GLOBAL MODE ANALYSIS OF IDEAL MHD MODES IN L = 2 HELIOTRON/TORSATRON SYSTEMS

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Abstract

By means of a global mode analysis of ideal MHD modes for Mercier-unstable equilibria in a planar axis $L = 2/M = 10$ heliotron/torsatron system with an inherently large Shafranov shift, the conjecture from local mode analysis for Mercier-unstable equilibria has been confirmed and the properties of pressure-driven modes have been clarified. According to the degree of the decrease in the local magnetic shear by the Shafranov shift, the Mercier-unstable equilibria are categorized into toroidicity-dominant (strong reduction) and helicity-dominant (weak reduction) equilibria. In both types of equilibria, interchange modes are destabilized for low toroidal mode numbers $n < M$, where M is the toroidal field period of the equilibria, and both poloidally and toroidally localized ballooning modes purely inherent to three-dimensional systems are destabilized for fairly high toroidal mode numbers $n \gg M$, For moderate toroidal mode numbers $n \sim M$, tokamak-like poloidally localized ballooning modes with a weak toroidal mode coupling are destabilized in toroidicity-dominant equilibria, and in contrast, in the helicity-dominant equilibria, interchange modes are destabilized. The interchange modes are localized on the inner side of the torus, because the Shafranov shift enhances the unfavorable magnetic curvature there rather than on the outer side of the torus. A continuous or quasi-point unstable spectrum is briefly discussed.

1. INTRODUCTION

From the local mode analysis of high-mode-number ballooning modes in an $L = 2/M = 10$ planar axis heliotron/torsatron system with an inherently large Shafranov shift (where L and M are the polarity and toroidal field period of the helical coils, respectively)[1], it was previously conjectured[2] that the global structure of pressure-driven modes for Mercier-unstable equilibria would have the following properties: Tokamak-like poloidally localized ballooning modes or interchange modes appear when their typical toroidal mode numbers are relatively low. As the typical toroidal mode numbers become higher, ballooning modes inherent to three-dimensional systems appear with larger growth rates and localized in both the poloidal and toroidal directions.

The purposes of this work are to prove the above conjecture and to clarify the inherent properties of pressure-driven modes, through a global mode analysis of the ideal MHD modes.

2. CATEGORIZATION OF MERCIER-UNSTABLE EQUILIBRIA

The changes in the local mganetic shear and the normal magnetic curvature by the Shafranov shift are related to toroidicity. The Shafranov shift decreases the local magnetic shear on the outside of the torus, leading to the reduction of the field line bending stabilizing effect on ballooning modes[1]. On the other hand, the Shafranov shift enhances (reduces) the local unfavorable normal magnetic curvature on the inner (outer) side of the torus. According to the degree of the decrease in the local magnetic shear by the Shafranov shift, the Mercier-unstable equilibria can be categorized into two types, namely, toroidicity-dominant and helicity-dominant equilibria. The former (latter) equilibria are characterized by properties that the local magnetic shear is strongly (weakly) reduced by the Shafranov shift, so that ballooning modes are easy (difficult) to destabilize. The former (latter) equilibria are basically created with peaked (broad) pressure profiles under the currentless condition.

3. GLOBAL MODE ANALYSIS IN MERCIER-UNSTABLE EQUILIBRIA

The global mode analysis of ideal MHD modes in Mercier-unstable equilibria is done by using the CAS3D2MN code[3] with the shift-and-invert Lanczos algorithm[4]. The three-dimensional equilibria considered here have toroidal field period M, and this is what mainly determines the toroidal period of the local magnetic curvature due to helicity. Therefore, we investigate the properties of pressure-driven modes for various cases of relative magnitude between the typical toroidal mode number n of the perturbation and the toroidal period M of the local magnetic curvature due to helicity: namely, $n < M$, $n \sim M$, and $n \gg M$.

For low toroidal mode numbers $n < M$, interchange modes occur, which feel the average magnetic curvature, for both types of Mercier-unstable equilibria. One of them is shown in Fig. 1. The radial distribution of the Fourier components of the normal displacement $\vec{\xi} \cdot \nabla\psi$ is shown in Fig. 1(a) with the origin of the poloidal angle on the inner side of the torus. Three resonant mode structures with n = 4 are visible and the amplitudes of modes with different toroidal mode numbers are quite small. The mode structure is similar to that of a ballooning mode except that each Fourier mode has both positive and negative parts, which means that the perturbed pressure $\tilde{P} = -\nabla P \cdot \vec{\xi}$ due to this type of interchange mode has a tendency to extend radially on the inner side of the torus and to change phase in the radial direction on the outer side of the torus through poloidal mode coupling as shown in Figs. 1(b) and (c). In other words, this type of interchange mode is anti-ballooning with respect to the poloidal mode coupling. This is because the normal magnetic curvature is more unfavorable on the inner side of the torus than on the outer side of the torus by the Shafranov shift.

For moderate toroidal mode numbers $n \sim M$, the modes begin to feel the local structure of the magnetic curvature. Since the Shafranov shift strongly reduces the stabilizing effects due to the local magnetic shear on the outer side of the torus in the toroidicity-dominant Mercier-unstable equilibrium, tokamak-like poloidally localized ballooning modes with weak toroidal mode coupling occur. This is shown in Fig. 2 with the origin of the poloidal angle on the outer side of the torus. The typical toroidal mode numbers are still so small that the modes can not feel the local structure of the magnetic curvature effectively, and hence the toroidal mode coupling is weak. Three groups of Fourier modes for the normal displacement $\vec{\xi} \cdot \nabla\psi$ with different toroidal mode numbers are visible, namely, n = 22, n = 32, and n = 42 in Fig. 2(a). Each group, however, consists of many Fourier modes with different poloidal mode numbers caused by the poloidal mode coupling, so that the structure of each group due to the poloidal mode coupling is quite similar to that of ballooning modes in tokamak plasmas. The corresponding contours of the perturbed pressure $\tilde{P} = -\nabla P \cdot \vec{\xi}$ are shown in Figs. 2(b) and (c). In contrast, for the helicity-dominant equilibria with a weak reduction of the stabilizing term of ballooning modes by the Shafranov shift, interchange modes still occur as shown in Fig. 3. The toroidal mode coupling of interchange modes becomes stronger as the toroidal mode number increases, as shown in Fig. 3(a) where the origin of the poloidal angle is on the inner side of the torus. Just as for interchange modes with $n < M$, the interchange modes with $n \sim M$ also have a tendency to be radially extended on the inner side of the torus and to change phase in the radial direction on the outer side of the torus, as shown in Figs. 3(b) and (c).

For fairly high toroidal mode numbers $n \gg M$, the modes can easily distinguish the local fine structure of the magnetic curvature. This results in the appearance of ballooning modes inherent to three-dimensional systems for both types of Mercier-unstable equilibria. These ballooning modes have such strong poloidal and toroidal mode couplings as to localize in both the poloidal and toroidal directions as shown in Fig. 4. There are eight groups of Fourier modes for $\vec{\xi} \cdot \nabla\psi$ with different toroidal mode numbers as shown in Fig. 4(a), where the origin of the poloidal angle is on the outer side of the torus. Neighboring groups of Fourier modes have opposite phase to each other, just as in

Fig. 2(a). This relative phase difference of the neighboring groups leads to the clear localization of the perturbed pressure in the toroidal direction, as shown in Figs. 4(b) and (c). On the outer side of the torus, the perturbed pressure, which localizes on the horizontally elongated poloidal cross section with the locally unfavorable magnetic curvature at the outside of the torus, almost disappears on the vertically elongated poloidal cross section with the locally favorable magnetic curvature at the outside of the torus. Moreover, the strong toroidal mode coupling causes a type of poloidal localization that is different from the kind only due to poloidal mode coupling as shown in Fig. 4(d). It shows the corresponding contours of the perturbed pressure on the (θ, ζ) plane with one period in the poloidal direction and one-tenth of a period (one field period) in the toroidal direction at the normalized radial coordinate r = 0.734. In Fig. 4(d), it can be seen that regions where the perturbed pressure has large amplitude (indicated by dark diagonal stripes) alternate with regions of quite small amplitude (denoted by white diagonal stripes). Judging from the value of the rotational transform on this flux surface, namely, ι = 0.58, we conclude that these high-amplitude and low-amplitude stripes are aligned along magnetic field lines, and that the strong toroidal mode coupling in addition to the poloidal mode coupling makes the perturbation be localized on selected flux tubes.

4. DISCUSSIONS

Since the interchange modes basically localize along mode rational magnetic field lines driven by the average unfavorable magnetic curvature, the toroidal mode coupling merely influences the localization in the toroidal direction but does not have an essential affect on the magnitude of the eigenvalues. This may lead to the existence of a narrow continuous or quasi-point unstable spectrum. In contrast, the ballooning modes basically localize near the locally unfavorable magnetic curvature, so that toroidal mode coupling significantly influences the eigenvalues. Thus, ballooning modes may be unable to have a continuous or quasi-point unstable spectrum, except perhaps for case with extremely high toroidal mode numbers ($n \to \infty$).

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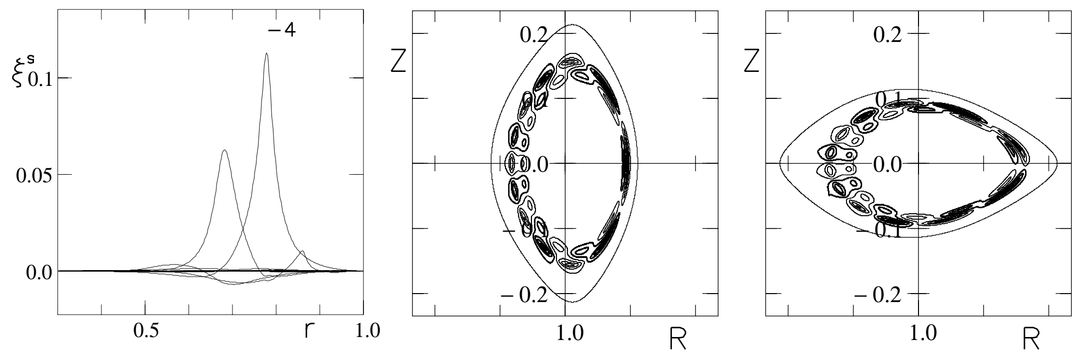


Fig. 1 (a) $\vec{\xi}\cdot\nabla\psi$ vs. r with the dominant toroidal mode number, and the corresponding contours of $\tilde{P} = -\nabla P\cdot\vec{\xi}$ on the vertically (b) and horizontally (c) elongated poloidal cross sections.

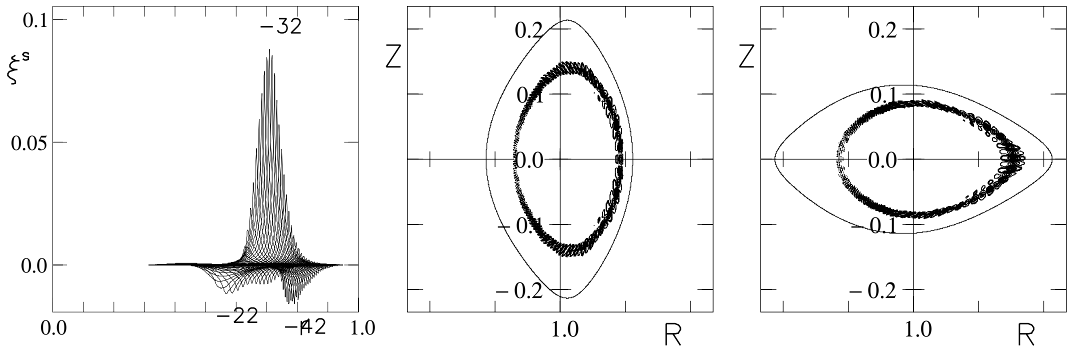


Fig. 2 The same quantities as in Fig. 1 for $n \sim M$ in a toroidicity-dominant equilibrium.

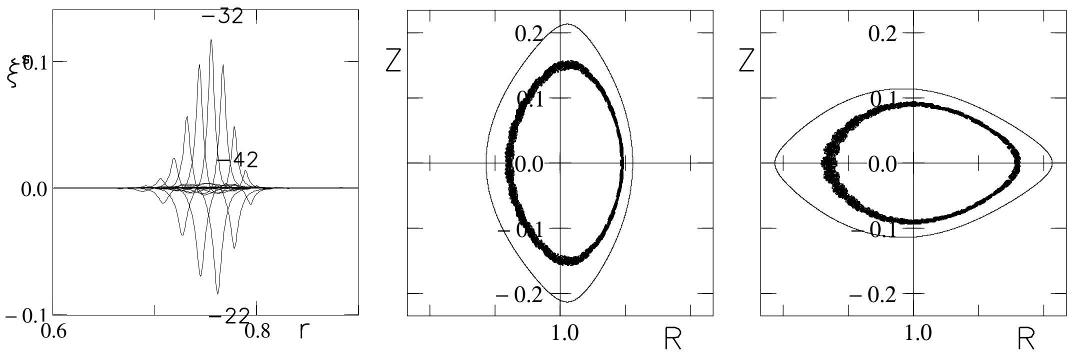
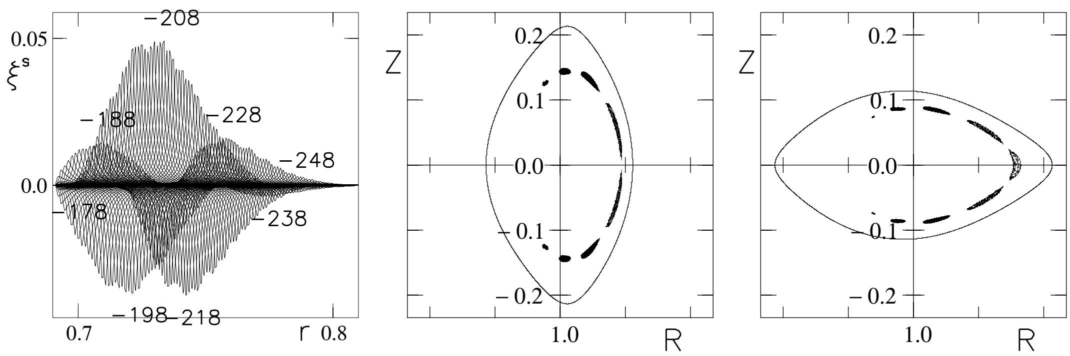


Fig. 3 The same quantities as in Fig. 1 for $n \sim M$ in a helicity-dominant equilibrium.



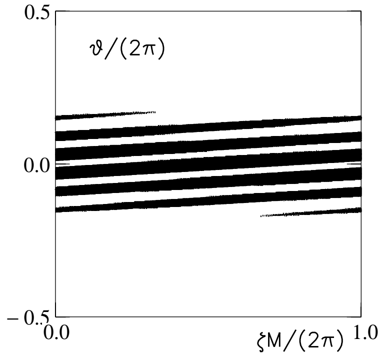


Fig. 4 (a) $\vec{\xi} \cdot \nabla\psi$ vs. r with the dominant toroidal mode numbers, and the corresponding contours of $\tilde{P} = -\nabla P \cdot \vec{\xi}$ on the vertically (b) and horizontally (c) elongated poloidal cross sections, and (d) on the (θ, ζ) plane at $r = 0.734$, where the Fourier mode with $n = 208$ has its maximum amplitude.