DYNAMICS OF ZONAL FLOWS AND SELF-REGULATING DRIFT-WAVE TURBULENCE

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Abstract

We present a theory of zonal flow - drift wave dynamics. Zonal flows are generated by modulational instability of a drift wave spectrum, and are damped by collisions. Drift waves undergo random shearing-induced refraction, resulting in increased mean square radial wavenumber. Drift waves and zonal flows together form a simple dynamical system, which has a single stable fixed point. In this state, the fluctuation intensity and turbulent diffusivity are ultimately proportional to the collisional zonal flow damping. The implications of these results for transport models is discussed.

1. INTRODUCTION

Zonal flows[1] are poloidally and toroidally symmetric $\left(k_\theta = k_z = 0\right)$, zero-frequency vortex modes with finite radial scale $\left(k_r \text{ finite}\right)$, and thus constitute a limiting case of the more general

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notion of a "convective cell"[2]. Since zonal flows are azimuthally symmetric, they are unable to directly tap expansion free energy stored in radial gradients and are thus excited exclusively via nonlinear processes, such as "inverse cascade" of drift wave turbulence. Zonal flows are of great significance to confinement physics, since they are, effectively, sheared $\underline{E} \times \underline{B_o}$ flow layers which strain and distort the drift waves they co-exist with[3]. Zonal flows have been observed in fluid, gyrofluid[4] and gyrokinetic[5] simulations of drift wave turbulence and in tokamak plasmas[6].

Noting that a spectrum of zonal flow shear layers are excited by the inverse cascade of drift waves, it is reasonable to suggest that the zonal flows constitute an ensemble of random shears which regulates the $k_\theta \neq 0$ component of the drift wave turbulence. This suggestion is confirmed by recent simulation results, which indicate that zonal flow shearing is the principle saturation mechanism operating in drift wave turbulence. Thus, generic drift (or drift-ITG) turbulence may be modeled as a self-regulating, two component system, consisting of:

a.) drift waves (with $k_\theta \neq 0$), which cause anomalous transport, and for which $\tilde{n}/n \sim e\tilde{\phi}/T$.

b.) zonal flows with $k_\theta = 0$, for which $\tilde{n}/n \sim \left(k_\perp^2 \rho_s^2\right) e\tilde{\phi}/T$, which share available gradient free energy.

The confinement regime quality is thus characterized by the branching ratio of zonal flow and drift wave energy. In particular, recent theoretical work has suggested that gyrofluid transport models over-estimate the long-time $\left(\omega_{bi} t > 1\right)$ damping of zonal flows, and thus over-predict levels of ITG-mode driven transport[7].

In this paper, we summarize recent developments in the theory of zonal flows and self-regulating drift wave turbulence. Section 2 presents the linear theory of zonal flows and discusses their damping by collisional processes. Section 3.1 addresses zonal flow generation. In particular, a gas of drift waves is shown to be unstable to modulation by a seed shear flow, which is subsequently amplified. Section 3.2 discusses the feedback of stochastic zonal flow shears on the drift wave spectrum. In particular, zonal flow shearing induces random refraction of drift waves, thus enhancing their coupling to small scale dissipation. Section 4 discusses the coupled system of equations for the zonal flow and drift wave spectra as a dynamical system in order to elucidate the states (fixed points) of self-regulating drift wave turbulence. Saturated fluctuation levels and transport coefficients are estimated. Section 5 contains a discussion and conclusions.

2. LINEAR THEORY OF ZONAL FLOW DYNAMICS

We have previously considered the decay of poloidal rotation as induced by neo-classical effects. We find that the rotation induced by a source at $t = 0$ can be given by $\phi(t) = \phi(0)k(t)$. $k(t)$ varies on two distinct time scales. For times less than the ion bounce time, ion Landau damping induces a rapid decay (possibly damped oscillations due to the excitation of Geodesic Acoustic Modes). However, this damping is incomplete and $k(t)$ approaches a value $.6\varepsilon^{1/2}/qze\tilde{\phi}/T$ which can decay further only via collisions for times longer than the ion bounce time, [8].

We note that Hammett-Perkins type gyro-fluid models as used in IFS-PPPL "predictions"[9] do not distinguish between time scales but are characterized by complete collisionless damping of zonal flows, thus possibly leading to unrealistically small values of damping and thus too high a drift wave level and predicted transport. Indeed more accurate GK codes are characterized by lower transport, and the near threshold transport is proportional to collision frequency as predicted here.

Note that for zonal flows interacting with an incoherent source such as ITG turbulence ϕ is of the form $\phi = \int k(t')S(t-t')dt'$ and the growth of $|\phi|^2$ is given by $\dfrac{d\phi^2}{dt} = \int_{-\infty}^{\infty} d\tau k(0)k(\tau)\langle S(0)S(\tau)\ \rangle$.

If the correlation time of S is long compared to an ion bounce time, as would be true not too far from threshold, then it is the long time scale collisional behavior of K which is of importance. For our purposes we will thus simply assume a simple collisional damping of the zonal flows.

3. NONLINEAR DYNAMICS OF ZONAL FLOWS

3.1 Generation

We seek an equation for the zonal flow intensity $U = \left|\phi_q\right|^2$ of the form:

$$\frac{\partial}{\partial t}U + \gamma_d U = [\text{Growth}]\ \text{U} \ + \ [\text{Noise}].$$

Here γ_d refers to the collisional damping discussed in the previous section, [Growth] U refers to amplification due to drift-wave coupling and [Noise] refers to incoherent emission of drift wave energy into the zonal flow. This section is concerned with the calculation of [Growth] and [Noise]. Zonal flow growth may be elucidated by considering the question of whether or not an ensemble or "gas" of drift waves is unstable to a shear perturbation. Since the drift wave gas must maintain a divergence-free radial current (composed of polarization and transport-induced currents) and since the zonal shear flows modulate the transport-induced current, it follows that zonal flow stability is determined by:

$$\rho^2 \frac{\partial}{\partial t}\left(\partial r^2 \phi\right) + \frac{\partial}{\partial r}\left[\frac{\delta\langle \text{J}_\text{r}\rangle}{\delta\phi}\phi\right] = 0\,. \tag{1}$$

Here ϕ is the zonal flow potential, $\langle \text{J}_\text{r}\rangle$ is the transport-induced radial current and ρ is the polarization screening length (i.e. $\rho = \rho_\text{s}$ for classical screening, $\rho = \left(1.6\varepsilon^{3/2}\right)^{1/2}\rho_\theta$ for neoclassical screening). The transport-induced current is caused by the local non-ambipolarity of the underlying instabilities (i.e. recall quasi-neutrality *does not* imply local ambipolarity)[10]. Since the radial current is simply the difference of electron and ion radial flux, it follows that the frequency Ω_q of a zonal flow mode with radial wavenumber q is

$$\Omega_\text{q} = -\left(q^2\rho^2\right)^{-1}\left(q\delta\Gamma_{-}\big/\delta\phi_\text{q}\right). \tag{2}$$

Here Γ_{-} is the relative flux. Γ_{-} is very detail-sensitive, since it involves the dissipative couplings of the various species. However, the generalized quasi-linear wave energy theorem (Poynting theorem) directly relates Γ_{-} to the radial wave energy density flux at stationarity. Thus, the zonal flow frequency can be written as:

$$\Omega_\text{q} q^2\rho^2 = -i\,q^2\frac{\delta}{\delta\phi_q}\left[\rho_s c_s \sum_{\underline{k}} k_\theta \left.\frac{\partial\in}{\partial k_r}\right|_{\omega_{\underline{k}}} \left|\frac{e\tilde{\phi}_{\underline{k}}}{T}\right|^2\right]. \tag{3}$$

Here $\underline{\text{k}}$ and $e\tilde{\phi}_{\underline{k}}\big/T$ refer to drift waves $\left(k_\theta \neq 0\right)$ and $\in$ is the difference of the real part of the ion and electron susceptibilities. For simple drift wave models, $\partial\in\big/\partial k_r \sim -k_r\rho_s^2$ so

$\Omega \sim k_\theta k_r |e\tilde{\phi}/T|^2 \sim \langle \tilde{v}_r \tilde{v}_\theta \rangle$, the Reynolds stress. Thus, zonal flows are seen to arise from modulations of the drift wave Reynolds stress.

Further progress is facilitated by recognizing that the separation between the spatio-temporal scales of the zonal flows and those of the drift waves suggests calculating the modulation using methods from adiabatic theory. The appropriate adiabatic invariant for drift wave turbulence[11] in azimuthally symmetric shear flows is

$$N(\underline{k}) = \left(1 + k_\perp^2 \rho_s^2\right)^2 \left| \frac{e\tilde{\phi}_{\underline{k}}}{T} \right|^2 . \tag{4}$$

Note that N is, in general, the (conserved) potential enstrophy Ω and is equal to the classical action only for the special case of zonal flow-drift wave interaction, for which k_θ is constant. Thus, the zonal flow growth rate may be written as

$$\Omega_q \left(q^2 \rho^2\right) = -i\, q^2 \rho_s c_s \sum_{\underline{k}} \left[\frac{k_\theta \partial \chi / \partial k_r}{\left(1 + k_\perp^2 \rho_s^2\right)^2} \right] \frac{\delta N}{\delta \phi_q} . \tag{5}$$

The quantity $\delta N/\delta\phi$ may be straightforwardly computed via linearization of the wave kinetic equation

$$\frac{\partial}{\partial t} N + \left(\underline{\mathrm{v}}_g + \underline{V}\right) \cdot \underline{\nabla} N - \frac{\partial}{\partial \underline{x}} (\omega + \underline{k} \cdot \underline{V}) \cdot \frac{\partial N}{\partial \underline{k}} = \gamma(k) N - \left(\Delta\omega(k)\right) N^2 . \tag{6}$$

Here $\underline{\mathrm{v}}_g$ is the drift wave group velocity, $\underline{V}$ is the zonal flow, $\gamma(k)$ is the drift wave growth rate and $\Delta\omega(k) N^2$ represents a damping of drift wave quanta due to local nonlinear coupling to damped scales. Linearizing Eqn. [6] finally yields the zonal flow growth rate

$$\gamma_q = -\frac{\rho_s^2}{\rho^2} q^2 c_s^2 \sum_{\underline{k}} \left[\frac{\left(k_\theta^2 \rho_s^2\right)\left(k_r \partial \langle N \rangle / \partial k_r\right)}{\left(1 + k_\perp^2 \rho_s^2\right)^2} R(\underline{k}, q) \left(1 - \frac{q^2 \rho^2}{1 + k_\perp^2 \rho_s^2}\right) \right], \tag{6a}$$

where $R(\underline{k}, q)$ is the resonance function

$$R(\underline{k}, q) = \gamma_{\underline{k}} \Big/ \left(\gamma_{\underline{k}}^2 + \left(\Omega_q - q V_g(k)\right)^2\right), \tag{6b}$$

and V_g refers to the radial component of the wave group velocity. Note that in practice, re $\Omega_q \sim (e\phi/T)^2$, so the zonal flow may be effectively regarded as a zero frequency mode.

Several features of this result merit discussion. First, note that $\gamma_q > 0$ for $\partial\langle N \rangle / \partial k_r < 0$ - no population inversion is required for modulational instability, in contrast to the familiar case of Langmuir turbulence. This is a symptom of the inverse cascade which occurs in a quasi-geostrophic fluid. In this vein, it is interesting to observe that $\gamma_q \sim q^2 D - q^4 \mu$, i.e. the growth rate can be written as the difference of a negative viscosity at larger scales and a positive hyper-viscosity at smaller scales. Two rather similar length parameters control the zonal flow scale - namely the polarization scale ρ and the wave mean free path V_g/γ. Also, it should be noted that for the relevant case of neoclassical polarization, $\gamma \sim B_\theta^2/B_T^2$ - i.e. poloidal field scaling appears explicitly through the zonal flow growth rate. Since zonal flows do not drive transport themselves and regulate drift waves by random shearing, the scaling of γ with B_θ is clearly favorable. Finally, it is useful to note that an estimate for the zonal flow growth rate (for classical polarization) is $\gamma_q \sim \left[\left(q^2 \rho_s^2\right)\left(c_s/L_\perp\right)/\delta\right]\langle n\rangle$ where $\langle n\rangle = \langle N\rangle/\left(\rho_s^2/L_\perp^2\right)$. We have taken $k\rho_s \sim 1$, δ is the non-adiabatic electron phase shift, and $L_\perp$ is the perpendicular scale length. A power-law wave number spectrum is assumed.

The noise emitted into the zonal flow may be determined by calculating the incoherent mode coupling into modes with q_r finite and $q_\theta = q_z = 0$. For a simple plasma model (classical polarization), the calculation is most efficiently done using two dimensional hydrodynamics. The result is:

$$\frac{\partial}{\partial t}\left|\phi_q\right|^2 \Bigg|_{\text{noise}} = \left(q^2 c_s^2\right) \sum_{\underline{k}} \rho_s^4 \frac{\left(k_r^2 k_\theta^2\right)^2}{\left(k_\perp^2\right)^2} \frac{\langle N\rangle^2 R(\underline{k},q)}{\left(1+k_\perp^2\rho_s^2\right)^2}. \tag{7}$$

Note that the noise resembles the square of the Reynolds stress, as it should. The zonal flow spectrum equation thus finally is:

$$\left(\frac{\partial}{\partial t} + \gamma_d\right)\left|\phi_q\right|^2 = \left(-q^2 c_s^2 \sum_{\underline{k}} \frac{\left(k_\theta^2 \rho_s^2\right) f(q\rho)}{\left(1+k_\perp^2\rho_s^2\right)^2} k_r \frac{\partial\langle N\rangle}{\partial k_r} R(\underline{k},q)\right)\left|\phi_q\right|^2$$
$$+ q^2 c_s^2 \sum_{\underline{k}} \rho_s^4 \frac{\left(k_r^2 k_\theta^2\right)^2}{\left(k_\perp^2\right)^2} \frac{\langle N(\underline{k})\rangle^2}{\left(1+k_\perp^2\rho_s^2\right)^2} R(\underline{k},\mathrm{q}). \tag{8}$$

3.2 Feedback on Drift Waves

Zonal flows shear and distort the drift wave spectrum which drives them. Thus, the zonal flows constitute a random strain field which randomly refracts drift waves, causing a diffusive increase in k_r which in turn enhances their coupling to small scale dissipation. The random shear by zonal flows can be constructively contrasted to mean flow shear by the observation that in the former case, $\left\langle \delta k_r^2 \right\rangle \sim D\tau$ while for the latter $\delta k_r \sim k_\theta V'_E \tau$ [12]. The *k*-space diffusion coefficient may be straightforwardly obtained by a quasi-linear analysis of the wave kinetic equation [Eqn. 6]. The quasi-linear equation is:

$$\frac{\partial\langle N\rangle}{\partial t} - \frac{\partial}{\partial k_r} D \frac{\partial\langle N\rangle}{\partial k_r} = \gamma(\underline{k})\langle N\rangle - \Delta\omega(\underline{k})\langle N\rangle^2. \tag{9a}$$

Here, the wave-vector diffusion coefficient is:

$$D = \sum_q k_\theta^2 q^2 \left(1 - \frac{q^2\rho^2}{1+k_\perp^2\rho_s^2}\right)^2 \left(\rho_s c_s q\right)^2 R(\underline{k},\mathrm{q})\left|\phi_q\right|^2. \tag{9b}$$

Anticipating the later need for estimates, a OD "toy model" of Eqns. [9] may be rigged up to be:

$$\frac{\partial\langle n\rangle}{\partial t}+\alpha|\phi|^2\langle n\rangle=\frac{c_s}{L_\perp}[\delta-\langle n\rangle]\langle n\rangle, \tag{10}$$

where $\alpha=\left[\left(q^2 c_s^2\right)\left(q^2\rho_s^2\right)/\delta c_s/L_\perp\right]$.

The physical process of work here is (random) shearing by a spectrum of zonal flows, which in turn refract k_r to higher values, where the drift wave packets are damped. The signature of this random-shearing-induced increase in the mean square k_r $\left(\bar{k}_r^2\right)$ is, of course, diffusion in k_r, i.e. $\left\langle\delta k_r^2\right\rangle=Dt$. The k_r-diffusion coefficient can be recovered from a simple random walk argument, i.e.

$$\frac{d}{dt}\delta k_r=\frac{-\partial}{\partial x}\left(k_\theta\tilde{V}_E\right),$$

so that integration along unperturbed rays yields D. It should be noted that while random shearing leads to a random walk in k_r (i.e. $\delta k_r\sim(D\tau)^{1/2}$, in comparison to deterministic shearing where $\delta k_r\sim\tau$), it is also the case that $\left\langle\tilde{V}'_{E^2}\right\rangle>>\left(\left\langle V_E\right\rangle'\right)^2$, on account of the contributions from smaller scale shear layers. Hence, it is not surprising that random shearing is a strong effect. Also, since the sum of drift-wave energy and zonal flow energy is conserved, the zonal flow growth rate may be calculated directly using the quasi-linear wave kinetic equation (Eqn. 9).

4. DYNAMICS OF DRIFT WAVE SELF-REGULATION

Equations [8] and [10], for the zonal flow and drift wave intensities, constitute the principal result of this paper. Obviously, these are coupled integro-differential equations, which require numerical solution. A tractable, zero-dimensional model for $|\phi|^2=U$ and $\langle n\rangle$ may be constructed from Eqs. [8, 10] by taking $k_\perp\rho_s=1$ and q to be a typical zonal flow scale. The OD analogue is:

$$\frac{\partial U}{\partial t}+\gamma_d U=\sigma\langle n\rangle U+[\text{Noise}], \tag{11a}$$

$$\frac{\partial n}{\partial t}=\frac{c_s}{L_\perp}[\delta-\langle n\rangle]\langle n\rangle-\alpha\langle n\rangle U. \tag{11b}$$

Here $\gamma_d\approx\nu_{ii}/\sqrt{\in}$, $\sigma\sim\left(q\rho_s\right)^2 c_s/L_\perp\delta$ as defined at the end of Section (IIIa), α is defined at the end of Section (IIIb) and the noise is defined in Section (IIIa). Note that these equations have the predator (U) - prey $(\langle n\rangle)$ structure. In the absence of noise, Eqs. [11a, b] are isomorphic to a heuristic model of the $L\to H$ transition, proposed previously[15]. In the absence of noise, the OD model given above has two fixed points, which are a no-flow state with $U=0$, and mean (drift wave) fluctuation level $\left|e\tilde{\phi}/T\right|^2\sim\delta\left(\rho_s/L_\perp\right)^2$, as well as a finite flow state with

$\left|e\tilde{\phi}/T\right|^2 \sim \delta\left(\rho_s/L_\perp\right)^2\left(\gamma_d/c_s/L_\perp\right)\left(\overline{k_\theta^2}/\overline{q^2}\right)$. Note this fluctuation level is set by the requirement of zonal flow marginality. The main effect of noise is to destabilize the no-flow state and to slightly shift the finite-flow fixed point. Hence, the only surviving stable fixed point is the finite flow state with:

$$|\tilde{n}/n|^2 \sim \delta\left(\rho_s/L_\perp\right)^2\left[\left(\gamma_d/c_s/L_\perp\right)\right]\left(\overline{k_\theta^2}/\overline{q^2}\right), \tag{12a}$$

which corresponds to a thermal diffusivity

$$\chi \sim D_{GB}\left[\delta^2 \frac{\gamma_d}{\left(c_s/L_\perp\right)} \frac{\overline{k_\theta^2}}{\overline{q^2}}\right]. \tag{12b}$$

Here $D_{GB} = \rho_s^2 c_s/L_\perp$, the gyro-Bohm diffusion coefficient. *A striking feature of these results is the explicit proportionality of the fluctuation level and transport coefficient to the collisional flow damping* $\gamma_d \sim \nu_{ii}$. This is a consequence of the fact that the flow is both driven by, and also regulates the strength of, the drift wave spectrum. Hence, increased collisional damping of the zonal flow makes it more difficult to excite the flow and to saturate the drift waves, thus leading to increased transport. It should be noted that the proportionality of χ to γ_d is consistent with results from recent gyrokinetic simulations and was previously suggested (though not demonstrated) by Zakharov and collaborators[14]. Finally, it should be clear that since the IFS-PPPL transport model over-estimates the zonal flow damping, it thus over-predicts the resulting transport.

It is impossible to over-emphasize the fact that this analysis depends heavily on the methodology of perturbation theory and thus will certainly lose its validity far from marginal stability. A symptom of failure would be the onset of equality between non-local and local (i.e. strong turbulence) interactions, thus vitiating the scale separation ordering fundamental to this work. Also, since the zonal flows tend to hover near marginality, the next-higher-order interaction, must be calculated. Since direct zonal flow - zonal flow interaction vanishes $\left(\underline{k}\cdot\underline{k}'\times\hat{z}=0!\right)$, the relevant coupling is mediated by drift waves, and corresponds to a zonal flow - drift wave - zonal flow interaction (6^{th} order). These will be discussed in a future publication.

5. SUMMARY AND DISCUSSION

In this paper, we have demonstrated that a drift wave "gas" is modulationally unstable to "test-shear" perturbations, resulting in the amplification of a spectrum of zonal flows. For $\omega_{bi}t>1$, linear zonal flow damping is due to collisions, only. The zonal flows feed back on the wave spectrum by random shearing of drift wave packets. Thus, the drift wave intensity $\langle n\rangle$ and zonal flow intensity U constitute a self-regulating, predator-prey system. When noise is included, this dynamical system has a single, stable fixed point, at which the drift wave spectrum adjusts to keep the zonal flow at (modulational) marginal stability. In this state, mean fluctuation levels and transport are ultimately proportional to the (*collisional*) zonal flow damping. Thus, an over-estimate of flow damping will result in an over-estimate of transport.

Further work on this model is clearly necessary. The coupled spectral equations require numerical solution. Since zonal flows are quasi-marginal, 6^{th}-order zonal flow - drift wave - zonal flow interactions should be calculated, as well. Transport estimates should be reconsidered in order to reflect the random Doppler shift produced by zonal flows, which will broaden the wave-particle resonances.

Other directions for further research in a related vein include, but are not limited to:

i.) studies of zonal flow and drift wave interaction with a *mean* electric field shear. Such a study requires consideration of profile effects, and a separate evolution equation for the mean field.

ii.) investigation of drift wave "trapping" by zonal flows via examining the structure of BGK solutions to the wave kinetic equation.

iii.) extension to electromagnetic drift wave regimes, where the mutual compensation of $E \times B$ and magnetic stresses results in a reduction in the effectiveness of flow generation.

iv.) examination of the modulational stability of streamers. Streamers are flows with $k_r \rightarrow 0$ and k_θ finite, and thus are the natural counterpart of zonal flows. Streamers are a possible plasma dynamic realization of the "avalanche" concept from self-organized criticality theory.

v.) renormalization group analyses of zonal flow - drift wave interaction. Such an approach would vitiate the rather arbitrary scale separation imposed here.

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