ITB oscillations: towards a limit cycle model

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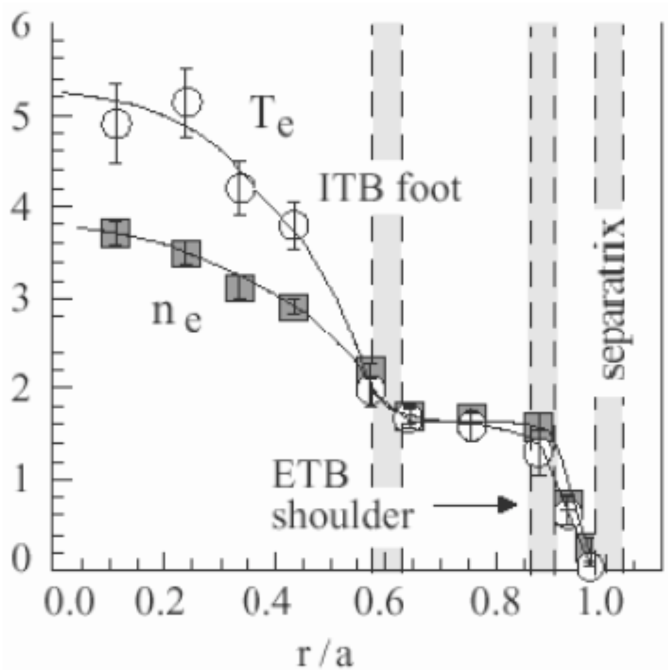
Introduction
What is an ITB ?

- An *Internal Transport Barrier* (*ITB*) is a localized region of improved particle and energy confinement that can appear in fusion devices.
- It gives rise to the local steepening of the pressure profile.
- It can induce a higher fraction of bootstrap current.



Experimentally measured electron density and temperature profiles

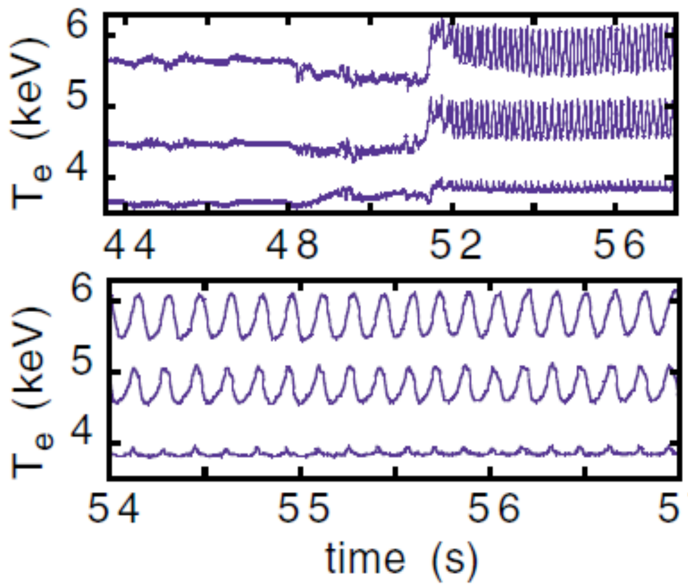
Connor *et al.*, *Nucl. Fusion* **44**, R1 (2004)

Introduction
Why do ITBs exist ?

- ITBs are known to be associated with micro-turbulence reduction.
- There is yet no ultimate explanation for their appearance,
  - it is a complex issue depending on several factors: type of machine, transport channel, ...
- A widely accepted mechanism for the formation of electron ITBs is that:
  - **an ITB is triggered when the plasma switches from a monotonic to a hollow q profile**,
  - the ITB foot shares the location of the minimum in the q profile.

Introduction
ITB oscillations

- Under certain conditions ITBs are seen to oscillate in time.
- It has been inferred that ITB oscillations could be the result of the nonlinear coupling between the **pressure** and **current density** profiles via the dependence of:
    - the **non-inductive current** on the **electron temperature gradient**,
    - the **thermal diffusivity** on the **magnetic shear**.

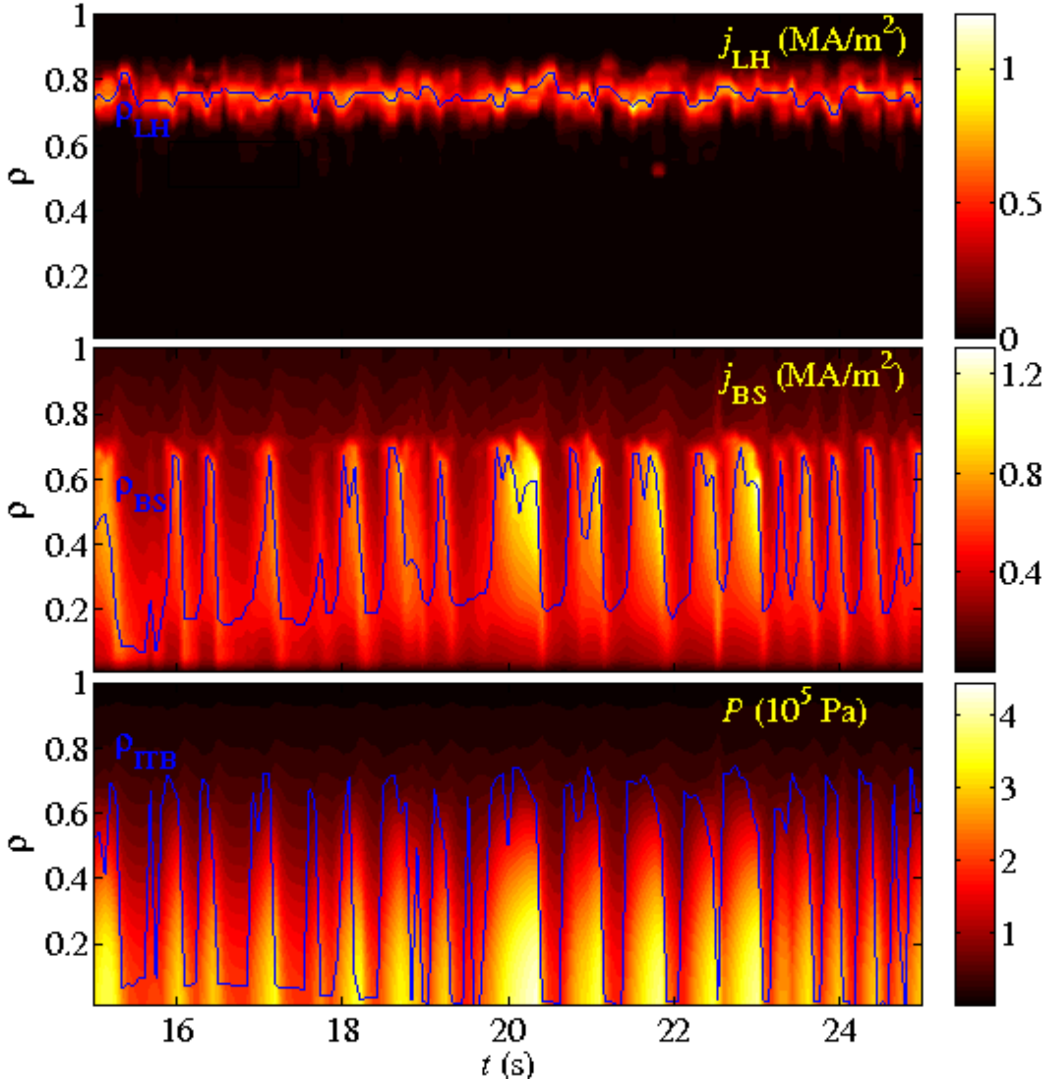


Time evolution of the electron temperature for a Tore Supra discharge
Giruzzi *et al.*, *Phys. Rev. Lett.* **91**, 135001 (2009)

ITB Oscillations in a JET-like AT Plasma

$I_P = 2.3$ MA

$P_{LH} = 4$ MW



J. P. S. Bizarro *et al.*, *IEEE Trans. Plasma Sci.* **36**, 1090 (2008)

Aim

Our aim is to attempt to capture and understand the essential physics underlying ITB oscillations, so that robust strategies can be defined to effectively control them.

We adopt a simple **local (0-D) model,** derived from the **transport equations** under a set of **simplifying assumptions**.

We analyse its stationary solutions with different tokamak regimes in view: a **pure Ohmic regime**, a **non-inductive regime** and, possibly, an **oscillatory regime.**

Reduction Method
Methodology

In order to reduce the standard 1-D transport equations for plasma energy and current,

$$\begin{cases} \frac{3}{2} n \partial_t T = \frac{1}{r} \frac{\partial}{\partial r} r n \chi \frac{\partial T}{\partial r} + S, \\ \mu_0 \partial_t j = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \left[\eta \left(j - j_{\mathrm{NI}} \right) \right], \end{cases}$$

to a 0-D model, we Taylor expand the profiles around the ITB foot r_b, adopt a set of assumptions, evaluate the equations at the origin, and retain the information relative to these two points (the origin and the ITB foot).

Reduction Method
Assumptions & Variables

In the reduction process, we adopt the following assumptions:

- T and j have null derivatives at the origin,
- T and j are constant at the ITB foot:

$$T(r = r_b) = T_b \text{ and } j(r = r_b) = j_b$$

- n is flat for $0 < r < r_b$,
- j_{NI} is null at the origin,
- η is the Spitzer resistivity,

Variables of the model :

T_0/T_b and j_0/j_b , where $T_0 = T(r = 0)$ and $j_0 = j(r = 0)$

Reduction Method
Transport Model

Taking the above referred mechanism for ITB formation, we adopt the following criterium:

whenever the q profile becomes hollow (i.e., has a local minimum), an ITB sets in and the diffusivity drops from a typical low- to a high-confinement value:

$$\chi = \chi_{\text{High}} + (\chi_{\text{Low}} - \chi_{\text{High}}) f(s)$$

where s is the magnetic shear and f models the transition from 0 to 1.

We choose f to be the logistic function:

$$f(s) = 1/(1 + e^{-\alpha s})$$

with α controlling the transition steepness.

Reduction Method
General Reduced system

The reduced form of the transport equations is then given by

$$\begin{cases} \tau_x \dot{x} = -\frac{8}{3}\tilde{\chi}(x,y)(x-1) + \tilde{S}_0, \\ \tau_y \dot{y} = \frac{4}{x^{3/2}}\left[-(y-1) + \frac{3}{2}\frac{y}{x}(x-1) - \tilde{S}_{\mathrm{NI}_b}\right], \end{cases}$$

where:

$x = T_0/T_b$ is the dimensionless temperature inside the barrier,

$y = j_0/j_b$ is the dimensionless current density inside the barrier,

$\tau_x = r_b^2/\chi_{\mathrm{Low}}$ is the transport time,

$\tau_y = \mu_0 r_b^2/\eta_b$ is the resistive time,

$\tilde{\chi}(x,y) = \chi_0(x,y)/\chi_{\mathrm{Low}}$ is the dimensionless diffusivity,

$\tilde{S}_0 = \tau_x S_0/(\frac{3}{2}nT_b)$ is the dimensionless heating power inside the barrier,

$\tilde{S}_{\mathrm{NI}_b} = j_{\mathrm{NI}_b}/j_b$ is the fraction of non-inductive current at the barrier's foot

Reduction Method
Heating & Current Drive

The system becomes:

$$\begin{cases} \tau_x \dot{x} = -\frac{8}{3}\left[\frac{\chi_H}{\chi_L} + (1 - \frac{\chi_H}{\chi_L}) f(y)\right](x-1) + {C_x}^{\mathrm{ohm}} \frac{y^2}{x^{3/2}} + {F_{\mathrm{Ext}}}^x(x,y) \\ \tau_y \dot{y} = \frac{4}{x^{3/2}}\left[-(y-1) + \frac{3}{2}\frac{y}{x}(x-1) - {C_y}^{\mathrm{BS}} \frac{x-1}{y+1} - {F_{\mathrm{Ext}}}^y(x,y)\right] \end{cases}$$

if the following different source terms are considered:

- The Ohmic heating: ${S_0}^{\mathrm{ohm}} = {C_x}^{\mathrm{ohm}} \; y^2/(x^{3/2})$
- The bootstrap current: ${S_b}^{\mathrm{BS}} = {C_y}^{\mathrm{BS}} \; (x-1)/(y+1)$
- The external heating power: ${F_{\mathrm{Ext}}}^x(x,y)$
- The external non-inductive current fraction: ${F_{\mathrm{Ext}}}^y(x,y)$

$${C_x}^{\mathrm{ohm}} = \tau_x \eta_b {j_b}^2 / (\tfrac{3}{2} n T_b) \qquad {C_y}^{\mathrm{BS}} = \alpha_{\mathrm{BS}} T_b / ({r_b}^{3/2} {j_b}^2)$$

Reduction Method
Parameters & External Sources

Parameters

We choose parameters pertinent to Tore Supra, a machine particularly suited for long pulse operation:

$$T_b = 4\ KeV,\ j_b = 1.5\ MA/m^2,\ \ \rho_b = 0.2$$

Choice of external sources

In order to model the external sources corresponding to a lower hybrid current drive (LHCD) system, we assume:

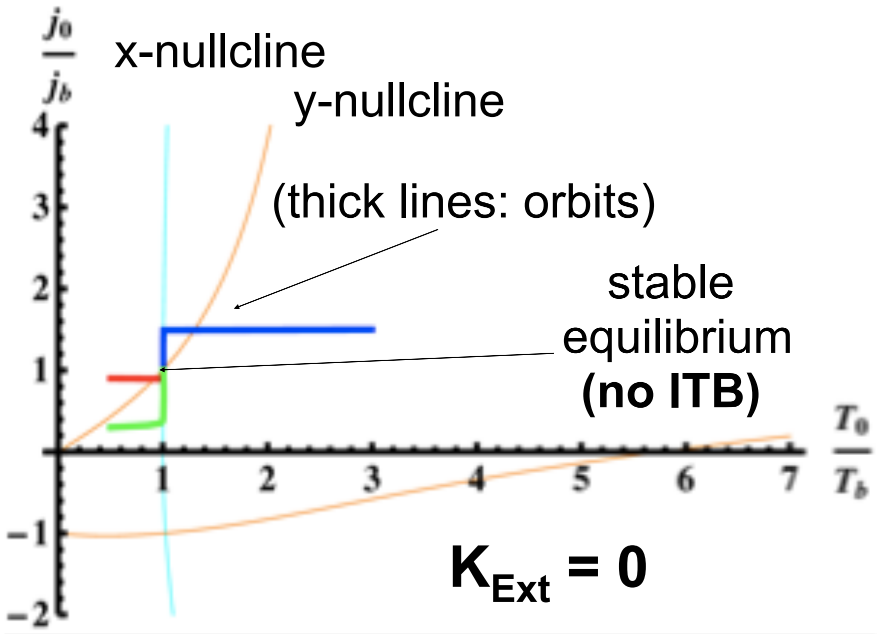
- a constant and well localized current deposition in r_b: $F_{\mathrm{Ext}}{}^{y}(x, y) = K_{\mathrm{Ext}}$
- a corresponding LH heating power in the plasma center, dependent on x and y: $F_{\mathrm{Ext}}{}^{x}(x, y) = K_{\mathrm{Ext}}\ x\ y$

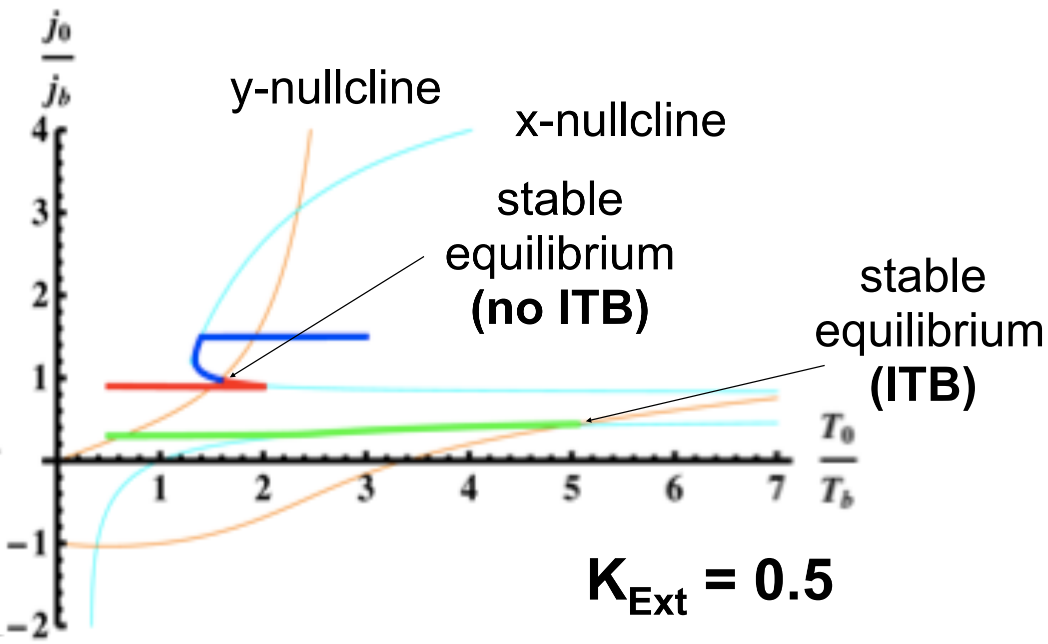
K_{Ext} is then the control parameter in our model.

Results
Numerical integration

Numerical integration for two different values of K_{Ext} :

Given the remarkably distinct time-scales, $\tau_x/\tau_y = \mathcal{O}(10^{-3})$, the orbits are rapidly attracted to the x-nullcline (line where $\dot{x} = 0$) and then approach the stable equilibria:

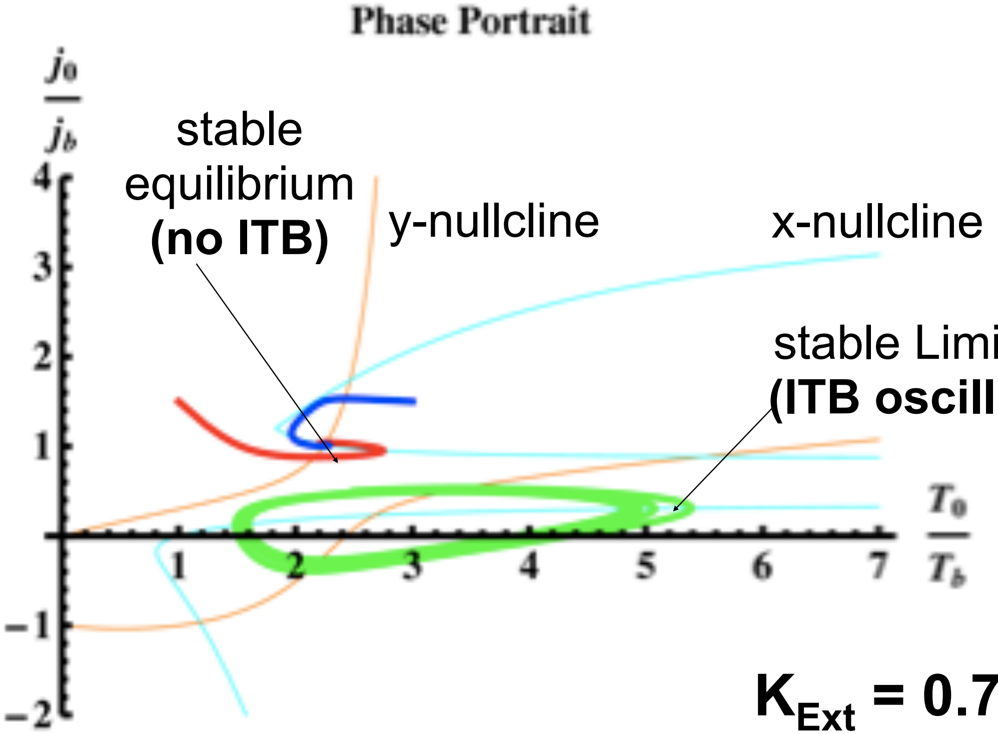




Results
Oscillatory regime

- Precedent parameter values: no oscillations observed.
- Singular perturbation theory: there can be *relaxation oscillations* but nullclines' shape and relative position must be compatible. In our case, they are not favorable to periodic solution.

We are, however, **able to observe ITB oscillations if we set** $\tau_x/\tau_y = 1$ **!**



$K_{Ext} = 0.7$

Conclusions

- Our model is consistent with experiments in the following points:
  - It captures a single equilibrium typical of a pure Ohmic regime ($y \sim 1$, $x \sim 1$) when no external sources are present - **ITB absence**;
  - For an external non-inductive current above a critical value, an additional stable equilibrium appears, typical of a steady state, advanced tokamak regime ($0 < y < 1$, $x >> 1$) - **ITB presence**;
  - The coexistence of these two distinct stable equilibria is consistent with the fact that careful plasma preparation is required for long non-inductive operation.
- For the type of discharges considered, **ITB oscillations** are captured when the characteristic transport and resistive times are similar.