand more physically the favorable transport, the topological properties of magnetic field strength B and second adiabatic invariant J_0 are examined as function of α^*. As the results, in the case of $\alpha^* = -0.2$, $N = 17$, in which helically trapped particles are completely collisionlessly confined, i) the $B_{\min}$(local minima of B along field lines in a given surface)-contours have the largest area of closed surfaces, which reveals good confinement of deeply trapped particles, ii) the J_0-contours are closer to magnetic surfaces and the B-contours have nearly equal angular spacing on a magnetic surface while the magnetic lines are far from quasihelical, showing to be nearly omnigenous, and iii) the $B_{\max}$(local maxima of B along field lines in a given surface)-contours have a relatively small area of closed surfaces and a large deviation from magnetic surfaces, which explains the observed loss particles to be transient particles and/or toroidally trapped particles. These topological properties of B and J_0 indicate that the configuration with reduced ε_T is nearly omnigenous. Actually, this reduced ε_T can be seen also in low aspect ratio stellarator/tokamak hybrid, which is optimized by aligning J^*(integral along φ_{Boozer}) contours with the flux surfaces[6]. This new approach to transport improvement gives rise to the possibility of stellarator design study in wider parameter domain than quasisymmetric approaches. The minimization of ε_T for the lower aspect ratio device is predicted to be attained by adjusting $B_{N,0}$ in accordance with the larger $B_{0,1}$ in the lower aspect ratio. Accordingly the reduction of ε_T term is required for improving transport in the optimization study and the increase of $B_{N,0}$ by its external superposition if necessary for lowering aspect ratio.

2.2. Suppression of magnetic island formation

Second is the suppression of magnetic islands, which are caused by resonant pressure driven currents in three-dimensional MHD equilibria[7]. There are two distinct resonant currents; direct currents caused by the variation of $\int d\ell/B$ in the vacuum field and indirect(nonlinear) currents caused by a variation of $\int d\ell/B$ that rises in the presence of finite β. The former can be minimized by proper design of the stellarator. The latter, on the other hand, are intrinsic to the 3-D nature of the equilibrium and give a fundamental β limit. Even at β smaller than the equilibrium β limit(β_{eq}) at which the magnetic axis shifts halfway to the wall, we generally expect the largest islands to form at the lowest-order rational surfaces, because they couple nonlinearly most readily to the nonresonant vacuum magnetic field Fourier components $\delta^V_{n,m}$=$\delta^V_{N,1}$(L=1 field) and $\delta^V_{0,1}$(toroidal effect) which are intrinsic to 3-D equilibria. At a resonant surface we have $\iota = n/m$ with $\iota/N < 1$ so that $n/N < m$, the finite β Fourier amplitude $\delta_{n,m}$ is then

$$\delta_{n,m} \sim (1 + (m-1)/3)\left\{\delta^V_{0,1} J_{n/N}(\theta_{N,1}) J_{m-n/N-1}(\theta_{0,1}) + \delta^V_{N,1} J_{n/N-1}(\theta_{N,1}) J_{m-n/N}(\theta_{0,1})\right\}$$

where J_ℓ is the Bessel function of order ℓ, n must be a multiple of N, $\theta_{n,1} = (3L_z^2\beta_0\delta^V_{N,1} /32\pi^2 a^2(n-\iota)^2)$, L_z is the length of the magnetic axis, β_0 is on-axis beta, a is plasma radius. At low β,$\theta_{j,k}$ is small and we can approximate $J_\ell\left(\theta_{j,k}\right) \sim (1/\ell!)\left(\theta_{j,k}/2\right)^\ell$, then $\delta_{n,m}$ is proportional to β^{m-1} and increases with decreasing N. Even though the vacuum field has no resonant terms in this case, the finite β field resonates with every rational surface. The islands do not overlap at small β because of the exponential decay of the $\delta_{n,m}$; but they increase in width with increasing β to give an equilibrium β limit. This behavior is intrinsic to the 3-D nature of the vacuum field having at least two Fourier components $\delta^V_{0,1}$ and $\delta^V_{N,1}$. For a given resonant $\delta_{n,m}$, the corresponding resonant field component is approximately given by

$$B_{1\rho\ n,m}/B_0 \sim \beta_0 L_z/4\pi a^2 d\iota/d\rho \cdot \delta_{n,m}\left[\ln\left\{(a-\rho_0)/\rho\right\} + \sum_{j=2}^{2m+1}(1/j) + a/\rho_0\right]$$

where ρ_0 corresponds to the rational surface. The corresponding island half-width at $\iota = n/m$ is approximately given by $W \sim \sqrt{2L_z B_{1\rho\ n,m}/\pi B_0 m d\iota/d\rho}$. All quantities here are evaluated at the rational surface. The width scales as $\beta^{m/2}$, and means the easier formation of indirect resonant islands in the smaller N helical magnetic axis systems, then the lower β_{eq} and/or the more deteriorate transport. In fact, L=1 systems with $N = 17$ and $\iota < 1$ are found to form negligibly small islands because they have higher-order rational surfaces with $m > N$. Then, the indirect resonant currents are much smaller than the direct resonant currents. The critical beta value β_{0cr}, above which beta the island width is equal to half the plasma radius, is much higher than unity, indicating negligibly small effect on β_{eq}. For the quasitoroidal symmetry approach with $N = 2$, aspect ratio of 4.5(HHM2)[3], however, the indirect island formation degrading β_{eq} and/or transport may be possible because relatively large $B_{0,1}$ and small $B_{2,1}$ couple to give a finite width of indirect resonant islands if there exists low m rational surface.

2.3.Field modifications for magnetic well improvement

Third is the magnetic well improvement by some field modifications proposed[8]; the negative pitch modulation in coil winding law and the superposition of relatively weak L=−1 field, multipole(dipole and quadrupole) and toroidal fields on standard L=1 systems. Furthermore, the smaller N system($N = 4$, $\alpha^* = -0.45$) forms a global magnetic well in vacuum to satisfy Mercier criterion up to a volume average beta $<\beta>\sim 4.5\%$, although the transport is worse than that in the larger N systems. The important features of this approach is to optimize N number into the compatibility of three key requirements at higher beta, with the combination of some field modifications improving magnetic well. The smaller N systems are the easier for the improvement of magnetic well, on the other hand the larger N systems for both the suppression of magnetic island formation and the transport improvement.

3. COMPATIBILITY OF TRANSPORT AND STABILITY

Up-to-date results obtained with the use of VMEC 3-D MHD equilibrium code, Mercier stability code and the numerical analysis of indirect(nonlinear) islands are summarized as follows. The large $N(=17)$ systems with field modifications, which improve a local magnetic well, satisfy three key requirements within the central beta value up to β_0=2% calculated, above which beta value it is not easy to obtain VMEC code solution with good convergence for the large N helical axis systems. That is, the plasma is marginally stable against Mercier localized modes, the nonlinearly resonant island width is negligibly small because of small beta effect on m mode number of rational surface, and the increase of beta value leads to the smaller effective toroidal curvature, then the more attractive transport features. The L=1 system with a large anti-well and without any field modifications is unstable against Mercier modes. The compatibility is observed to be better for L=1 system with N=17, $\alpha^* = -0.2$ and for L=±1 system with N=17, coil current ratio of $|I_{L=-1}/I_{L=1}| = 0.3$. The toroidal shift of magnetic axis at β_0=2% is 15%∼ 50% of minor radius depending on field modifications. Next problem to be solved is to optimize N number into the compatibility of three key requirements at higher beta and lower aspect ratio by applying field modifications for magnetic well improvement and superposing $B_{N,0}$ if necessary for transport improvement in lower aspect ratio.

4.CONCLUSIONS

A new approach to the transport improvement in helical magnetic axis stellarator is proposed. First of all, the proposal is tried for L=1 helical systems, then the compatibility with the equilibrium and stability are optimized considering three key requirements. The important features of this optimization study is to find the magnetic well improving field modifications to satisfy the reduction of effective toroidal curvature ε_T for localized trapped particles and the suppression of magnetic island formation. The reduced ε_T configuration is near the omnigeneity condition, the property whereby the bounce averaged cross-flux-surface drift vanishes, then giving rise to the possibility of stellarrator design study in a wider parameter domain than quasisymmetry approaches. Up-to-date results show both modifications of negative coil pitch modulation and the superposition of L=−1 field to satisfy the compatibility for L=1 systems with a relatively large value of field period number N within the central beta value up to β_0=2% calculated(above which beta value it is not easy to obtain VMEC code solution with good convergence for the large N helical magnetic axis systems). The better compatibility at higher beta and lower aspect ratio is progressing by optimizing N, R/R_C(coil aspect ratio), $B_{N,0}$(one of the nearest satellite harmonics of L=1 field)with the combination of some field modifications improving magnetic well.

References

[1]YOKOYAMA, M.,NAKAMURA, Y.,WAKATANI, M.,AIZAWA, M.,SAITO, K.H.,SAITO, K.N.,KAWAKAMI,I.,SHIINA,S. , Proc.16th IAEA Fusion Energy Conference 2(1996)175.
[2]NUHRENBERG,J.,ZILLE,R.,Phys.Lett.A.129 (1988)113.
[3]GARABEDIAN,P.R., Phys.Plasmas 3(1996)2483.
[4]CARY,J.R. and SHASHARINA,S.G. ,Phys.Rev.Lett. 78(1997)674.
[5]SHIINA,S.,AIZAWA,M.,SAITO,K.H.,Proc.24th EPS on Controlled Fusion and Plasma Physics P2.088.
[6]SPONG,D.A.,HIRSHMAN,S.P.,WHITSON,J.C., Plasma Physics Reports 23 (1997)483.
[7]REIMAN,A.H.,BOOZER,A.H., Physics Fluids 27(1984)2446.
[8]AIZAWA,M.,SAITO,K.H.,SAITO,K.N.,KAWAKAMI,I.,SHIINA,S.,Proc.14th IAEA Conference on Plasma Physics and Controlled Thermonuclear Fusion Reseach 2(1992)439.