DIMENSIONLESS ENERGY CONFINEMENT SCALING IN W7-AS

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Abstract

Energy confinement in W7-AS has been analyzed in terms of dimensionally exact form free functions employing Bayesian probability theory. The confinement function was set up as a linear combination of dimensionally exact power law terms as already proposed very early by Connor and Taylor. Generation of this expansion basis is dictated by the basic plasma model which one assumes. Based upon data accumulated in W7-AS, which contains the energy content for a wide variety of variable settings, predictions for single variable scans are made. The scaling functions for density and power scans, respectively, are in quantitative agreement with data collected in W7-AS. The result of a single variable scan is therefore already hidden in the data obtained for arbitrary variable choices and can be extracted from the latter by a proper data analysis. Furthermore, the optimal model for the description of the global transport in W7-AS is identified as the collisional low beta kinetic model.

1. INTRODUCTION

Fusion plasma behavior has been described for about twenty years by energy confinement scaling functions [1]. Such confinement relations serve presently primarily two purposes: first, they constitute a convenient summary of machine operation. This allows inter-machine comparisons and the characterization of conditions for enhanced confinement regimes. Second, energy confinement scaling provides the basis for the design of future experiments such as ITER representing the tokamak line or W7-X and LHD in the stellarator branch.

Confinement scaling relations have been used to predict L-mode tokamak performance with some success [2]. This is notable for several reasons. First, the popular power law functional form of the confinement scaling function was originally assumed for reasons of convenience and simplicity and lacks a physical foundation. This initial choice has subsequently been justified by a surprisingly good characterization of data trends. There have also been, now and then, attempts at improved data representation by more complicated functions as for example the class of offset linear scalings [3]. Since they were not really superior to a single power law term, the latter has accumulated considerable credit just by experience.

Another major shortcoming of the unconstrained power law scaling function is its dimensional incorrectness. Connor and Taylor [4] tried to interpret experimental scaling functions, established by Hugill and Sheffield [1], in terms of constraints derived from the requirement of physical invariance under similarity transformations of the basic equations describing plasma behavior. These attempts were accompanied by considerable frustration since they found that

CT model	x_1	x_2	x_3	$p(M_j\|\boldsymbol{W^{exp}},\boldsymbol{\sigma},I)$
1. collisionless low-β	x	0	0	$4\times10^{-12}\%$
2. collisional low-β	x	y	0	99.7%
3. collisionless high-β	x	0	z	0.25%
4. collisional high-β	x	y	z	0.025%

Table I: Parameter of the Connor Taylor kinetic plasma models

the experimental scaling was incompatible with any of their plasma models. Based on this experience they suggested that the theoretically derived dimensional constraints be incorporated directly in the power law ansatz. This proposal has been followed subsequently on many occasions [5]. It consists of expressing the energy content W of the toroidal magnetic confinement device by [4]

$$\begin{aligned} W^{theo} &= cna^4RB^2\left(\frac{P}{na^4RB^3}\right)^{x_1}\left(\frac{a^3B^4}{n}\right)^{x_2}\left(\frac{1}{na^2}\right)^{x_3} \\ &= c' g\,(n,B,P,a;\mathbf{x})\ , \end{aligned} \tag{1}$$

where n is the average density, a and R minor and major radius of the torus, B the magnetic field and P the deposited heating power. The particular values of x_1, x_2, x_3 specify the plasma kinetic model as shown in Table I. Since we concentrate in this paper on data from a single machine parameters which are constant within the examined data set (e.g. R) are absorbed in c'. The number of degrees of freedom in (1) varies – depending upon the model – between one and three, whereas the unconstrained ansatz for a single device with $R = const$ would have four (n, B, P, a). Imposing physical constraints on the power law ansatz reduces the flexibility and leads necessarily to an increased misfit. We will demonstrate in this paper, however, that a dimensionally exact power law type of ansatz can be formulated which leads to a significantly reduced misfit.

Interestingly enough, already Connor and Taylor [4] proposed to express a general form free energy confinement scaling function as a series of terms of the form (1) for properly chosen $\mathbf{x}_k$ with expansion coefficients c_k. In mathematical terms this is nothing but the expansion in a basis which is dimensionally exact. In this paper we shall exploit this suggestion.

2. DIMENSIONALLY EXACT BASIS FUNCTION

Consider a set of measurements of the plasma energy content for N different values of the experimental input variables (n, B, P, a). We represent the theoretical prediction for the energy content by an N-dimensional vector $\boldsymbol{W^{theo}}$ which may be described for all plasma models by

$$\boldsymbol{W^{theo}} = \sum_{k=1}^{N} c_k \boldsymbol{f}\,(\mathbf{x}_k)\ . \tag{2}$$

The i-th component of the expansion vector $\boldsymbol{f}\,(\mathbf{x}_k)$ corresponds to the i-th measurement and reads according to (1) $f_i(\mathbf{x}_k) = g\,(n_i, B_i, P_i, a_i; \mathbf{x}_k)$. In general, N such linearly independent vectors form a complete basis in the N-dimensional data space and would therefore allow a pointwise reconstruction of the data. This is neither desirable, nor with respect to physics correct, since the corresponding vector of measured energy contents $\boldsymbol{W^{exp}}$ is corrupted by noise. What we really want is an expansion, truncated at some appropriate upper limit E and describing the physics, while the residual $N-E$ terms in the expansion (2) fit only noise. Further, we would like to identify the plasma physics model which describes the data best. An important topic are single variable scans (e.g. the variation of the energy content as function of the density with all other variables fixed) which constitute a very stringent test on any energy confinement function. Such scans are not directly accessible from published databases. On the other hand, single variable scans are experimentally cumbersome, and expensive experiments have to be performed for each and every input variable of interest. It is therefore highly desirable to extract single variable scans from existing databases by employing improved data analysis techniques. A comprehensive answer to these and other questions is provided by Bayesian probability theory [6]. While the explicit calculation is explained in greater detail in [7] we give here only a short presentation of the results.

3. RESULTS

The data which we have used in our calculations are the 153 $\iota \approx 1/3$ W7-AS data from the international stellarator energy confinement data base [8] (ι: rotational transform). We have selected the $\iota \approx 1/3$ data only since the single variable scans which we shall present have been performed at this value of the rotational transform.

The calculated model probabilities are depicted in the last column of table I. We see that the $\iota \approx 1/3$ W7-AS data are best described by the collisional low beta Connor Taylor model. The high beta models follow in second and third place with much lower probability. The collisionless low beta model is clearly inappropriate to describe the transport physics in W7-AS.

To obtain this result only up to three terms in the expansion (2) were used. Though the inclusion of higher terms leads to a further decrease of the misfit between data and model prediction the probability for a given E decreases rapidly, so that contributions of higher expansion orders become very small. This is a demonstration of Occam's razor automatically included in Bayesian theory. This principle dictates that a simpler model should be preferred unless a more complicated one leads to a substantially better fit to the data. The present optimum three term expansion reduces the misfit to about 65% of its initial value, where the initial value corresponds to a simple one term least squares fit. The possibility for such an effect has previously been pointed out by Kaye et al. [2].

Finally we show the result for density and power scans obtained on the one hand from the present theory and on the other hand from experiments at W7-AS. Because these data are not included in the stellarator confinement data base $\boldsymbol{W^{exp}}$ the analysis is based on, this test shows the predictive power of our approach.

The full circles in Fig. 1a represent experimental results for the density scan. Representative error bars signify the precision level of these data. The continuous curve depicts the result of the present semi-empirical theory along with the confidence range indicated by the gray shaded area. The stellarator energy confinement data base is represented by the open circles which are spread all over since they were obtained for various settings of the variables (B, P, a). The histogram at the base line indicates the number of shots at the respective density and gives an impression about the range our result for the single variable scan is best supported by the data base. Last but not least the dashed curve represents the density dependence as inferred from an unrestricted single power law conventional least squares fit resting on the same data $\boldsymbol{W^{exp}}$ as our Bayesian result. It hits the progression of the data at two points only, while staying out of the data scatter (of the full circles) most of the time. Within the density range of the single variable scan the prediction of the semi-empirical theory runs straight through the data and exhibits clearly the previously supposed density saturation [8, 9], which can never be obtained by a single power law term at all. Outside this range the data set $\boldsymbol{W^{exp}}$ is too sparse, which is reflected in the rapidly widening error band. In contrast to the robust but erroneous power law scaling the present theory indicates where the extrapolation becomes unreliable. It might be an unfortunate but honest conclusion that an extrapolation beyond the parameter regime

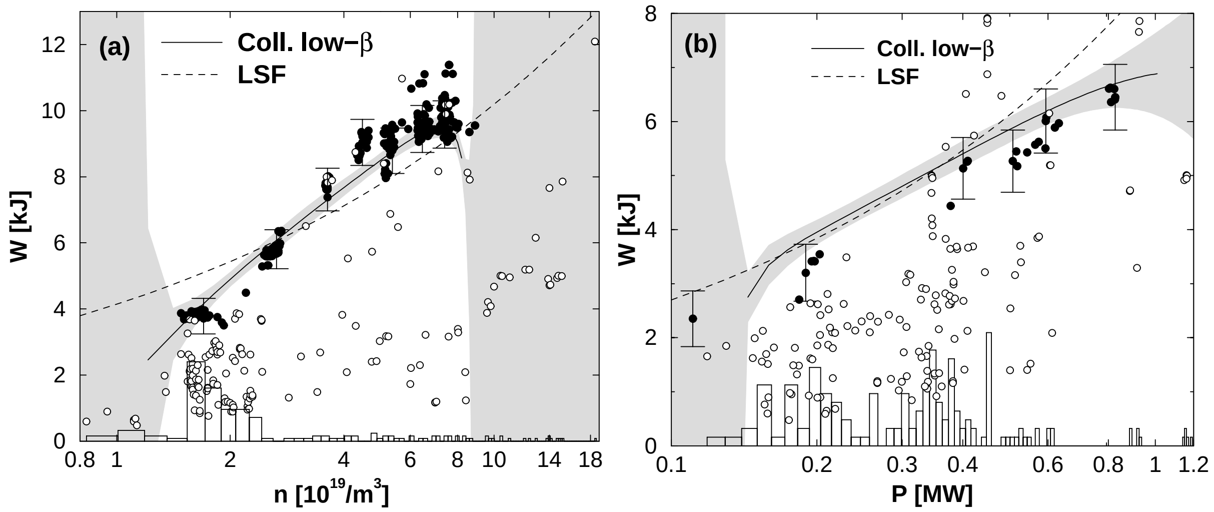

Fig. 1: a) Experimental results of a single variable density scan (full circles) compared to the predictions of the present semi-empirical theory (continuous line, shaded area represents the error) for $B = 2.5T$, $P = 0.45MW$, $a = 0.176m$. The input data (open circles) are shown regardless of additional variations in B, P, a and are therefore spread all over. The histogram accounts for their distribution over the density axis. A least squares fit (LSF) of the input data would yield $W \sim n^{0.39}$ (dashed line). b) Single variable power scan with $n = 2.4 \cdot 10^{19}\mathrm{m}^{-3}$, $B = 2.5T$, $a = 0.176m$. A least squares fit of the input data would yield $W \sim P^{0.5}$ (dashed line).

supported by the data base is not possible. However, one has to consider that the seeming predictability of the commonly used power law scaling performs even worse, producing an ever increasing function which misses the saturation entirely. Note that the comparison between the single variable scan and the prediction of our analysis holds on absolute scales! Neither in Fig. 1a nor in the by-standing Fig. 1b adjustable scale parameters are necessary. This means that experiments in W7-AS have an impressive reproducibility.

Fig. 1b displays a similar comparison for a power scan in W7-AS. Again the semi-empirical theory shown as the continuous line predicts the measured energy content - on an absolute scale - within experimental error and corroborates the experimentally observed power degradation. The dashed curve is from a power law fit and is, as for the density scan, convex while the present model shows concave dependence in both cases.

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