COLLISIONAL TRANSPORT IN NONNEUTRAL PLASMAS[*]

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Abstract

Classical transport theory grossly underestimates collisionally-driven cross-field transport for plasmas in the parameter regime of $r_c << \lambda_D$, where $r_c \equiv \bar{v}/\Omega_c$, $\lambda_D^2 \equiv T/4\pi e^2 n$. In current experiments operating in this regime, cross-field test particle transport is observed to be a factor of 10 larger than the prediction of classical theory. Heat conduction is enhanced by up to 300 times over classical theory, and viscosity is up to 10^4 times larger. New guiding center theories of transport due to long-range collisions have been developed that agree with the measurements. Theory also predicts that emission and absorption of plasma waves may further enhance the thermal conduction and viscosity, providing a possible mechanism for anomalous thermal conductivity in the electron channel of fusion plasmas.

1. INTRODUCTION

Recent theory and experiments on magnetized non-neutral plasmas have investigated the rate at which collisions cause transport of particles, momentum, and energy across an imposed magnetic field. It may be surprising that anything new can be learned in this area, since this subject was exhaustively studied in the early days of plasma physics, resulting in what has come to be known as the classical theory of collisional transport[1-4]. However, recent experiments have observed collisional transport that is much larger than the classical theory predicts. Test particle diffusion is ten times larger than classical theory [5,6]; thermal transport is up to 300 times larger and is independent of magnetic field strength [7-9]; and viscosity is up to 10^4 larger, and actually increases rather than decreases with magnetic field strength [10,11]. Note that the enhanced transport is *not* due to turbulent fluctuations; the nonneutral plasmas are very quiescent. Also, the measured fluxes are in substantial agreement with new theories of transport due to long-range **E** x **B** drift collisions being developed by the UCSD group [7-8,12-16].

The basic problem with classical transport theory is that it ignores an entire class of collisions. The elementary cross-field step envisioned in classical theory occurs as a result of a close collision that scatters the particle velocity vectors. This scattering causes the particle guiding centers to change position by $\Delta r \approx r_c$ [see Fig. 1(a)]. Such scattering occurs only for small impact parameters $\rho < r_c$. However, there are also long-range collisions with impact parameter $\rho > r_c$, and these dominate in plasmas for which $r_c << \lambda_D$. For these collisions, the interaction fields cause the particles to **E** x **B** drift across the magnetic field [Fig. 1(b)], and also to exchange energy associated with velocity components parallel to the field.

The experiments that observe the enhanced transport caused by these long-range collisions were carried out on single-species nonneutral plasmas confined in a Malmberg-Penning trap. The confinement geometry is displayed in Fig. 2. The trap consists of cylindrical electrodes immersed in a uniform magnetic field. The plasma rotates through this magnetic field, creating an inward **v** x **B** force that confines the plasma radially. Axial confinement is provided by application of voltages to the end electrodes, creating an axial potential well.

In the next section we discuss thermal conduction, and in section 3 we examine test particle diffusion. In order to save space we do not consider the plasma viscosity in detail. Interested readers are directed to the references [8,10,11,13,14].

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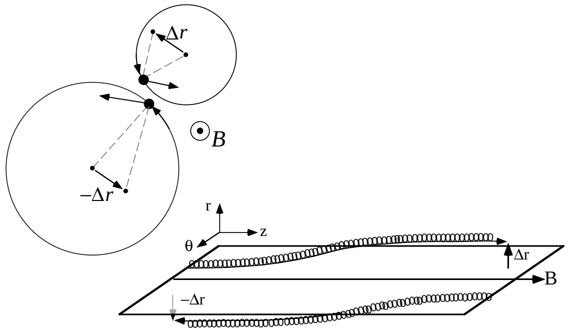

FIG. 1. a) Short-range and b) long-range collisions.

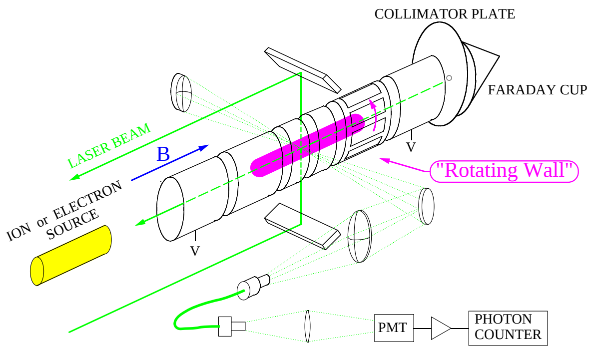

FIG. 2. Electrodes and laser paths.

HEAT TRANSPORT

Preliminary measurements of cross-field collisional heat transport on a pure ion plasma show a strong enhancement (up to 300× classical) due to long-range collisions [8,9]. A radial thermal gradient is created in the Mg^+ ion column by shining a laser along the magnetic field (Fig 2). After turning off the heating, the relaxation of the temperature gradient is measured using a low intensity probe beam. The measurements show that the heat flux is diffusive, and some results for the thermal diffusivity are displayed in Fig. 3. Our evaluation [7] of the heat conductivity κ from long-range collisions with impact parameters in the range $r_c < \rho \lesssim \lambda_D$ gives

$$2\kappa / 5n \; = \; 0.48 \nu_0 \lambda_D^2 \; = \; 0.039 \frac{e^2}{m\bar{v}}, \text{ with } \nu_0 \equiv n\bar{v}b^2 \text{ and } b \equiv e^2 / T \; . \qquad (1)$$

The theory has no adjustable parameters, and agrees well with measurements over a range of $1\frac{1}{2}$ decades in temperature, 2 decades in density and for two magnetic field strengths. (Only a portion of this data is displayed in Fig. 3, for $n = 1.4 \times 10^7 \, \text{cm}^{-3}$ at $B = 4T$.) In this theory, particles exchange parallel velocities, even though they are on field lines separated by up to λ_D. For comparison, the classical prediction is $\kappa / n = \nu_{\perp\|} r_c^2$, where $\nu_{\perp\|} \equiv (8\sqrt{\pi}/15)\nu_0 \ln(r_c / b)$.

Surprisingly, the heat transport from long-range interactions is independent of density and magnetic field, scaling only with temperature as $T^{-1/2}$. All three scalings (*n,B,T*) are quite different than the classical theory predicts [4].

In addition, the new theory predicts a contribution to κ from wave-mediated interactions between particles separated by very long distances ($\rho >> \lambda_D$.) . This mechanism was originally considered by Rosenbluth and Liu [17] as a possible explanation of the anomalous heat loss through the electron channel in fusion plasmas. More recently, Ware has discussed the enhancement of the wave transport for a non-Maxwellian particle distribution (e.g., a high energy tail) [18]; such distributions can be produced in a trapped nonneutral plasma. Of course, the advantage of using a nonneutral plasma with $r_c \ll \lambda_D$ for such studies is that the mechanism of interest dominates the heat transport.

3. TEST PARTICLE DIFFUSION

The test particle diffusion data is obtained from *steady-state* ion plasmas maintained near thermal equilibrium by the “rotating wall” drive [19]. A localized density n_t of “test” particles is created by placing some Mg^+ ions in a different atomic spin state than the rest of the plasma, using laser light. The test particles are observed to diffuse radially, and the diffusion coefficient *D* is obtained [5,6].

Classical short-range velocity-scattering collisions [1-3] give

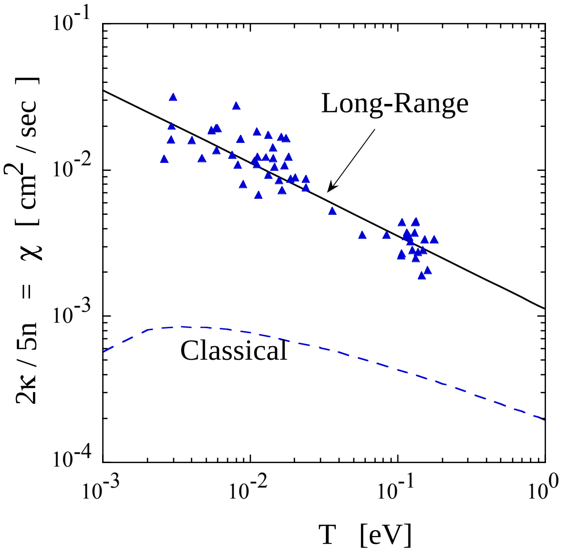

FIG. 3. Thermal diffusivity vs. temperature.

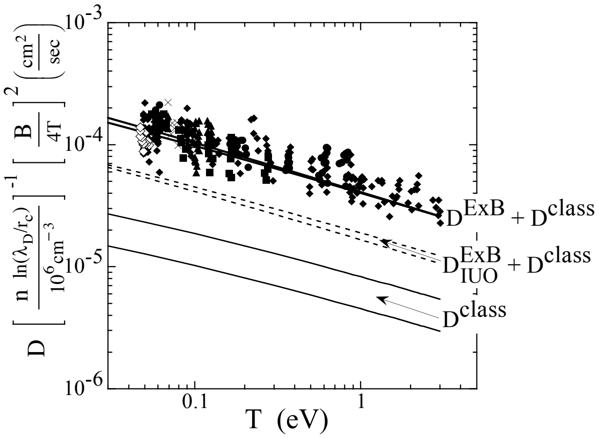

FIG. 4. Test particle diffusion vs. temperature.

$$D^{\text{class}} = \frac{4}{3}\sqrt{\pi}\ \nu_0 r_c^2\ \ln\left(\frac{r_c}{b}\right) \equiv \frac{5}{2}\nu_{\perp\|}\, r_c^2\ . \tag{2}$$

Experiments with neutral plasmas have investigated this classical transport in various regimes [20-22].

A detailed calculation [5] of long-range **E** x **B** drift collisions [23] using the Integration along Unperturbed Orbit (IUO) technique yields

$$D_{\text{IUO}}^{\mathbf{ExB}} = 2\sqrt{\pi}\ \ \nu_0 r_c^2\ \ln\left(\frac{\bar{\text{v}}}{\text{v}_{\text{min}}}\right)\ln\left(\frac{\lambda_D}{r_c}\right). \tag{3}$$

The strongest long-range collisions occur between particles with near-zero relative axial velocity; the velocity v_{min} is the minimum relative velocity for which the unperturbed orbit analysis is still valid, determined by either shear or collisions. In $D_{\text{IUO}}^{\mathbf{E\times B}}$, the $\ln(\lambda_D / r_c)$ term arises because guiding center theory for the dynamics breaks down for impact parameters $\rho < r_c$ and the interaction is Debye-shielded for $\rho > \lambda_D$.

Figure 4 shows the experimentally measured diffusion compared to the initial theory prediction, $D = D_{\text{IUO}}^{\mathbf{E\times B}} + D^{\text{class}}$, for a wide range of densities, temperatures, and magnetic fields. The measurements are about 3× larger than this prediction. This might be considered "reasonable" agreement, but it led to re-examination of the theory and to the discovery of a new plasma effect.

We have determined that this 3× discrepancy in D is caused by a novel effect in kinetic theory which causes the IUO technique to fail [12]. In IUO, two colliding particles are assumed to move along the magnetic field on unperturbed trajectories, interacting only once. However, velocity-scattering collisions with surrounding particles eventually cause the relative axial velocity of the interacting pair to *reverse*, and the particles may make another collision; in fact, they may collide *several times.* This surprising "collisional caging" effect is neglected in IUO, and it leads to a factor-of-three increase in the test particle diffusion, i.e. $D^{\mathbf{E\times B}} = \alpha\, D_{\text{IUO}}^{\mathbf{E\times B}}$, with $\alpha = 3$ in a shear-free plasma. This new prediction is shown by the upper dashed line in Fig. 3, where it can be seen to match the experiment within the scatter of the data.

This $\alpha = 3$ enhancement occurs only if the collisional dynamics is effectively 1-dimensional. Thus, the enhancement will be smaller if particles become spatially separated in (r,θ) before they can suffer a second collision, e.g. due to shear in the rotation rate $\omega_E(r)$. In the experiments to date, the plasma was near thermal equilibrium, with minimal shears; but if the shear were made larger, one would expect the diffusion coefficient to decrease to near $D_{\text{IUO}}^{\mathbf{E\times B}}$.

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