Effects Of Transverse Electric Field, Couple Stress And Heterogeneity Of A Poorly Electrically Conducting Fluid Saturated Nano Porous Zeolites Acquiring Smart Material Properties

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Abstract

In this paper we explain a Mathematical Model involving Darcy linear drag, Forchheimer quadratic drag, horizontal density gradient and the variation of electrical conductivity due to organic substances dissolved in a heterogeneous Boussinesq poorly conducting couple stress fluid flow (PCPCSFF) through Nano Porous Zeolites regarded as densely packed porous media. Initially, the flow is at rest and set in motion due to initial piecewise horizontal concentration gradient.

Analytical solutions, for electric potential using the Maxwell field equations and for velocity and density using nonlinear Darcy – Forchheimer equation in the presence of couple stress and electric force are obtained using the method of time series evolution. The analytical solutions for streamlines and density are computed for different values of time, t, for a particular value of electric number W_1 and couple stress parameter β and the results are depicted graphically in figures 1 and 2. From these figures we found that the streamlines are closer in the region of x<0 than that of x>0 and the density profiles are concentrated in the lower region and develop curvature in the presence of electric field and couple stress parameter. The physical reason for the nature of streamlines and density profiles are given in the last section and some important conclusions are drawn.

1. Introduction

To supply adequate food for the increase in population and modern way of living farmers have put arbitrarily more fertilizers into the soil. This has resulted in reduction in quality and quantity of food due to soil resource degradation and caused distress among farmers. There is a necessity to find mechanisms to reverse this trend. For this, it is necessary either to use more land or intensify crop production per unit of available land. We know that scarcity and increase in the cost of land in our country, the poor farmers cannot afford to buy more land and hence they have to resort to intensify crop production per unit of available land. It is a known fact that in our country our soils are either low fertile or made less fertile due to the intensive removal of nutrients without caring for substituting fresh nutrients. In that case intensify crop production is not viable proposition and we have to resort to alternate method. In this paper we propose to use nanoporous zeolites which are more amicable.

Zeolites are microporous aluminosilicate materials. Microporous material is a material containing pores with diameter less then 2 nm. Aluminosilicate minerals are minerals composed of aluminum, silicon and oxygen. The hydrated aluminosilicates minerals are zeolites which have pore structure having naturally occurring materials. These nanoporous zeolites were originally found in 1756 by the Swedish mineralogist Axel Fredrik Cronstedt. He called the material zeolite coined from the Greek Language meaning "boiling stone". The organic minerals dissolved in water in micropores forming poorly conducting fluid with electrical conductivity, σ, increasing with temperature. Further, the organic minerals and other substances are freely suspended in water in voids of porous zeolite and they will execute microrotation forming micropolar fluid developed by Eringen [8]. A particular case of micropolar fluid when microrotation balances with the natural vorticity of the fluid is called a couple stress fluid. The nanoporous zeolites have high void volume, low density and high cation exchange capacity. In these, the two process namely particle diffusion within the zeolite and diffusive transport through the liquid film surrounding the particles have been identified as the most important processes in an ion exchange property. These processes cause the difference in temperature between high surface temperature of nanoporous zeolite and the low temperature of water in the void volume of nanoporous zeolite producing the variation of electrical conductivity, $\Delta\sigma$. This $\Delta\sigma$ releases the electric charges from the nuclei forming distribution of charge density, ρ_e. In addition, the motion of electric charges induces an electric field, $\vec{E}_i$. In case, high strength of electric field is needed in the design of artificial zeolites, it can be generated called applied electric field, $\vec{E}_a$ by segmented electrode of different potentials set up at the upper and lower surfaces of the nanoporous zeolite. The total electric field, $\vec{E}(=\vec{E}_i+\vec{E}_a)$ not only produces the current density, $\vec{J}$, according to Ohm's law, and also produces, along with ρ_e, the force $\rho_e\vec{E}$. This current acts as sensing and the force acts as actuation, which are the two important properties to make a material to be a smart material. These smart nanoporous zeolite can be widely used in industries for water purification, as catalysts, nuclear reprocessing, for the production of laundry detergents and so on. They can also be used in the design of artificial organs like cartilages in synovial joints, endothelium the walls of the coronary arteries, the trachea (i.e., wind pipe) in biomedical engineering. In particular, these smart nanoporous zeolites may be used in agriculture as fertilizers to grow more quality and quantity of food and as well as to increase soil fertility. Numerous soil and land quality indicators have been used to asses the degradation statues (see Ramesh etal [7]).

Zeolites have porous structure which can accommodate a wide variety of cat ions. These positive ions are loosely held so that they can readily be exchanged for others in the contact solution. Natural zeolites found in volcanogenic sedimentary rocks have been not only used in agriculture and they have also been used as building stone, as light weight aggregate and pozzolans in cements and concretes, as filler in paper, as soil amendments in agronomy and horticulture, in the removal of ammonia from municipal, industrial, agriculture and hospital wastes, drinking water, as energy exchangers in solar refrigerators, as dietary supplements in animal diet, in pet litters, in taking up ammonia from animal manures and as ammonia fillers in kidney – dialysis units. In addition to their recent success in the healing of cuts and wounds, natural zeolites are now

considered to be full fledged minerals commodities, the use of which promise to expand even more in the future (see Mumpton [6]).

Natural zeolites are hydrated aluminosilicates. They consist of open three dimensional cage-like structures and a vast network of open channels extending throughout. Loosely bound positively charged ions called cations are attached at the juncture of the negatively charged aluminosilicates lattice structure. This frame work provides exceptional strength and stability to the lattice structure.

The channels, typically 0.3 to 0.7 nanometers in diameters (3 to 7 angstroms slightly larger than a water molecule), selectively screen molecules according to size and exchangeable cations.

Molecules too large to pass through the entry channel are excluded, thus giving rise to the term "Molecular Sieve". The molecular structure, surface area, surface charge density and cation exchange capacity (CEC) of each particular zeolite will determine its loading, shrinking, swelling and stability under various conditions. The zeolites have a rigid three – dimensional crystalline structure (similar to honeycomb) consisting of a network of interconnected tunnels and cages. Zeolites in general have high specific surface areas and their rigid framework limits shrinking and swelling.

Natural zeolites form in a condition where volcanic rocks and other layers react with alkaline groundwater. Zeolites are also crystallizing in post-depositional environments over the years ranging from thousand of years in swallow marine basins. Naturally occurring zeolites are therefore rarely pure. It is for this reason, naturally occurring zeolites, without uniformity and purity, are excluded from many important commercial applications.

Since naturally occurring zeolites are rarely pure, several types of synthetic zeolites have been synthesized. One of the important processes used in zeolite synthesis is sol-gel in which other elements like metals and metal oxides can easily be incorporated. It is known that the silicate sol formed by the hydrothermal method is very stable.

The literature on nanoporous zeolites is silent about synthesizing them having the properties of smart materials. Therefore, the main objective of this paper is to explain how to make Nanoporous smart zeolites using organic minerals dissolving in water in the voids of nanoporous zeolites making that water poorly conducting with conductivity σ increasing with temperature as explained above. Such smart nanoporous zeolites will have several advantages as explained earlier in this section. However, their advantages are more useful in agriculture to grow more food using the property of sensing whenever the crop is needed water or nitrogen or both and the actuation property provide them. To achieve this objective, this paper is planed as follows. Formulation of the problem is given in section 2. The solutions of the problem using time evolution are given in section 3. Conclusions are drawn in section 4.

2. Formulation of the Problem

In this section, we give the required basic equations, following Rudraiah and Ng [4], for a couple stress poorly conducting incompressible heterogeneous two dimensional fluid saturated densely packed Nanoporous Zeolites using ensemble averages.

In Cartesian form, they are:

The Conservation of Mass for incompressible Boussinesq fluid

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0, \tag{2.1a}$$

Condition for heterogeneity

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+w\frac{\partial \rho}{\partial z}=0 \tag{2.1b}$$

Conservation of Momentum for flow regarded as densely packed porous media

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}+\frac{C_b}{\sqrt{k}}|\bar{q}|u=-\frac{1}{\rho_0}\frac{\partial p}{\partial x}-\frac{\upsilon}{k}\left(1+\frac{\lambda}{\mu k}\right)u+\frac{\rho_e E_x}{\rho_0} \tag{2.2}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}+\frac{C_b}{\sqrt{k}}|\bar{q}|w=-\frac{1}{\rho_0}\frac{\partial p}{\partial z}-\frac{\upsilon}{k}\left(1+\frac{\lambda}{\mu k}\right)w-\frac{\rho g}{\rho_0}+\frac{\rho_e}{\rho_0}E_z \tag{2.3}$$

where x and z are the horizontal and vertical Cartesian coordinates.

Conservation of Electric Charges:

$$\frac{\partial \rho_e}{\partial t}+u\frac{\partial \rho_e}{\partial x}+w\frac{\partial \rho_e}{\partial z}+\sigma\frac{\rho_e}{\varepsilon_0}+E_z\frac{\partial \sigma}{\partial z}=0 \tag{2.4}$$

The Maxwell Equations, in the absence of Magnetic field

$$\nabla.\vec{E}=\rho_e/\varepsilon_o, \quad \nabla\times\vec{E}=0, \quad \vec{E}=-\nabla\varphi \tag{2.5 a, b, c}$$

Following Rudraiah [1, 2], we have

$$J=\sigma\vec{E}, \quad \sigma=\sigma_0\left[1+\alpha_h\left(T-T_0\right)+\alpha_c\left(C-C_0\right)\right] \tag{2.6 a, b}$$

where, $\bar{q}=(u,0,w)$ is the mean filter velocity, φ electric potential, ρ_e the volume distribution of charge density, $\rho_e, \vec{g}\ and\ \vec{E}$ are the gravitational field and the electric field, $\vec{J}$ the current density, σ the electrical conductivity of PCCFF, αh and αc are the volumetric coefficients of electrical conductivity, υ the kinematic viscosity of fluid, λ the couple stress coefficient, k the permeability, Cb the quadratic drag coefficient of a nanoporous zeolite, ε_o the dielectric constant of free space, σ_0 is that of σ at $T=T_0$ and $C=C_0$. T_0 and C_0 are respectively the reference temperature and concentration distributions. The poorly conducting fluid is initially at rest and since $\sigma \ll 1$, any perturbation on σ is negligible and hence it depends on the conduction temperature, $\frac{d^2T}{dz^2}=0$ and conduction diffusion $\frac{d^2C}{dz^2}=0$.

The solutions for T and C satisfying the conditions $T=T_0$, $C=C_0$ at $z=0$ and $T=T_1$ and $C=C_1$ at $z=h$, the width of the nanoporous zeolite, are $T=\Delta T\,z/h+T_0$ and $C=\Delta C\,z/h+C_0$

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Then, σ, given by eqn (2.6b) takes the form

$$\sigma/\sigma_0 = 1 + \gamma z \approx e^{\gamma Z} \ (\because \gamma << 1), \gamma = \alpha_T \Delta T / h + \alpha_c \Delta C / h \tag{2.7}$$

For base flow (i.e. $\vec{q} = 0, \dfrac{\partial}{\partial t} = 0$), eqn (2.4) becomes

$$\sigma \frac{\rho_e}{\varepsilon_0} + E_z \frac{d\sigma}{dz} = 0$$

This, using (2.5c) and (2.7), becomes

$$\frac{\partial^2 \varphi}{\partial z^2} + \gamma \frac{\partial \varphi}{\partial z} = 0 \tag{2.8}$$

Its solution, satisfying the conditions $\varphi = \dfrac{Vx}{h}$ at $z = 0$ and $\varphi = \dfrac{V(x - x_0)}{h}$ at $z = h$

is $\varphi = \dfrac{Vx}{h} - \dfrac{Vx_0(1 - e^{-\gamma Z})}{(1 - e^{-\gamma h})}$ \hfill (2.9)

Then eqns (2.5 a, b, c) take the form

$$\rho_e = -\varepsilon_0 \nabla^2 \varphi = -\varepsilon_0 \frac{Vx_0}{h} \frac{\gamma^2 e^{-\gamma Z}}{(1 - e^{-\gamma h})}, \ E_x = -V\!\!\Big/\!h, \ E_Z = \frac{Vx_0 \gamma e^{-\gamma Z}}{h(1 - e^{-\gamma h})} \tag{2.10 a, b, c}$$

where V is the uniform potential at the boundaries due to segmented electrodes, E_x and E_z are the components of the electric field in x and z directions.

3. Solution Using Time Evolution

We consider a two – dimensional motion of a poorly conducting heterogeneous couple stress fluid saturated densely packed nanoporous zeolite with x – axis horizontal and z – axis vertical and velocities u and w respectively in x and z directions. A non-uniform initial horizontal density gradient sets in a vertical motion denoted by w in addition to horizontal motion. We make the above equations (2.1a) to (2.4) dimensionless using the scales $v^2 / \alpha g C_b k^{3/2}, k/v, v/C_b \sqrt{k}, \sigma_0, V, V/\sqrt{k}$ and $\varepsilon_0 V/k$ for length, time, velocity, conductivity, potential, electric field and density of charges respectively. Eliminating the pressure between eqns (2.2) and (2.3) and expressing them interns of stream function ψ defined by

$$u = -\partial \psi / \partial z = -\psi_z \ and \ w = \partial \psi / \partial x = \psi_x, \tag{3.1}$$

we get

$$\nabla^2 \psi_t - \sigma_1 \psi_z \nabla^2 \psi_x + \sigma_2 \psi_x \nabla^2 \psi_z + \sigma_3 \nabla^2 \psi + \left(\psi_x^2 + \psi_z^2\right)^{1/2} \nabla^2 \psi$$
$$-\left(\psi_x^2 + \psi_z^2\right)^{-1/2} \left(2\psi_x \psi_z \psi_{xz} + \psi_x^2 \psi_{xx} + \psi_z^2 \psi_{zz}\right) + \frac{\sigma_1 \partial \rho}{\partial x} - w_1 e^{-\gamma z} = 0 \tag{3.2}$$

where

$$\sigma_1 = g C_b k^{3/2} / v^2, \sigma_2 = \alpha g k^2 / v^2, \sigma_3 = 1 + \beta \sigma^2,$$
$$\beta = \lambda / \mu h^2, \sigma = h/\sqrt{k} \ and \ w_1 = \varepsilon_0^2 V^4 \gamma^2 x_0$$

is the electric number. We solve eqn (3.2) using the time series evolution of the form

$$\psi = \psi_0 + \psi_1 t + \psi_2 t^2 + \psi_3 t^3 \tag{3.3}$$
$$\rho = \rho_0 + \rho_1 t + \rho_2 t^2 + \rho_3 t^3 + .. \tag{3.4}$$

Since the fluid is initially at rest, it follows that $\psi_0 = 0$ and $\rho_0 = \rho_0(x,z)$ which is the initial density. Equation (3.2), using eqns (3.3) and (3.4) and equating the coefficients of like powers of t to zero and assuming w $\ll$ u so that

$$\left(\sqrt{u^2 + w^2}\right)u \approx u^2, because\left(\sqrt{u^2 + w^2}\right) \approx u \quad \text{we get}$$

At o(t^0): $\nabla^2\psi_1 = -\sigma_1\rho_{0x} + w_1 e^{-\gamma z}, \rho_1 = 0$ (3.5 a, b)

At o(t): $2\nabla^2\psi_2 + \sigma_3\nabla^2\psi_1 + \sigma_1\rho_{1x} = 0$

This, using equations (3.5 a, b) becomes

$$\nabla^2\psi_2 = -\sigma_3/2, \nabla^2\psi_1 = -\sigma_1\left(-\sigma_1\rho_{0x} + w_1 e^{-\gamma z}\right)/2 \quad (3.6)$$

From eqn (2.4) we see that its derivative may be approximated, as in Simpson and Linden [5] by

$$\rho_{xt} = -\sigma_2\left(u\rho_x\right)_x \quad (3.7)$$

From this it follows that the evaluation of the density gradient is determined by the sign of $\left(u\rho_x\right)_x$. This amounts to an increase in horizontal density gradient which will occur only when the horizontal motion in the region of stronger density gradient is towards the region of the weaker density gradient. To know the effect of inertia we have to go up to o(t^2) in eqn (3.2) and obtain, by equating the terms of o(t^2) to zero, the equation

$$\nabla^2\psi_3 = \frac{1}{3}\left[\begin{array}{l}-\sigma_2\left(\psi_{1x}\nabla^2\psi_{1z} - \psi_{1z}\nabla^2\psi_{1x}\right) - \sigma_3\nabla^2\psi_2 - \sigma_1\rho_{2x} - 2\psi_{1z}\psi_{1zz} \\ -\psi_{1z}\psi_{1xx} - 2\psi_{1x}\psi_{1xz}\end{array}\right] \quad (3.8)$$

Further, in dimensionless form, we have

$$\rho_1 = 0, \rho_2 = \left(\psi_{1z}\rho_{0x} - \psi_{1x}\rho_{0z}\right)/2, \rho_3 = \left(\psi_{2z}\rho_{0x} - \psi_{2x}\rho_{0z}\right)/3 \quad (3.9)$$

$$\rho = \rho_0 + \frac{1}{2}\left(\psi_{1z}\rho_{0x} - \psi_{1x}\rho_{0z}\right)t^2 + \frac{1}{3}\left(\psi_{2z}\rho_{0x} - \psi_{2x}\rho_{0z}\right)t^3 \quad (3.10)$$

We note, as in Simpson and Linden [5], to find an increase in horizontal density gradient we have to consider a non-uniform horizontal density gradient of the form

$$\rho = \begin{cases} 1 - \gamma_1\sigma_1 x & x < 0 \\ 1 - \gamma_2\sigma_1 x & x > 0 \end{cases} \quad (3.11)$$

where $\gamma_1 = \alpha_1/\alpha, \gamma_2 = \alpha_2/\alpha, \alpha_1$ and α_2 are the values of α in $x<0$ and $x>0$ respectively. This reveals a discontinuity in the density gradient at $x=0$ although the density is continuous. Equations (3.5a), (3.6) and (3.8), using eqn (3.11), become

$$\nabla^2\psi_1 = \begin{cases} \gamma_1 + w_1 e^{-\gamma z}, x < 0 \\ \gamma_2 + w_1 e^{-\gamma z}, x > 0, \end{cases} \nabla^2\psi_2 = \begin{cases} -\sigma_3\gamma_1/2 + w_1 e^{-\gamma z}, x < 0 \\ -\sigma_3\gamma_2/2 + w_1 e^{-\gamma z}, x > 0 \end{cases} \quad (3.12, a, b)$$

$$\nabla^2\psi_3 = \frac{\sigma_3^2}{6}\begin{cases} \gamma_1 + w_1 e^{\gamma z}, x < 0 \\ \gamma_2 + w_1 e^{\gamma z}, x > 0 \end{cases} + \frac{1}{3}\left[\begin{array}{l}\psi_{1z}\left(2\psi_{1zz} + \psi_{1xx}\right) - 2\psi_{1x}\psi_{1xz} \\ -\sigma_2\left(\psi_{1x}\nabla^2\psi_{1z} - \psi_{1z}\nabla^2\psi_{1x}\right)\end{array}\right] \quad (3.13)$$

Solving eqns (3.12a, b) we get

$$\psi_1 = \begin{cases} \gamma_1 z^2/2 + \gamma_2 h^2/2 + \left((\gamma_1 + \gamma_2)h^3/\pi^2\right)\sum_{i=0}^{\infty} a_i & x < 0 \\ \\ \gamma_2 z^2/2 + \gamma_1 h^2/2 - \left((\gamma_1 + \gamma_2)h^3/\pi^2\right)\sum_{i=0}^{\infty} b_i & x > 0 \end{cases} \quad (3.14)$$

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$$\psi_2 = \begin{cases} \gamma_1 z^2/4 + \gamma_2 h^2/4 + \left((\gamma_1+\gamma_2)h^3/\pi^2\right)\sum_{i=0}^{\infty} a_i & x < 0 \\[3mm] \gamma_2 z^2/4 + \gamma_1 h^2/4 - \left((\gamma_1+\gamma_2)h^3/\pi^2\right)\sum_{i=0}^{\infty} b_i & x > 0 \end{cases}$$

(3.15)

In eqns (3.14) and (3.15) a_i and b_i are given by

$$a_i = 1/(1+2i)^2 \cos(1+2i)\pi z/h\, e^{(1+2i)\pi x/h} \qquad\qquad (x<0)$$

$$b_i = 1/(1+2i)^2 \cos(1+2i)\pi z/h\, e^{-(1+2i)\pi x/h} \qquad\qquad (x>0)$$

Similarly, the solutions of eqns (3.13) are determined but for want of space are omitted here because the expressions are lengthy but they are included in computing

$$\psi = \psi_1 t + \psi_2 t^2 + \psi_3 t^3$$

and the results are graphically represented and discussed in section 4. Similarly, we obtain

$$\rho = \begin{cases} (1-\gamma_1\sigma_1 x) - (1+t)t^2\left[\gamma_1^2 z/2 - \left((\gamma_1-\gamma_2)\gamma_1 h/\pi^2\right)\right]\sum_{i=0}^{\infty} C_i e^{-\alpha_i x} & (x<0) \\[3mm] (1-\gamma_2\sigma_2 x) - (1+t)t^2\left[\gamma_1^2 z/2 - \left((\gamma_1-\gamma_2)\gamma_2 h/\pi^2\right)\right]\sum_{i=0}^{\infty} C_i e^{\alpha_i x} & (x>0) \end{cases}$$

(3.16)

where $C_i = \left(\sin(2i+1)\pi z/h\right)/(2i+1)$, $\alpha_i = (1+2i)\pi/h$

From eqn (3.16) it follows that the maximum value of ρ_x occurs at $x = 0$ and as $x \to \infty$ we have

$$\rho_x = -\sigma_1\gamma_2 + \frac{(1+t)t^2}{2}(\gamma_1-\gamma_2)\gamma_1\gamma_2 \log\left[\tan\frac{\pi}{4}(1+2z)\right]$$

(3.17)

Since $\gamma_1 > \gamma_2$, it is clear from (3.17) that ρ_x increases with time from $z < 0$ and decreases for $z > 0$.

4. Conclusions

The effects of electric field, couple stress parameter, β, Forchheimer quadratic drag, the variation of initial electrical conductivity and density in the horizontal and vertical directions in a heterogeneous poorly conducting fluid flow through nanoporous zeolite are investigated analytically. The instantaneous streamlines for the flow with $\gamma_1 > \gamma_2$ for different values of t and for a particular value of electric number, w_1, and couple stress parameter, β, are drawn in Fig 1. From this figure we conclude that the streamlines are closer in the region of $x < 0$ than that of $x > 0$. Physically this is attributed to the fact that more intensive flow is produced by the combined effect of electric field, couple stress parameter, β, and density gradient in that region. The time evolution of density is numerically computed using eqn (3.16) for different values of time t and for a particular value of w_1 and β the results are presented in Fig. 2.

From this figure we observe that the density profiles are concentrated in the lower region and hence there will be an increase in density gradient in that region. We also observe that the density profile develops curvature near $x = 0$ instead of being straight line. From this we conclude that this curvature due to smart properties of nanoporous zeolite in the presence of electric field and couple stress parameter, β, sets up a circulation in the transverse plane and hence the magnitude of the density gradient

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increases with time. This is favorable to grow more quality and quantity of food, because if crops needed either water or nitrogen or both, then the sensing property of soil material sense it and its actuation property supply the requisite of these materials.

Acknowledgement

References

[1] Rudraiah N, Mathematical Model to purify poorly conducting polluted ground water through porous medium using the concept of smart materials. Accepted for presentation and for including in the proceeding of 13thACFM, 2010, Dhaka, Bangladesh.

[2] Rudraiah N, Modeling of nano and smart materials, book published by book paradise, Bangalore, India, 2003.

[3] Eringen A.C Theory of micropolar fluids, J Math Mech 1966;16;1.

[4] Rudraiah N, and Ng.C.O, A model for manufacture of nanosized smart material free from impurities, Current Science 2004; 86, 1076.

[5] Simpson JE and Linden PF, Frontogenesis in a fluid with horizontal density gradient, J Fluid Mech 1989; 202; 1.

[6] Mumpton, A.F. (1998), "Uses of natural zeolites in agriculture and industry", a paper presented at National Academy of Science Colloquim "Geology, Mineralogy And Human Welfare", held on Novemebr 8-9, 1998 at the Arnold and Mabel Beckman Centre in Irvine, CA, Colleoquim paper Laroca Magica.

[7] Ramesh K, Biswas A.K, Somasundaran. J, and Annangi Subba Rao (2010). A review article, Current Science, vol 99, No 6, 25 September, pp 760 – 764.

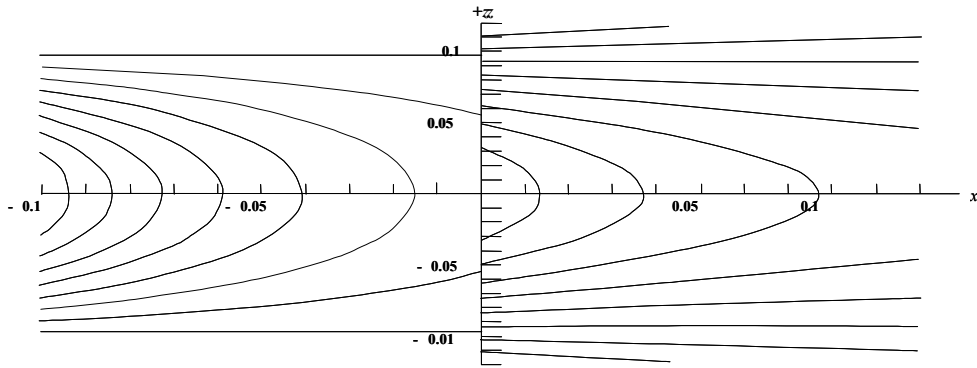[8] Eringen A.C Simple micropolar fluids, Int. J. Engng. Sci. 2(2), pp 205-17.

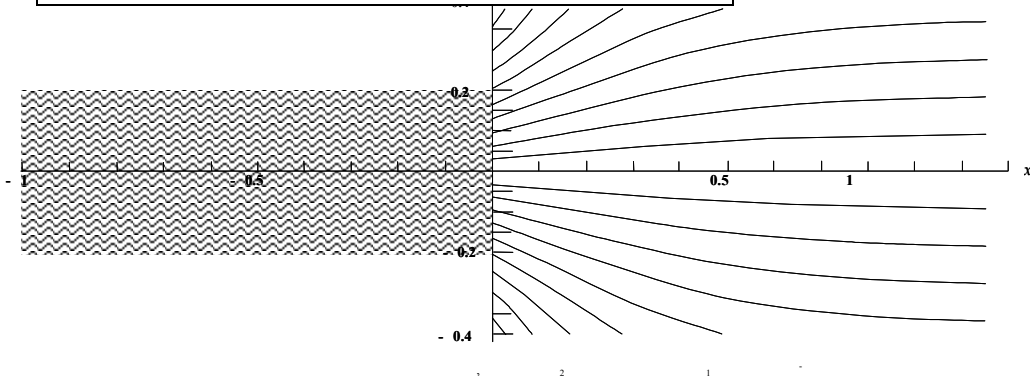Fig.1 Isopyenals for $\gamma_1 = 1.05 \, and \, \gamma_2 = 0.06$

Fig.1 Isopyenals for $\gamma_1 = 1.05, \gamma_2 = 0.06 \, and \, \sigma_1 = 4 \times 10^4$