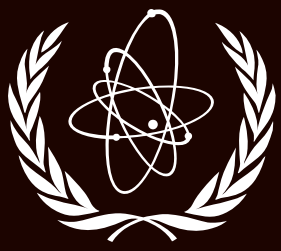

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# Introduction to Water Cooled Reactor Theory with the Micro-Physics Simulator Lite Edition

VIENNA, 2019

TRAINING COURSE SERIES 70

# INTRODUCTION TO WATER COOLED REACTOR THEORY WITH THE MICRO-PHYSICS SIMULATOR LITE EDITION

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TRAINING COURSE SERIES No. 70

# INTRODUCTION TO WATER COOLED REACTOR THEORY WITH THE MICRO-PHYSICS SIMULATOR LITE EDITION

INTERNATIONAL ATOMIC ENERGY AGENCY
VIENNA, 2019

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INTRODUCTION TO WATER COOLED REACTOR THEORY WITH THE MICRO-PHYSICS SIMULATOR LITE EDITION
IAEA, VIENNA, 2019
IAEA-TCS-70
ISSN 1018–5518



Printed by the IAEA in Austria
June 2019

## FOREWORD

The IAEA offers its Member States a suite of basic principle nuclear power plant simulators for education and training in various advanced technologies. These simulators allow hands-on, interactive learning on a wide range of topics for different types of reactor technologies. The IAEA assists Member States in the use of simulators through training courses and by publishing related documentation.

One simulator available to Member States is the Micro-Physics Nuclear Reactor Simulator Lite Edition, or MPSL. The IAEA provides users of the MPSL access to a user manual developed by Nuclear Engineering Ltd of Osaka, Japan, containing information on simulator operation, features, models, benchmarks and specifications. The present publication is intended to support the user manual with a substantive detailing of reactor theory fundamentals, providing sufficient background information and examples to enable readers to understand and use the MPSL. Users may refer to this introductory text to gain a conceptual understanding of simulator responses or to build foundational knowledge for further education on reactors or the use of other IAEA simulators. The content is closely linked with the MPSL so that inexperienced users can become more familiar with, and better understand, the key concepts and fundamental theory behind it; conversely, the simulator can be used as an effective means to reinforce concepts already learned.

The publication provides basic information that can be applied directly in training courses, as lecture material or as student or trainee reference material, and that can be incorporated into curricula on a variety of nuclear related topics. It is expected to be useful to students or trainees as a basic reference text on reactor theory fundamentals and is not intended as an exhaustive textbook or an extensive reference work. The concepts covered are intended to directly support the use and demonstration of the MPSL, providing a comprehensive introduction to reactor theory through an accessible, hands-on learning experience.

The IAEA thanks Nuclear Engineering Ltd for providing the Micro-Physics Nuclear Reactor Simulator Lite Edition. The IAEA acknowledges the valuable contributions of M. Tatsumi (Japan) and R. Schow (United States of America). The IAEA officers responsible for this publication were C. Takasugi and T. Jevremovic of the Division of Nuclear Power.

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**CONTENTS**

# 1. INTRODUCTION

## 1.1.BACKGROUND

Educational basic principle nuclear power plant (NPP) simulators support a hands-on learning approach to education on a variety of nuclear engineering topics. While many of these simulators support reactor operations, reactor safety, and reactor systems education directly with interactive, full plant models, the Micro-Physics Simulator Lite (MPSL), a water cooled reactor (WCR) basic principle simulator provided to the IAEA by Nuclear Engineering Ltd., offers a different approach based largely on the observation of reactor core behaviour. The MPSL has a valuable position among IAEA NPP simulators in its capacity to support education on fundamentals of reactor theory with relevant NPP simulator exercises.

This publication is intended to provide users with a 'big picture' and basic principle understanding of reactor operation. It is intended to serve as a basis of knowledge for use of the MPSL, though much of the content can be generally applied to NPP simulators or used in general nuclear engineering education.

## 1.2.OBJECTIVES

The objectives of this publication are to:

- Serve as a reference for lectures on basic WCR theory, operating principles and fundamentals on reactor dynamics.
- Support Member States intending to use IAEA NPP simulators in national education and training programmes with a resource for necessary background information on WCR theory.
- Serve as a reference for students and trainees, who may be unfamiliar with reactor operation (e.g. engineering safety, physical sciences, non-nuclear engineering fields, and including non-technical fields) to understand reactor theory at a basic principle level.
- Provide MPSL exercises for use in IAEA education and training courses, use by Member State institutions, or direct use by students or trainees.

## 1.3.SCOPE

The scope of this publication is to provide basic level information on reactor theory within the framework of knowledge necessary to understand the MPSL. This includes basic coverage of nuclear physics in relation to reactor theory, as well as of basic reactor dynamics and explanations of a variety of interrelated operating parameters. The level of explanation is restricted to conceptual knowledge and basic principles, with simplified models — though it is sufficiently comprehensive to allow users to understand the fundamentals of the MPSL and other IAEA simulators and to support further learning.

Discussions in this publication are focused on theory related to pressurized water reactors (PWRs); however, many of the same concepts also apply to other WCR technologies.

The MPSL, as with other IAEA educational basic principle NPP simulators, is intended for education purposes only and not for design, licensing, technology assessment, or safety analysis.

## 1.4.STRUCTURE

This publication contains four sections:

- **Water Cooled Reactor Designs** provides an overview of WCR designs, such as the basic configuration and function of systems and components, relevant to later explanations.
- **Nuclear Physics: Fundamentals of Interest to Water Cooled Reactor Operation** provides background information on relevant nuclear principles, neutron classifications, and neutron interactions.
- **Water Cooled Reactor Operating Parameters** is comprised of an overview of basic reactor kinetics, followed by an explanation of conditions contributing to reactivity and core life.
- **Micro-Physics Simulator Lite Exercises** contains examples of depletion analysis and transient analysis functions of the MPSL, with explanations of parameter trends and detailed discussion on the observed trends.

## 2. WATER COOLED REACTOR DESIGNS

One method of classifying reactors is according to their fuel, coolant, and moderator materials. These materials can have a major effect on the reactor design, included systems and considerations for operation. This section describes the basic concepts of the common WCR designs, distinguished by these materials, and highlights systems and considerations important to reactor operation.

Reactor fuel serves as both a source of neutrons and as a material for fission, producing additional neutrons and releasing energy. Common materials for WCR fuel include uranium and/or mixed oxide (MOX) fuel. Uranium can be used in a reactor either in its natural composition or can be enriched to have a higher relative content of $^{235}U$ (as opposed to $^{238}U$), which readily undergoes fission by slow, or thermal, neutrons. MOX fuel consists of more than one type of nuclide that can undergo fission in oxide form, but often refers to fuel which has both uranium and plutonium oxides [1].

It is common for WCR fuel to be manufactured as small pellets and loaded into tubes called fuel rods or fuel pins. These pins are then arranged together with a support structure to form an assembly or bundle. These assemblies are placed in a core plate, typically in a circle to form a cylindrical reactor core.

In WCRs, coolant functions to remove heat produced by fission and transfer energy to drive a turbine–generator system to produce electricity. The WCRs include those cooled by either light water ($H_2O$) or heavy water ($D_2O$).

Moderator is used in thermal reactors to reduce neutron speed. Neutrons originating from fission often have a high energy and are unlikely to interact with fuel materials. At the lower, 'moderated' speed, the likelihood of neutrons interacting with fuel resulting in fission is, importantly, increased. Moderator fills the spaces in the core between the fuel pins, so that neutrons leaving one fuel component will be slowed prior to re-entering fuel materials. For water cooled reactors, water or graphite is used as a moderator.

Currently operating WCRs are split into four general categories, as follows:

- **Pressurized water reactors (PWRs)** may be fuelled by either enriched uranium or MOX fuel and are both cooled and moderated by $H_2O$ that is kept at high pressure to prevent boiling at high temperatures.
- **Boiling water reactors (BWRs)** may be fuelled by either enriched uranium or MOX fuel and are both cooled and moderated by $H_2O$ that is allowed to boil in the core under normal operating conditions.
- **Pressurized heavy water reactors (PHWRs)** may be fuelled by natural uranium, moderated by $D_2O$ and cooled by either $H_2O$ or $D_2O$ that is pressurized to prevent boiling at high temperatures.
- **Light water cooled graphite moderated reactors (LWGRs)** may be fuelled by either enriched uranium or MOX fuel, moderated by graphite and cooled by $H_2O$.

Another type of reactor commonly cooled by water, not used for power generation, is water cooled research reactors. These reactors are typically used for training and research. There are great variations in both design and operating principles, depending on the intended application. Other WCR technologies are under development, such as supercritical water cooled reactors which operate with coolant pressurized above the light water critical pressure.

## 2.1.PRESSURIZED WATER REACTORS (PWRs)

PWRs are light water moderated and light water cooled reactors, and the most widely used power generating reactor type. The typical PWR design utilizes two, completely separated circuits of coolant water. These are referred to as the 'primary' and 'secondary' coolant loops. The primary loop is filled entirely with high pressure liquid water and the secondary loop has both liquid water and steam. By separating into two loops, minimal radioactive shielding is required on the secondary loop. A simplified two loop PWR coolant system with all the discussed components is illustrated in Figure 1.

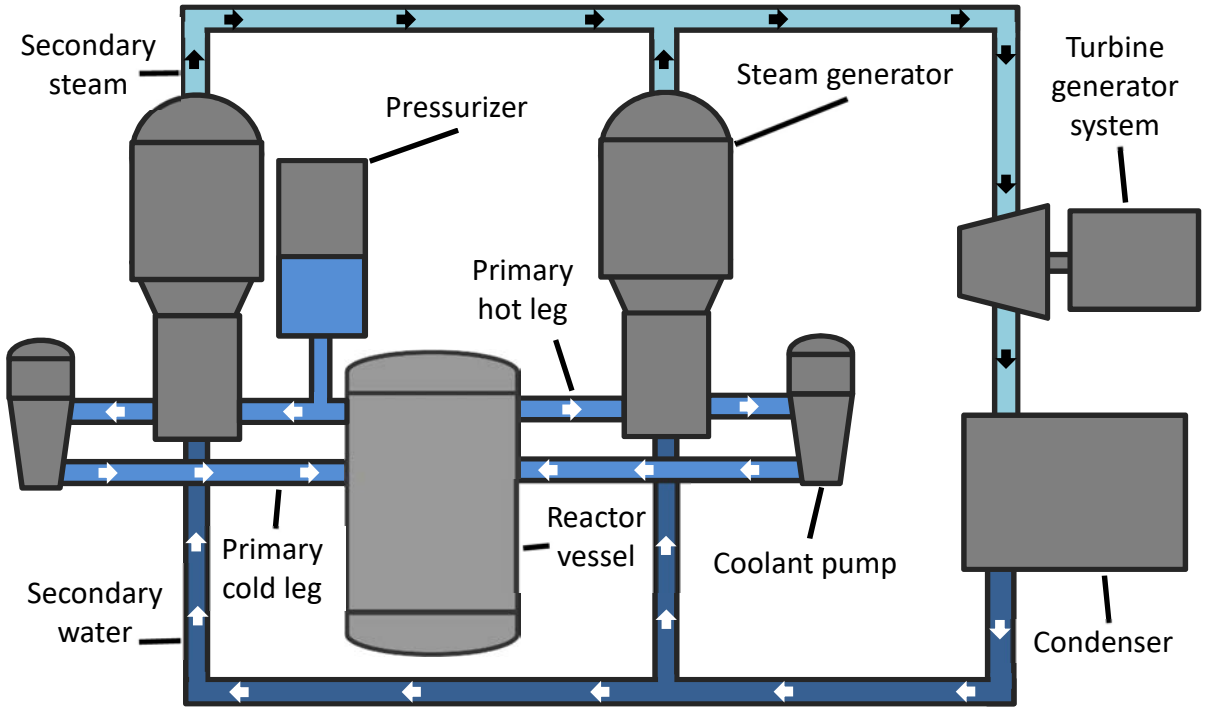


*FIG. 1. Simplified two loop PWR plant layout.*

The primary loop consists of the flow path from the reactor vessel, where the reactor core is located, through piping to steam generators and back to the reactor vessel. Water is heated when passing through the core in the reactor vessel and cooled in the steam generator. For this reason, the portion flowing from the reactor vessel, where the coolant water is hottest, to the steam generator is known as the 'hot leg'; the portion flowing from the steam generator back to the reactor vessel is known as the 'cold leg'.

The temperature throughout the primary loop is typically well above the boiling temperature of water at atmospheric pressure, and water is heavily pressurized to prevent boiling. This pressure

control is facilitated by the ‘pressurizer’, typically found on the hot leg of one of the steam generators. The pressurizer is partially filled with water and partially filled with steam. When a pressure increase is required, heaters in the pressurizer boil water into steam. Conversely, when a pressure decrease is required, sprays in the pressurizer condense steam into water. Additional features of the primary loop include reactor coolant pumps, which force the flow of water in the primary loop and are typically located on the cold leg.

Steam generators serve as the boundary between the primary and secondary loop. Primary fluid enters from the hot leg, flows through tubes in the steam generator, and exits into the cold leg. Secondary fluid flows into the steam generators and over the tubes of primary fluid.

Secondary fluid is at a lower pressure, so it boils from the heat transferred from the primary fluid. Steam then flows from the steam generators and is used to drive the turbine–generator system to generate electricity. Following this, it passes through a condenser and is returned as liquid to the steam generator. It is common for PWR systems to include multiple steam generators.

## 2.2.OTHER WATER COOLED REACTOR TECHNOLOGIES

Because PWRs are the focus of this publication, other types of WCRs are described only in terms of their major operating principles and their differences from PWRs. Descriptions in later sections of this publication assume a PWR, though much of the basic theory and many of the operating principles remain the same.

### 2.2.1. Boiling water reactors (BWRs)

The main, defining characteristics of BWRs are that they allow boiling within the reactor vessel and have only a single coolant loop. After PWRs, they are the second most widely used nuclear electricity generation reactor type. Water is boiled as it passes through the core, and through steam separators and dryers in the top of the reactor vessel, and steam flows to the turbine–generator system where electricity is generated. Steam is then condensed and returned to the reactor vessel. BWRs often have recirculation pumps, which force the flow of water over the core. A simplified two loop BWR coolant system with all the discussed components is illustrated in Figure 2.

Because there is no secondary loop to separate the coolant that passes over the core from the turbine–generator system, the entire loop becomes radioactive and requires shielding. Additionally, any pressure changes or challenges to flow directly impact the conditions in the core. BWRs do not require steam generators or the pressurizer that are typical of PWRs, as the function of each is performed by boiling within the reactor vessel.

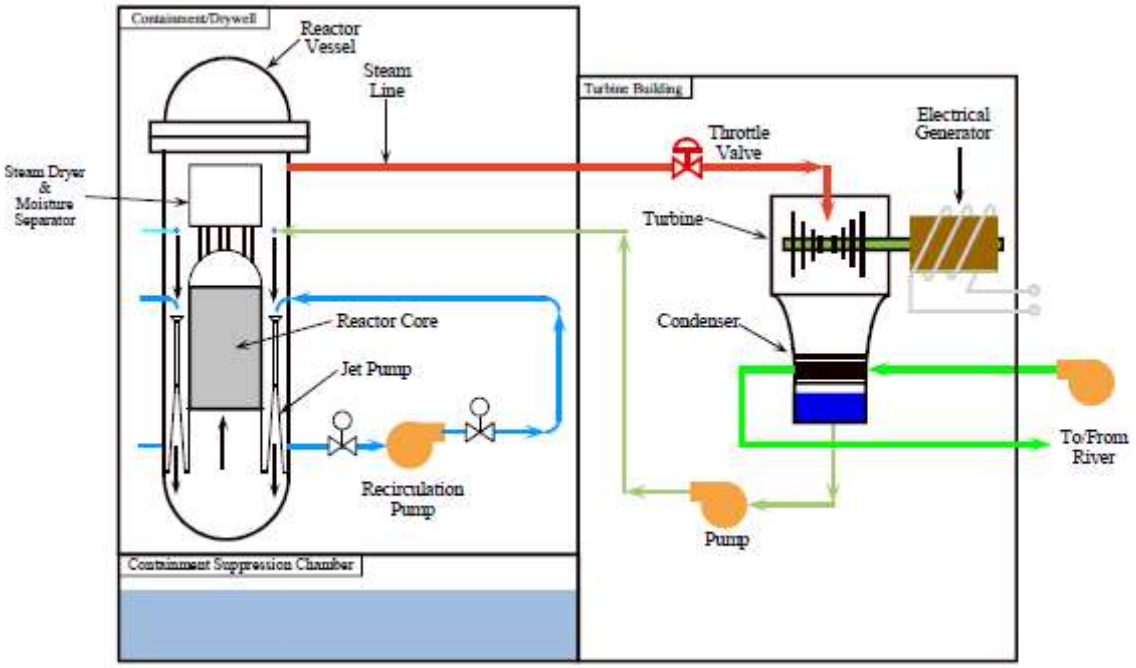


*FIG. 2. Schematic of boiling water reactor.*

### 2.2.2. Pressurized heavy water reactors (PHWRs)

The third most widely used nuclear electricity generation reactor type is PHWRs. These reactors are fuelled by natural enrichment uranium and moderated by heavy water. They may be cooled by either light or heavy water. For reasons which are explained in more detail later, heavy water improves the efficiency of the neutron life cycle, thereby allowing the reactor to be fuelled by non-enriched uranium.

PHWR systems operate similarly to PWRs and most of the same systems are used. The major difference, however, is seen within the reactor vessel. Common PHWR designs load natural enrichment uranium fuel in short fuel rods arranged into cylindrical bundles. These are then loaded horizontally into the reactor vessel end to end in several large, pressurized tubes. To refuel, these tubes are depressurized and fuel bundles are pushed through the opposite end.

### 2.2.3. Light water cooled graphite moderated reactors (LWGRs)

LWGRs are cooled by light water, moderated by graphite, and fuelled by low enrichment uranium. Graphite moderation is more effective than water; however, it is less effective than heavy water. As such, fuel must be enriched, but not as significantly as in light water PWRs and BWRs. Coolant is partially boiled as it passes through the core region, and the mixture of liquid water and steam is passed to a steam separator system. The steam separator directs steam to a turbine–generator system followed by a condenser and returns liquid water to the core.

In comparison to PWRs, the fuel is less enriched and the core design greatly changed to accommodate graphite moderation. Similar to BWRs, LWGRs create steam within the core, but separate it from water away from the vessel. As a result, there is no steam generator or pressurizer, and no boundary between water cooling the core and steam reaching the turbine–generator.

## 2.3.SAFETY SYSTEMS

Reactor safety systems are designed to accomplish three safety functions [2]:

(1) Control reactivity;
(2) Remove heat from the core;
(3) Contain radioactive material.

Reactivity and related systems are discussed later in this publication, as some key concepts must first be explained.

Cooling of radioactive material is essential for preventing damage or even melting of fuel. Different reactor designs use different safety systems to address this issue. Safety cooling systems primarily focus on 1) maintaining coolant inventory in the event of leaking, 2) providing a heat sink for coolant to transfer heat to, and 3) providing electrical power to any essential systems (i.e. generators to power pumps to cool the core). The following are some examples, which may or may not be included depending on the design:

— Safety injection systems that pump additional water into the primary coolant loop and restore coolant inventory. These may require significant power to run the pumps.
— Depressurization systems consist of a series of valves which may be opened to vent steam from the coolant system. The pressure in the coolant system drops, allowing safety injection systems to supply fresh, cool water.
— Gravity drain tanks are elevated containers of coolant, which will drain into the primary coolant loop to restore coolant inventory.
— Heat exchange loops can allow coolant to flow away from the reactor to heat exchangers submerged in coolant pools. These can be either forced circulation with pumps or naturally circulating.
— Accumulators are pressurized tanks of coolant separated from the primary coolant loop by ‘check valves’ which open when the coolant system falls below a certain pressure. Once opened, coolant flows from the accumulator and into the coolant system.
— Sump recirculation systems. In the event of leaking, water may accumulate beneath the reactor system. Sump recirculation collects this water and pumps it back into the coolant system to maintain the coolant inventory.

Containment of radioactive material consists of several layers to prevent its release to the environment, typically including the following [3]:

— Fuel matrix is individual pieces of fuel, such as ceramic fuel pellets, and contains some of the radioactive material produced.
— Fuel cladding is the tube in which fuel is loaded to make fuel elements or fuel pins.
— Coolant system boundary consists of the systems and components of the primary coolant system, including the reactor pressure vessel, steam generators, primary system piping, primary system coolant pumps and the pressurizer.

— Containment is a large structure, often steel and reinforced concrete, built over the full primary coolant system.

Maintenance of the earlier boundaries is largely performed by the same safety systems which provide core cooling. As the temperature rises, they are the mechanism for preventing fuel melting or cladding damage.

Containments have a design pressure and if the atmosphere inside the containment rises above this value, then its capacity to contain radioactive material is reduced. Containments incorporate safety systems, such as water sprays or heat exchangers, which reduce pressure by cooling and condensing steam that has been released from the coolant system boundary. The containment atmosphere may also be vented through a series of filters to reduce pressure and minimize the radiation released.

In some accident scenarios, flammable hydrogen may be produced and released into the containment. In certain conditions, this hydrogen poses a risk of explosion and destruction of the containment. Some designs include safety systems for removing hydrogen from the atmosphere or igniting hydrogen before it rises to a level which may cause damage.

There are a large number and variety of safety systems. This publication is focused primarily on the theory of water cooled operating parameters which affect the core. The most important safety systems considered in this publication are those which control reactivity, discussed later, and those which result in temperature and pressure changes in the primary coolant system.

## 2.4.OPERATIONAL LIMITS AND MARGINS

Reactor operators must maintain the reactor parameters within acceptable limits to ensure safe operation. These boundaries are known as operational limits and conditions (OLCs) and include the following [4]:

— **Safety limits** are limiting values for parameters such as fuel and cladding temperatures or coolant pressure to prevent significant radioactive material from being released. For example, high fuel temperatures can lead to damage or even melting of fuel and removal of barriers to radioactive material release, so a safety limit is set at the temperature at which damage may begin to occur.
— **Limiting safety system settings** are the parameter values, which are used to initiate safety systems or other means of preventing a safety limit from being reached. For example, indications that fuel temperatures are nearing the safety limit may activate safety systems to provide additional cooling.
— **Limits and conditions for normal operation** are measures taken to ensure safe operation, within the assumptions of a safety analysis report. This may include a minimum staffing requirement, minimum amount of functional equipment, or a maximum amount of radioactive material that can be released as part of a procedure.
— **Surveillance requirements** include regular inspections, monitoring, calibration and testing of equipment to ensure that all other OLCs are being met.

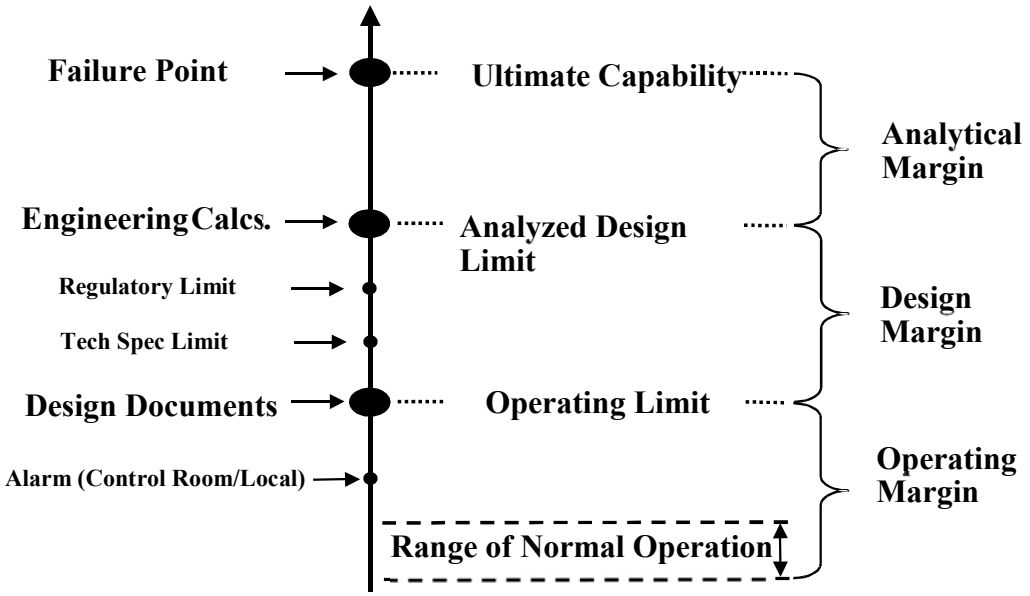


*FIG. 3. Analytical, design and operational margins.*

These limits are designed for specific reactors and based on analysis of the reactor design. A safety margin, including analysis uncertainty and conservative restrictions, is applied in addition to the limits to provide confidence that safety is ensured during operation and the safety limit is never reached (see Figure 3). These limits are typically defined in a plant's technical specifications, along with rules for minimum staffing and qualifications, administrative reporting and responsibilities, plant specific design information, etc. as strict rules and conditions for operation and set the boundaries of acceptable plant operation.

# 3. NUCLEAR PHYSICS: FUNDAMENTALS OF INTEREST TO WATER COOLED REACTOR OPERATION

This section provides the necessary background for understanding the fundamentals of nuclear phenomena occurring within a reactor core. This includes the mass–energy balance, radioactive decay, various neutron interactions, and neutron classification.

## 3.1.NUCLEAR REACTIONS

Nuclear power production is based on energy produced from atomic nuclei. Fission reactors utilize neutron–nucleus fission interactions in order to produce large quantities of energy, but this is just one of many possible neutron interactions. It is essential to first establish the relationship between mass and energy as well as the ways in which neutrons or nuclei may be produced and lost. The mass energy balance and radioactive decay are discussed in this section.

### 3.1.1. Mass and energy balance

Nuclear interactions are comparable to chemical interactions. Much like chemical interactions, energy must be conserved in every nuclear reaction; nuclear interactions do, however, allow for conversions of mass to energy and of energy to mass. The following relation shows this conservation for stationary particles:

$$\sum_i [E_i + m_i \mathrm{c}^2] = \sum_i [E_i' + m_i' \mathrm{c}^2] \tag{1}$$

In this equation, $E$ is the energy and $m$ is the mass of the interacting particles. The square of the speed of light, c, is used as the conversion between mass and energy. A simple conversion can be done by changing the units by the square of the speed of light; the result is the relation $\mathrm{c}^2 = 931.5\ \mathrm{MeV/amu}$.

The amount of kinetic energy gained, or equivalently the amount of rest mass energy lost, in an interaction is described as the $Q$ value, given by the following two equations:

$$Q = \sum_i E_i' - \sum_i E_i \tag{2}$$

$$Q = \left[\sum_i m_i - \sum_i m_i'\right] \mathrm{c}^2 \tag{3}$$

A reaction with a positive $Q$ value is known as an exothermic reaction (releasing energy), and a negative $Q$ value as an endothermic reaction (absorbing energy).

The following example $Q$ value calculation shows the case of an interaction in which a neutron interacts with a boron nucleus to produce an alpha particle and a lithium nucleus, which is in an excited state 0.48 MeV above its ground state:

$$\mathrm{n} + {}^{10}_{5}\mathrm{B} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{7}_{3}\mathrm{Li}^{*} + Q \tag{4}$$

$$Q = [\mathrm{m_n} + \mathrm{m_{B-10}} - \mathrm{m_{He-4}} - \mathrm{m_{Li-7}}]\mathrm{c}^2 - 0.48\ \mathrm{MeV}$$

$$Q = [1.0086649\ \mathrm{amu} + 10.012937\ \mathrm{amu} - 4.0026032\ \mathrm{amu} - 7.0160040\ \mathrm{amu}]\left(931.5\frac{\mathrm{MeV}}{\mathrm{amu}}\right) - 0.48\ \mathrm{MeV}$$

$$Q = 2.310\ \mathrm{MeV} \tag{5}$$

Since the $Q$ value in this example is positive, this is an exothermic reaction.

When protons and neutrons are combined to form a nucleus, the resultant nucleus has a mass less than the sum of its parts. This phenomenon is known as mass defect. The mass defect can be calculated as the difference between the summed nucleon mass and the resultant nucleus:

$$\Delta m = Z \cdot \mathrm{m_p} + (A - Z) \cdot \mathrm{m_n} - m \tag{6}$$

This energy equivalent of the mass defect is known as the binding energy and can be found by using the 931.5 MeV/amu conversion. Another important value is the binding energy per nucleon, which is found by dividing the binding energy by the number of protons and neutrons in the nucleus. The trend for the binding energy per nucleon as a function of mass number is shown in Figure 4.

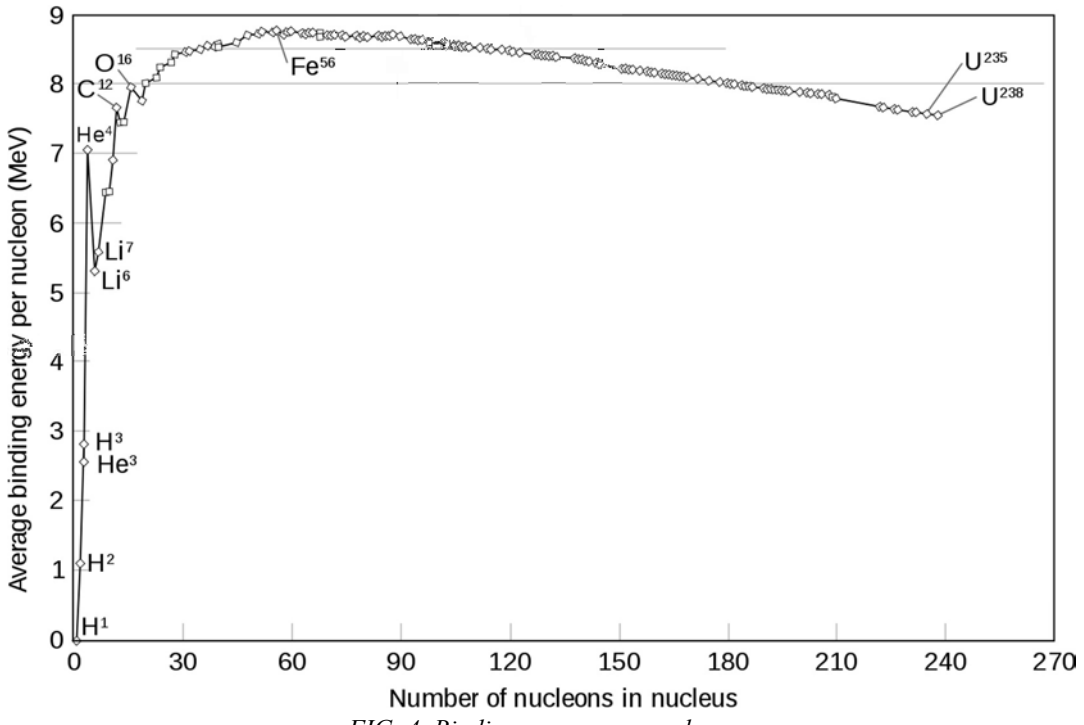


*FIG. 4. Binding energy per nucleon.*

A peak binding energy per nucleon is located at $^{56}$Fe and represents an important boundary. In general, nuclei which are lower in mass number can be combined in a process called fusion to release energy, while those with higher mass number can be broken apart in a process called fission to release energy.

An example calculation for the mass defect of a $^{238}$U atom, with atomic mass number of 92, is calculated as follows:

$$\Delta m = 92 \cdot \mathrm{m_p} + (238 - 92) \cdot \mathrm{m_n} - m$$

$$\Delta m = 92 \cdot 1.007825032 \text{ amu} + 146 \cdot 1.008664923 \text{ amu} - 238.050785 \text{ amu}$$

$$\Delta m = 1.9342 \text{ amu} \tag{7}$$

The energy equivalence can be found for this value, by using the 931.5 MeV/amu conversion, to be 1801.7 MeV. Furthermore, the average binding energy per nucleon is 7.6 MeV, found by dividing the binding energy by the total number of protons and neutrons (238).

### 3.1.2. Radioactive decay

Radioactive decay occurs in unstable nuclei. Most often discussed are those which decay from unstable nuclei to stable nuclei and result in emission of alpha (helium nuclei), beta (electron or positron), or gamma particles (electromagnetic waves originating from a nucleus). In addition to these, the decay of some nuclei may result in neutron emission.

As is detailed later in this section, a large variety of unstable nuclei are generated as a result of fission in a reactor core and can have a large impact on reactor operation. Therefore, a thorough understanding of radioactive decay is necessary. The rate of decay, or activity, of a sample is given by the equation:

$$A = \lambda \cdot N \tag{8}$$

In this equation, $N$ is the number of nuclei of the isotope that is undergoing radioactive decay. $\lambda$ is known as the decay constant and is different for every radioactive isotope. Eq. (8) can also be used to derive the number of atoms remaining in the sample by relating the activity to the change in number of nuclei over time:

$$A = -\frac{dN}{dt} = \lambda \cdot N$$

$$\int \frac{1}{N} dN = \int -\lambda \, dt$$

$$\ln N = -\lambda \cdot t + \mathrm{C}$$

$$N = \mathrm{e}^{\mathrm{C}} \mathrm{e}^{-\lambda t}$$

$$N = N_0 \mathrm{e}^{-\lambda t} \tag{9}$$

In this equation, a boundary condition is used that ensures the number of nuclei at a time of zero seconds is equal to the initial number of nuclei. This boundary condition ($\mathrm{e}^{\mathrm{C}} = N_0$) replaces the term including the constant of integration to give the final equation. The result of this

derivation is an equation for the number of nuclei as a function of the original number of nuclei, the decay constant, and time.

Another term which is used to describe radioactive decay is the half-life, which describes the amount of time it takes for half of the nuclei to decay. A relation for the half-life can be derived from Eq. (9) by solving for time where the number of nuclei is half the initial value:

$$\frac{1}{2}N_0 = N_0 \mathrm{e}^{-\lambda t_{\frac{1}{2}}}$$

$$\ln\left(\frac{1}{2}\right) = -\lambda t_{\frac{1}{2}}$$

$$t_{\frac{1}{2}} = \frac{\ln(2)}{\lambda} \tag{10}$$

From Eq. (9), the amount of an isotope remaining, and resulting the amount of activity, is shown to fall exponentially. This, however, is not as straightforward if the isotope that is decaying is also being generated at the same time, as several isotopes in a reactor are. Radioactive decay is revisited later in this publication, as additional concepts must be introduced prior to further discussion.

### 3.2.NEUTRON INTERACTIONS

Neutrons can undergo several different interactions with nuclei. The probability of an interaction is described as a cross-section, which depends on both the neutron energy and the target nucleus. Microscopic cross-sections are quantified in units of barn, where one barn is $10^{-24}$ cm$^2$, and describe the interaction probability as a material characteristic. Alternatively, macroscopic cross-sections are given in units of inverse length to take into account the density of target nuclei.

The following interaction descriptions include: elastic scattering, inelastic scattering, radiative capture, and fission. Microscopic cross-sections for the included interactions are grouped as shown in Figure 5 and represented by the Greek lowercase letter $\sigma$ with subscripts corresponding to different interactions.

In practical applications, it is useful to describe the probability of interaction using the macroscopic cross-section. While the microscopic cross-section describes the probability of interaction with a single nucleus, the macroscopic cross-section considers materials with some density of nuclei. Macroscopic cross-sections, represented by the Greek uppercase letter Σ, are given by:

$$\Sigma = N\sigma \tag{11}$$

where $N$ is the number density of the nuclide in a material being considered.

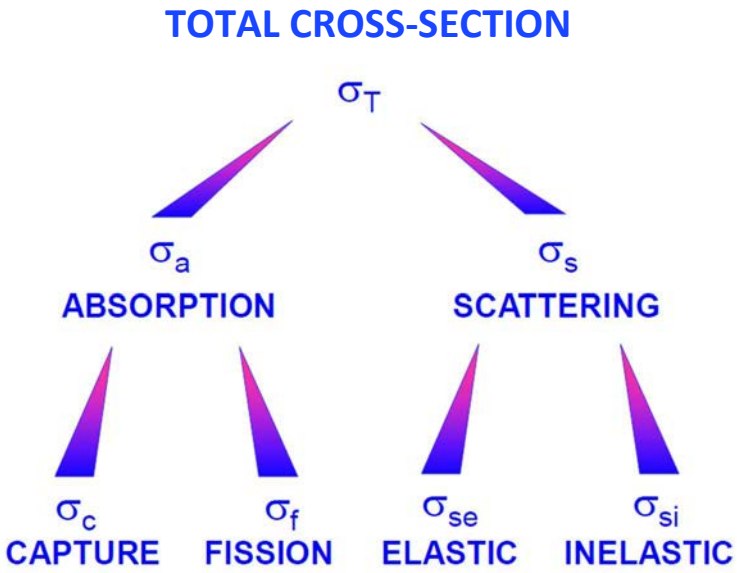


*FIG. 5. Cross-section categorization and denotation.*

Because of the effect on interaction cross-sections, it is useful to classify different types of neutrons according to energy. There are very precise definitions for different energy ranges; however, for the sake of simplicity, three regions are considered: fast, epithermal, and thermal. Fast neutrons have large energies, on the order of 1 MeV (or higher). The slowest, lowest energy neutron category considered are thermal neutrons. Between fast and thermal is the epithermal range. This region is often characterized by resonance peaks in absorption cross-sections; in other words, there is a high likelihood of interaction for a particular energy of neutron. These three regions are shown in Figure 6, with boundaries at 1 eV and 0.1 MeV.

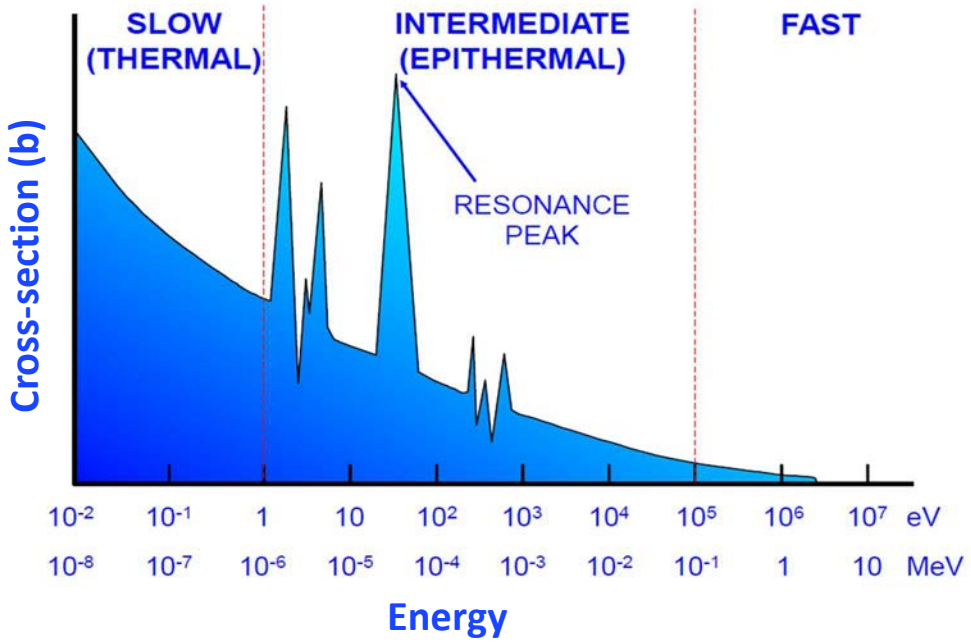


*FIG. 6. Illustrative trend of microscopic cross-section for absorption as a function of neutron energy.*

Neutron interactions can be separated into direct and compound interactions. Direct interactions occur over a very short time, on the order of $10^{-22}$ seconds, where the neutron does not disturb the target nucleus. In compound interactions, the neutron forms a 'compound nucleus', which

has a half-life of approximately $10^{-16}$ seconds. This compound nucleus then disintegrates or 'decays' by one of a few pathways, each resulting in different overall interaction products. The pathway of this decay occurs independent of the compound nucleus formation. Resonance peaks in interaction cross-sections occur at energies corresponding to the energies resulting in formation of a compound nucleus.

### 3.2.1. Elastic scattering

Elastic scattering refers to a neutron interaction where the neutron interacts with the target nucleus, with the result of the same neutron and nucleus following the interaction. The neutron may transfer some kinetic energy to the nucleus in this process. Figure 7 shows the elastic scattering interaction of a neutron with a target nucleus.

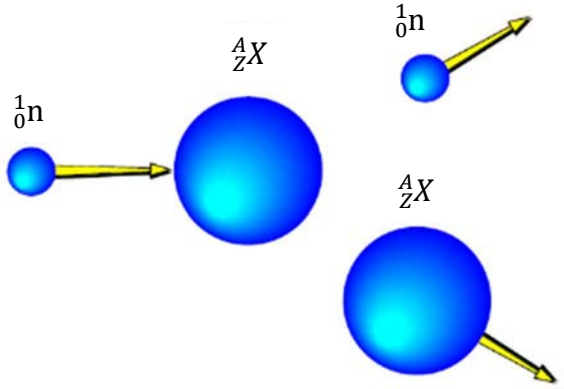


*FIG. 7. Elastic scattering of neutron and nucleus X.*

Elastic scattering may or may not occur with the formation of a compound nucleus. This leads to two possible interaction types, potential elastic scattering and resonance elastic scattering, respectively given by:

$$^{1}_{0}\mathrm{n} + {}^{A}_{Z}X \rightarrow {}^{1}_{0}\mathrm{n} + {}^{A}_{Z}X \tag{12}$$

$$^{1}_{0}\mathrm{n} + {}^{A}_{Z}X \rightarrow {}^{A+1}_{Z}X^{*} \rightarrow {}^{1}_{0}\mathrm{n} + {}^{A}_{Z}X \tag{13}$$

In Eq. (13), the compound nucleus is denoted by an asterisk.

The probability for potential elastic scattering is nearly constant across neutron energies. Resonance elastic scattering, by way of compound nucleus formation, leads to several interaction resonance energies. The result of these two is a relatively stable interaction cross-section in the low neutron energy range, with resonances peaks appearing at higher energies. The elastic scattering cross-section then has two regions, as shown in Figure 8.

Elastic scattering is important for reactors as it allows fast neutrons to 'thermalize' or lose energy until they are in the thermal energy region as shown in Figure 9. This is important in reactors as many common reactor fuel materials have large fission cross-sections for thermal neutrons.

The amount of kinetic energy lost by the neutron is dependent on the target nucleus, due to conservation laws of energy and momentum. The more massive the target nucleus is, the less energy is lost. The less massive the target nucleus, the more energy is lost. A neutron 'moderator' is used to thermalize neutrons in a reactor and is typically made of low mass nuclei. For most WCRs, water is used as both the coolant and the moderator. Water is a good moderator due to the large presence of hydrogen, the least massive nuclei, and convenient due to water's wide availability.

$D_2O$ is a particularly good moderator because of the large number of hydrogen nuclei present. The reason PHWRs can use non-enriched fuel is because of the highly effective moderation provided by heavy water

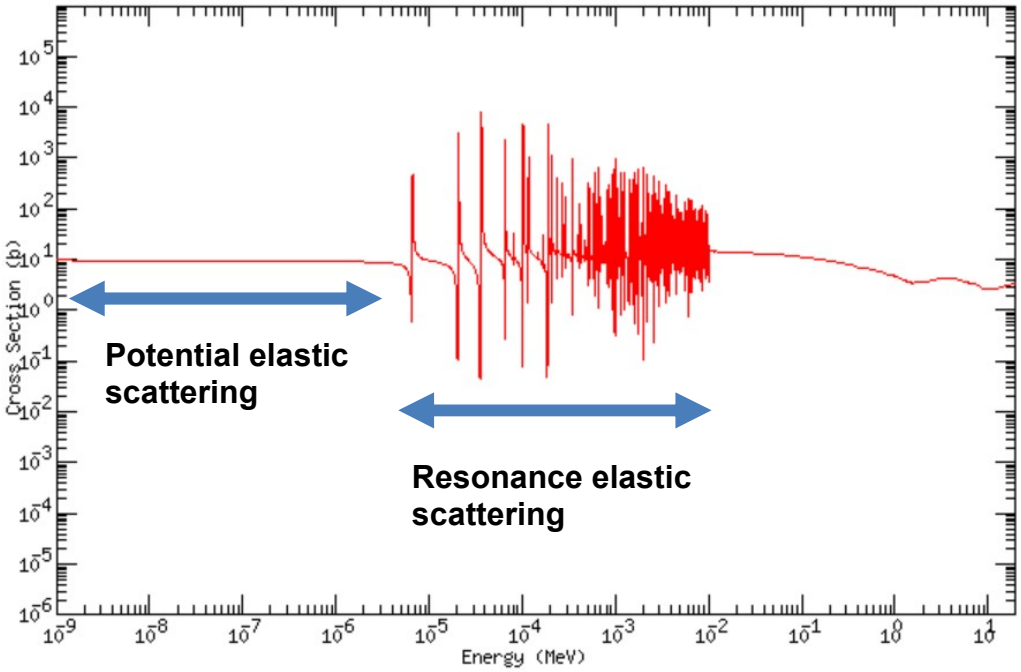


*FIG. 8. Microscopic cross-section for elastic scattering of $^{235}U$ [5].*

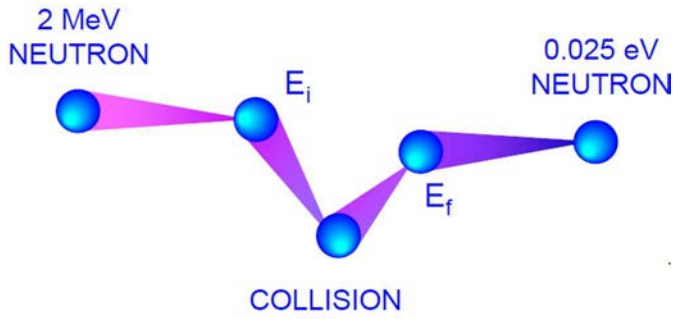


*FIG. 9. Thermalization of a neutron by elastic scattering.*

### 3.2.2. Inelastic scattering

In neutron inelastic scattering a compound nucleus is formed, and results in emission of a neutron so that the product nucleus is left in an excited state. The excited nucleus then returns to its ground state through emission of gamma ray(s). This is illustrated in Figure 10.

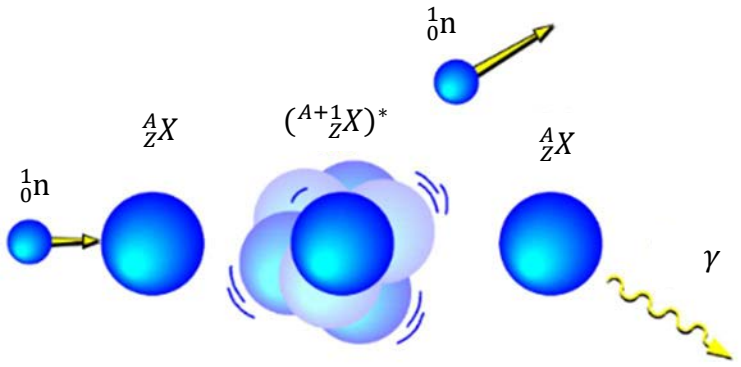


*FIG. 10. Inelastic scattering of neutron and nucleus X.*

The interaction equation is as follows:

$$^{1}_{0}\mathrm{n} + ^{A}_{Z}X \rightarrow (^{A+1}_{Z}X)^{*} \rightarrow ^{1}_{0}\mathrm{n}' + ^{A}_{Z}X + \gamma \qquad (14)$$

In Eq. (14), an apostrophe indicates the significant energy loss of the neutron used to place the nucleus in an excited state. Because a large amount of energy is required to excite the nucleus, this is a 'threshold' interaction, which can only occur with neutrons that have greater energy than the energy of the resultant excited nucleus state. As a result, the interaction cross-section for inelastic scattering is zero for low energy neutrons.

The excitation energy and inelastic scattering cross-sections of excited states are different for various nuclei, depending on mass. Low mass nuclei have very small cross-sections and typically higher energy thresholds. Inelastic scattering is thus more common in massive nuclei. Additionally, inelastic scattering can have multiple cross-sections, corresponding to the different excited nucleus states. An example for the first three inelastic scattering cross-sections is given in Figure 11.

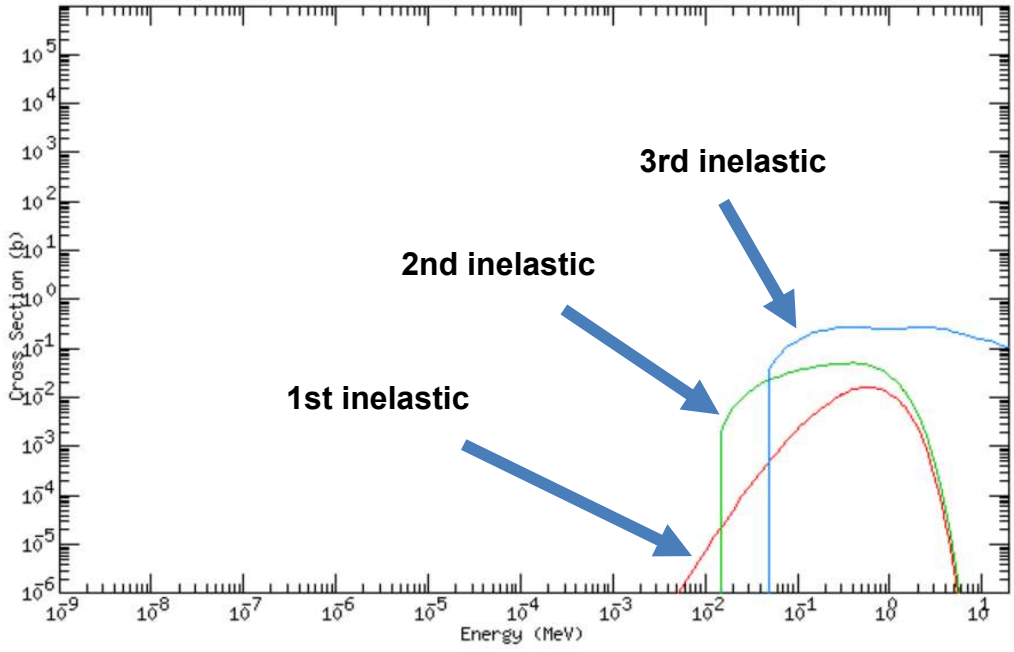


*FIG. 11. Microscopic cross-sections for first three inelastic scatterings of $^{235}$U [5].*

In reactors, inelastic scattering is important for the slowing of neutrons, particularly in the massive nuclei of the fuel. Neutrons of high energy, greater than the threshold interaction energy of fuel materials such as $^{238}$U, can lose significant energy because of inelastic scattering.

### 3.2.3. Radiative capture

In radiative capture interactions, neutrons are absorbed in nuclei to form a new nucleus, which then decays by emission of gamma ray(s). This is shown in Figure 12.

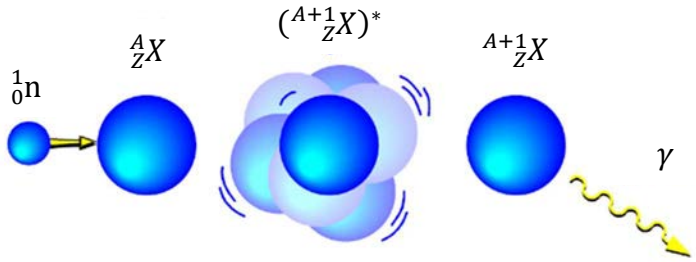


*FIG. 12. Radiative capture of neutron by nucleus X.*

The interaction equation for radiative capture is as follows:

$$^{1}_{0}\mathrm{n} + {}^{A}_{Z}X \rightarrow (^{A+1}_{Z}X)^{*} \rightarrow {}^{A+1}_{Z}X + \gamma \qquad (15)$$

Unlike inelastic scattering, radiative capture can occur for any energy of neutron.

There are three distinct energy regions when considering the interaction cross-section. In the low energy region, the cross-section is inversely proportional to the neutron velocity. At higher energies, resonance peaks are observed in the resonance region. Above this, radiative capture cross-sections fall to a relatively low and stable level. A qualitative interaction cross-section plot for a radiative capture is represented in Figure 13.

Radiative capture causes a significant number of neutron losses in a reactor. The neutrons undergoing radiative capture are lost from fission, thereby reducing the overall neutron population.

Nuclides with large non-fission absorption cross-sections, such as boron or hafnium, are therefore used as control rod materials to control the population of neutrons in the reactor; by changing the position of control rods within the reactor core, these materials are able to absorb enough of the neutrons such that the neutron count is at a stable level, keeping the produced power at a constant level (called the steady state condition of a reactor operation).

Because radiative capture results in the formation of new nuclei, neutron capture produces various isotopes in a reactor core. This includes many radioisotopes from neutron irradiation of structural materials or moderator impurities as well as new fissile material. One important example of this is that when $^{238}$U present in the fuel absorbs a neutron it becomes $^{239}$U, which undergoes two beta decays to become $^{239}$Pu. $^{239}$Pu, like the fuel $^{235}$U, has a large fission cross-section in the thermal neutron energy region and may contribute significantly to the neutron

population in a reactor. Radioactive isotopes that are generated may also decay by emission of alpha, beta, gamma or neutron radiation.

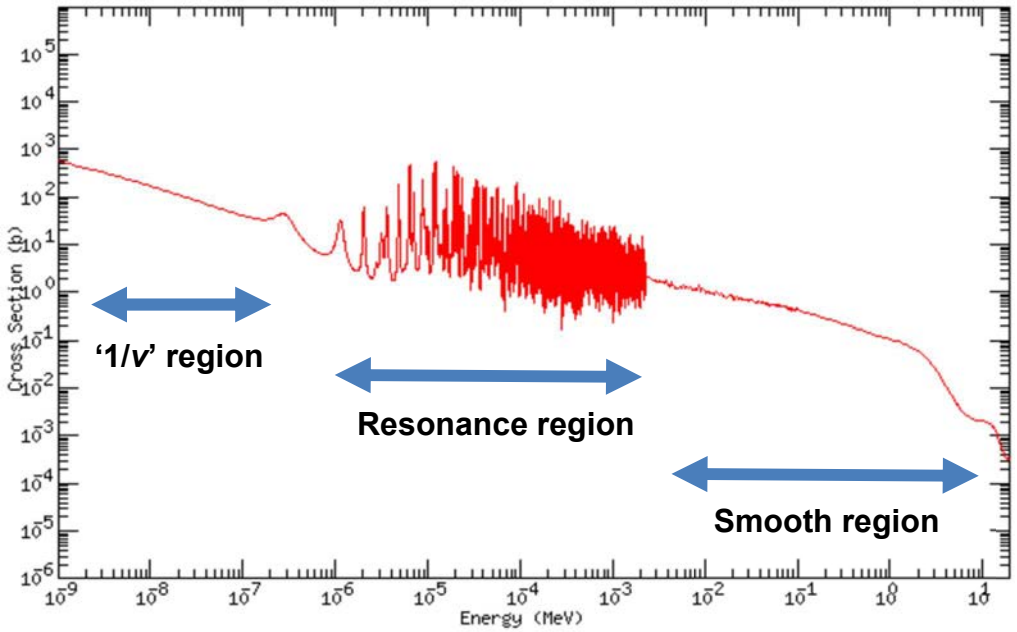


*FIG. 13. Microscopic cross-section for radiative capture of $^{235}U$ [5].*

### 3.2.4. Fission

Nuclear fission occurs when a nucleus is split into two or three smaller parts. By breaking up heavy nuclei, some mass is converted into energy. Fission can occur spontaneously in a few nuclei or be induced by introduction of energy such as by an incident neutron. Spontaneous fission is relatively rare, occurring in very few isotopes, and is a very slow process time wise. Induced fission, however, is the primary source of energy generated in a reactor. The general steps of induced fission are outlined in Figure 14 and an example of induced fission of $^{235}U$ is illustrated in Figure 15.

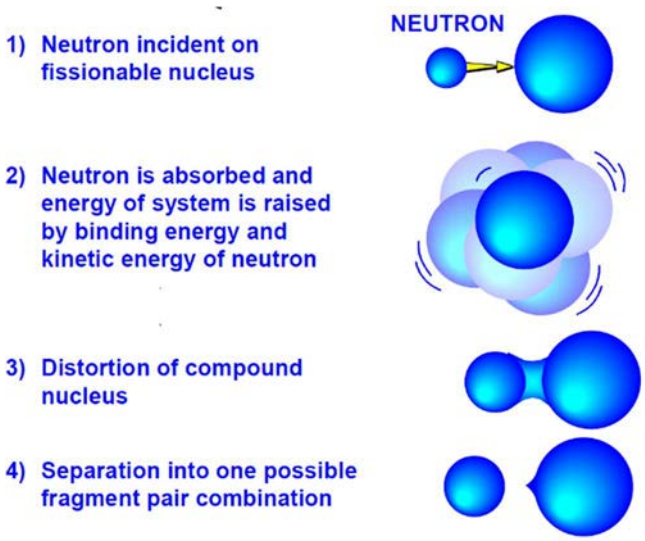


*FIG. 14. Induced fission process.*

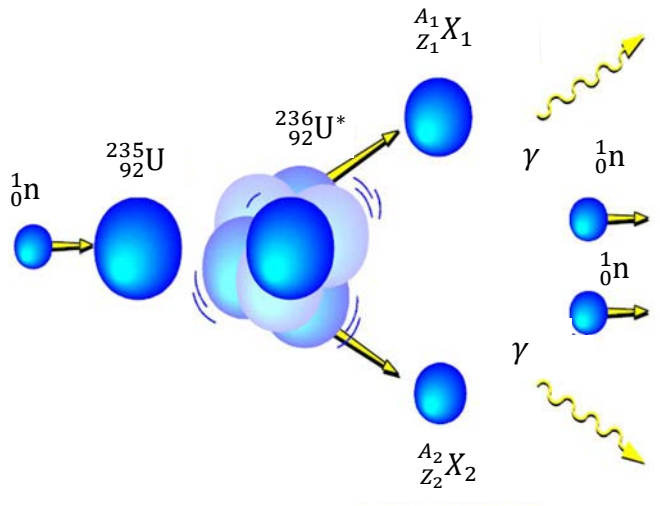


*FIG. 15. Fission of $^{235}U$ resulting in two fission products, two new neutrons and gamma radiation.*

The interaction equation for induced fission is given by:

$$^1_0n + ^A_ZX \rightarrow (^{A+1}_ZX)^* \rightarrow ^{A_1}_{Z_1}X_1 + ^{A_2}_{Z_2}X_2 + \nu \cdot ^1_0n + Q \tag{16}$$

where $\nu$ is the number of neutrons to maintain nucleon conservation for the formation of isotopes $X_1$ and $X_2$. The two generated isotopes are known as 'fission fragments' or 'fission products', because they are the result of fission interaction. In general, fission results in the one light and one heavy fission product. These, however, can be any of several nuclei within a range of masses. It is therefore useful to describe the relative production of various isotopes as a 'yield', or percent of total production of fission products. The yield distribution for fission products can then be illustrated as shown in Figure 16. The relative yield of different isotopes varies between different target nuclei undergoing fission.

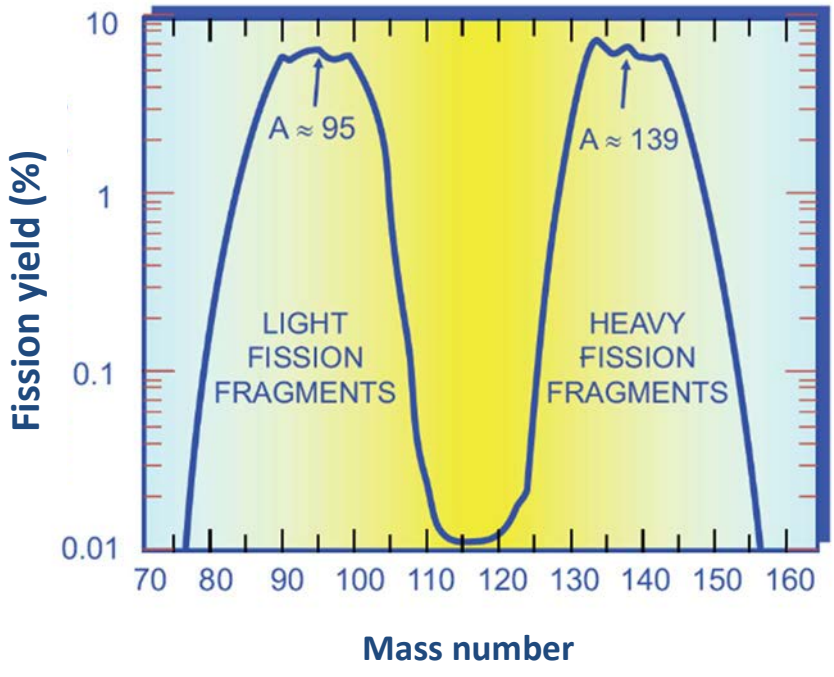


*FIG. 16. Illustration of fission product yield distribution as a function of mass number.*

While in theory most nuclei are able to undergo fission, only heavy nuclei can fission with reasonable yield. This is because the amount of energy necessary to fission nuclei tends to decrease for massive nuclei. Therefore, heavy materials such as $^{235}U$ or $^{239}Pu$ are good fission reactor fuel materials. In relation to those nuclei, which can be reasonably induced to fission, three terms must be defined to describe different cross-section behaviours [6]:

— **Fissile** nuclei require little energy and can sustain a nuclear chain reaction of fissions predominantly by slow, thermal neutrons. Fissile materials, such as $^{235}U$ and $^{239}Pu$, have fission cross-sections qualitatively similar to the radiative capture cross-section shown in Figure 13. That is, the cross-section shows an inversely proportional relationship to the neutron velocity at low energies, with the resonance region at intermediate energies, and with a smooth trend at high energies.

— **Fissionable** nuclei require more energy and can sustain a nuclear chain reaction of fissions, including those by fast neutrons. While this definition includes fissile nuclei, certain fissionable nuclei, such as $^{238}U$ and $^{240}Pu$, are 'fast fissionable' and have no fission cross-section for low or intermediate neutron energies. Fissions may only occur at energies greater than the resonance region and above a given threshold energy. This results in a fission cross-section trend as shown in Figure 17.

— **Fertile** nuclei are not fissile but may absorb neutrons and decay to then create fissile nuclei. One example is $^{238}U$, which can absorb a neutron and decay twice to become fissile $^{239}Pu$.

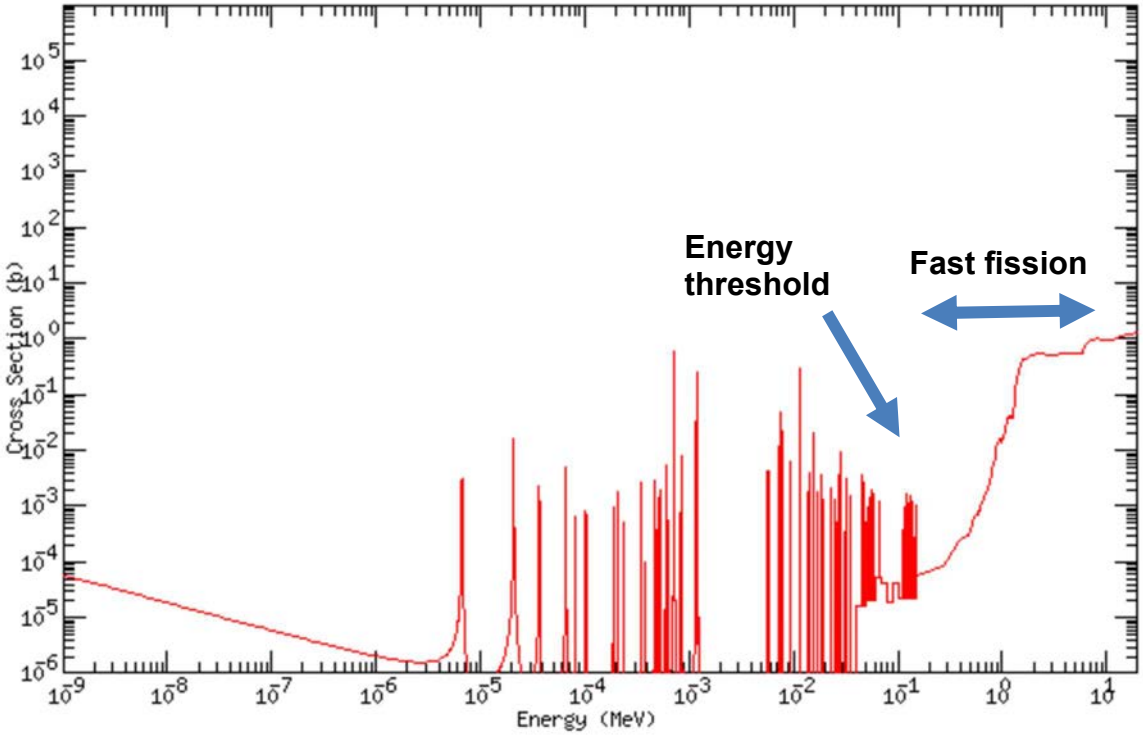


*FIG. 17. Microscopic cross-section for fission of $^{238}U$ [5].*

## 3.3.THE ROLE OF PROMPT AND DELAYED NEUTRONS IN REACTOR OPERATION

Neutrons originating from fission can be emitted either as prompt or delayed neutrons. Prompt neutrons are released at the time of a fission event, typically on the order of $10^{-14}$ seconds following fission of a nucleus. Delayed neutrons result from radioactive decay of the created fission products and are generated on average approximately 12.58 seconds following the initial fission event. The drastic difference in these time scales is important to reactor operation.

Following generation in a reactor, both neutrons share similar lifetimes of thermalization and reabsorption in approximately a $10^{-4}$ second timespan. However, the main difference is the prompt and delayed neutron 'generation time', which includes the release time difference. These generation times for prompt and delayed neutrons are denoted as $l^*$ and $l_d$, respectively, and illustrated in Figures 18 and 19.

Delayed neutrons are produced by the decay of several different fission products, called delayed neutron precursors, each of which has a different relative production from fission, rate of decay, and decay energy. These nuclides are often grouped, based on these differences. Table 1 [7] shows the properties of the common six delayed neutron groups.

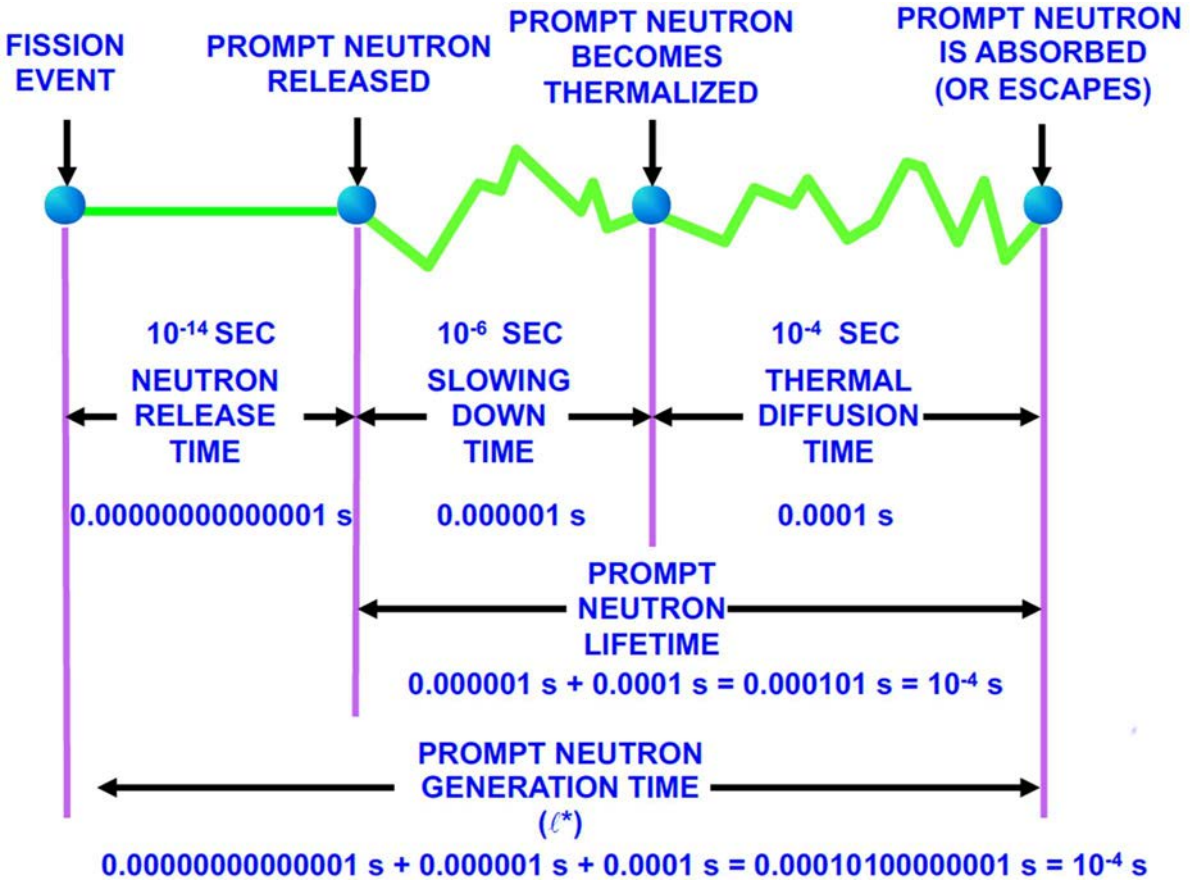


*FIG. 18. Prompt neutron lifetime and generation time.*

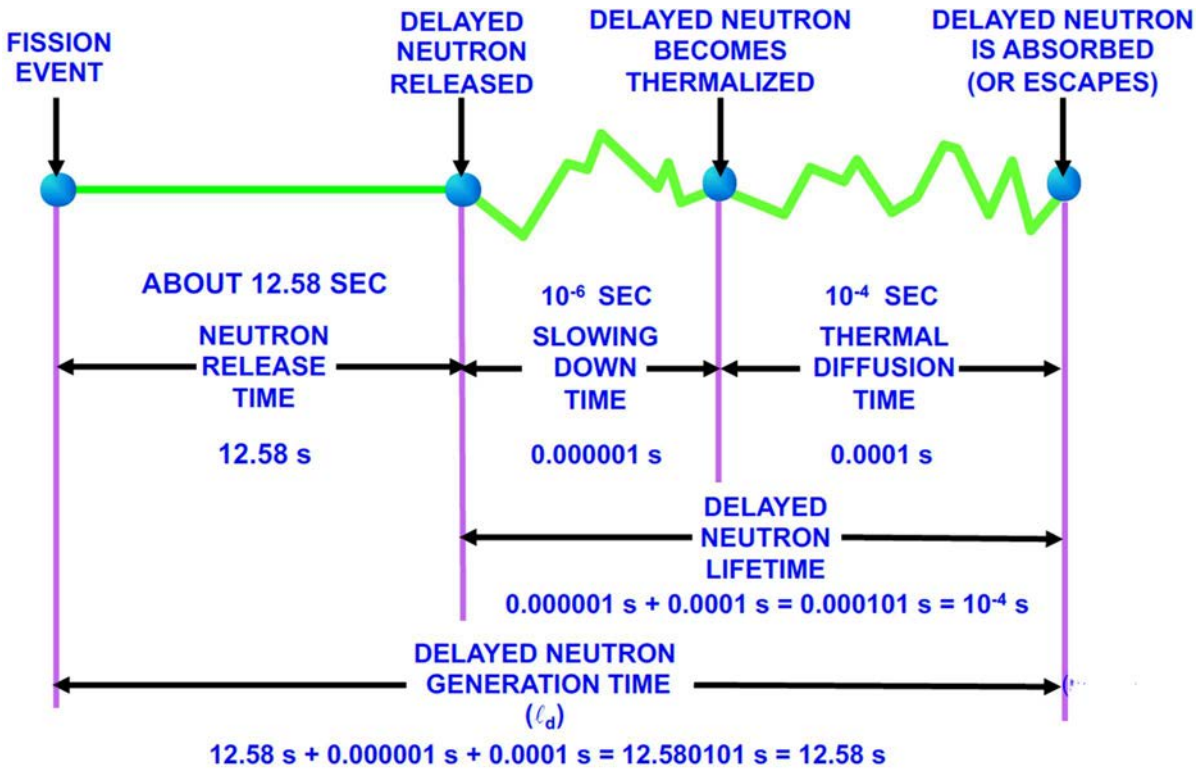


*FIG. 19. Delayed neutron lifetime and generation time.*

The delayed neutron fraction $\beta$, represents the total effective yield of delayed neutrons, and different nuclei (such as $^{238}$U or $^{239}$Pu) in a reactor core have different delayed neutron fractions and different fission yields. As the reactor operates, the relative quantity of materials undergoing fission changes. This is a result of some materials being removed through interactions such as fission and others being produced by absorption or decay. The relative production of each neutron precursor changes in response to this. An example of changing contribution to the overall core delayed neutron fraction is given in Table 2.

The overall delayed neutron fraction of the core, or the total delayed neutron fraction, is represented as a weighted sum of contributing delayed neutron fractions:

$$\beta_{\text{core}} = (\beta_{\text{U}-235})(\gamma_{\text{U}-235}) + (\beta_{\text{U}-238})(\gamma_{\text{U}-238}) + (\beta_{\text{Pu}-239})(\gamma_{\text{Pu}-239}) \qquad (17)$$

Using the example values given in Table 2, this would result in a total delayed neutron fraction of 0.0070 at the beginning of core life and 0.0054 at the end of life. The beginning of core life is characterized by fresh fuel consisting largely of uranium and which has not accumulated $^{239}$Pu through neutron absorption in $^{238}$U. The end of core life, however, is characterized by a significant amount of $^{239}$Pu created by this absorption process and a lessened amount of $^{235}$U fuel due to fission. With consideration of both prompt and delayed neutrons, a mean neutron generation time can be established by weighting the relative number of prompt and delayed neutrons, indexed by energy group, to their corresponding generation times:

$$\bar{l} = (1 - \beta)l^* + \sum_i \beta_i \cdot l_i \qquad (18)$$

Because the prompt neutron generation time is very short, and difficult to control in a reactor core, reactors typically operate in such a way that delayed neutrons are essential to maintaining the fission chain reaction. The details of how the neutron population is affected by the delayed neutron fraction are discussed in the following section.

TABLE 1. DELAYED NEUTRON PRECURSOR GROUPS EMITTED IN THERMAL FISSION OF $^{235}U$

| GROUP | PROBABLE PRECURSOR | % OF DELAYED NEUTRONS | HALF-LIFE (SECONDS) | MEAN LIFE (SECONDS) | EFFECTIVE YIELD | NEUTRONS DELAYED PER FISSION |
|---|---|---|---|---|---|---|
| 1 | Br-87 | 3.23 | 55.60 | 80.21 | 0.00021 | 0.00052 |
| 2 | I-137 | 21.7 | 22.00 | 31.74 | 0.00141 | 0.00346 |
| 3 | Br-39 | 19.55 | 6.20 | 8.94 | 0.00127 | 0.00310 |
| 4 | I-139 | 29.3 | 2.30 | 3.32 | 0.00255 | 0.00624 |
| 5 | As-85 | 11.4 | 0.60 | 0.87 | 0.00074 | 0.00182 |
| 6 | Li-9 | 4.16 | 0.23 | 0.33 | 0.00027 | 0.00066 |
| | | | | TOTAL | 0.00645 | 0.00158 |

TABLE 2. DELAYED NEUTRON FRACTION AND FISSION YIELD

| | U-235 | U-238 | Pu-239 |
|---|---|---|---|
| DELAYED NEUTRON FRACTION | 0.0064 | 0.0156 | 0.0021 |
| FISSION YIELD (%) | | | |
| BEGINNING OF LIFE | 0.93 | 0.07 | 0.00 |
| END OF LIFE | 0.55 | 0.07 | 0.38 |

# 4. WATER COOLED REACTOR OPERATING PARAMETERS

The large number of neutrons in a reactor core and potential neutron interactions necessitates the use of more macroscopic models for reactor behaviour than presented in the previous section. This section therefore explains a point kinetics simplified model for the neutron population and describes several factors which can change the probability of various neutron interactions to occur. The scope of this section is to provide a basic understanding of core lifetime and transient behaviours.

## 4.1.REACTOR KINETICS

Reactor kinetics describes the time dependent behaviour of neutron flux, and thus the power, in a reactor. The models which are introduced in this section are point kinetic models which ignore the shape of the neutron flux distribution, and effectively reduce the reactor system to a point.

While the reactor kinetics models presented in this section are relatively simple, significant rigour can be introduced to develop more accurate models of reactor behaviour (for more discussion on this point see [8]).

### 4.1.1. Criticality

One method of quantifying the changing number of neutrons in a reactor core is with equations which consider the different ways neutrons can be produced or lost. The model defined with Eq. (19) is known as the six factor formula for the neutron life cycle, and the output is known as the effective neutron multiplication factor. The effective neutron multiplication factor describes the relation given in Eq. (20).

$$k_{\text{eff}} = \varepsilon \cdot L_{\text{f}} \cdot p \cdot L_{\text{th}} \cdot f \cdot \eta \tag{19}$$

$$k_{\text{eff}} = \frac{number\ of\ neutrons\ in\ a\ generation}{number\ of\ neutrons\ in\ the\ previous\ generation} \tag{20}$$

The fast fission factor, denoted as $\varepsilon$, is the ratio between the number of fast neutrons in the previous generation plus those produced from fast fissions and the fast neutrons originating from the previous generation. In this way, it accounts for the production of neutrons, which result from fast fission.

The fast non-leakage probability, denoted as $L_{\text{f}}$, is the probability that a fast neutron does not escape from the reactor system.

The resonance escape probability, denoted as $p$, is the probability that neutrons are not absorbed in a resonance peak during the thermalization process. Neutrons output from this term are successfully thermalized.

The thermal non-leakage probability, denoted as $L_{\text{th}}$, is the probability that a thermal neutron does not escape from the reactor system.

The thermal utilization factor, denoted as $f$, is the ratio of neutrons which are absorbed in fuel to those absorbed in all reactor core materials (including the fuel). This can be described in terms of macroscopic absorption cross-sections:

$$f = \frac{\Sigma_{\mathrm{a}}^{\mathrm{fuel}}}{\Sigma_{\mathrm{a}}} \tag{21}$$

The thermal fission factor, denoted as $\eta$, is the average number of fast neutrons produced for each thermal neutron absorbed in fuel material. These neutrons are then passed to the next generation. The thermal fission factor can be described from fuel cross-sections and the average number of neutrons produced from a single fission, denoted as $\nu$, to be:

$$\eta = \frac{\nu\sigma_{\mathrm{f}}^{\mathrm{fuel}}}{\sigma_{\mathrm{a}}^{\mathrm{fuel}}} \tag{22}$$

The fast non-leakage factor, resonance escape probability, thermal non-leakage factor, and thermal utilization factor all reduce the number of neutrons in the generation, while the fast and thermal fission factors increase the number of neutrons. Figure 20 is an illustration of a stable neutron generation cycle, with an example set of six factors as given in Table 3.

The output of the six factor neutron life cycle is the effective neutron multiplication factor, $k_{\mathrm{eff}}$. When this factor is at unity, the number of neutrons in each cycle is constant. When above unity, more neutrons are produced than in previous generations. Conversely, when below unity, fewer neutrons are produced in each generation. These three regions of $k_{\mathrm{eff}}$ define the reactor criticality with characteristics and effects on neutron flux as shown in Table 4. This table also shows the reactivity, $\rho$, for these various states.

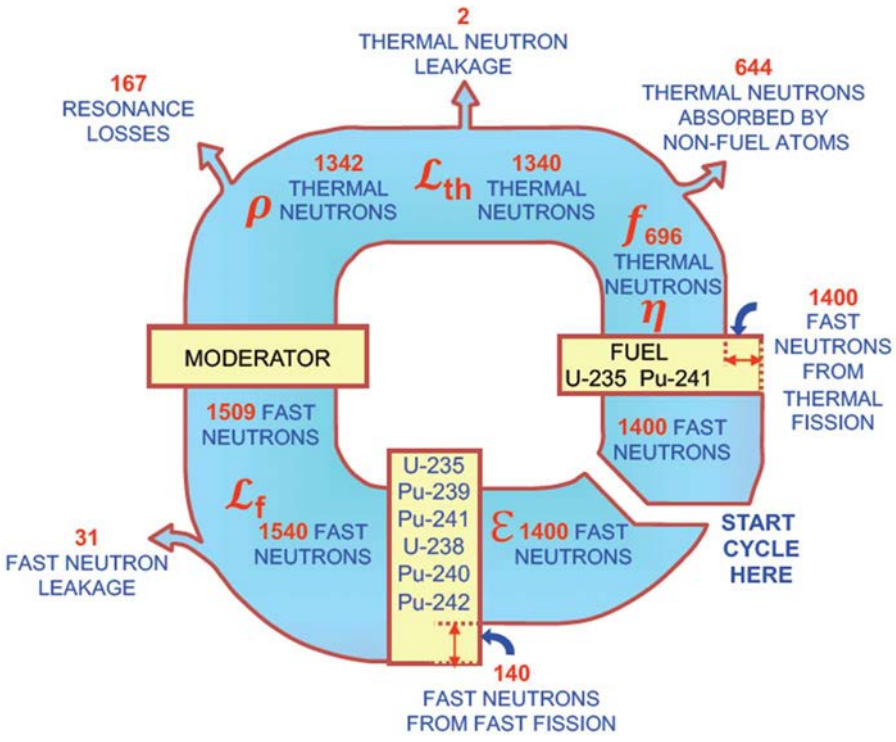


*FIG. 20. Application of six factor formula in a neutron generation cycle.*

TABLE 3. SIX FACTORS FOR EXAMPLE REACTOR WITH $k_{\text{eff}} = 1$

| FACTOR | VALUE | INPUT NEUTRON COUNT | OUTPUT NEUTRON COUNT |
|---|---|---|---|
| Fast fission factor ($\varepsilon$) | 1.100 | 1400 | 1540 |
| Fast non-leakage probability ($L_{\text{f}}$) | 0.980 | 1540 | 1509 |
| Resonance escape probability ($\rho$) | 0.889 | 1509 | 1342 |
| Thermal non-leakage probability ($L_{\text{th}}$) | 0.999 | 1342 | 1340 |
| Thermal utilization factor ($f$) | 0.519 | 1340 | 696 |
| Thermal fission factor ($\eta$) | 2.011 | 696 | 1400 |

TABLE 4. REACTOR CRITICALITY STATES

| $k_{\text{eff}}$ | $\rho$ | REACTOR STATE | CHARACTERISTICS | EFFECT ON NEUTRON FLUX |
|---|---|---|---|---|
| < 1 | < 0 | Subcritical | The number of neutrons in one generation is less than the number in the previous generation. | Decreasing neutron flux, or stabilized/stabilizing to a constant subcritical level. |
| = 1 | = 0 | Critical | The number of neutrons in one generation is equivalent to the number in the previous generation. | Stable neutron flux. |
| > 1 | > 0 | Supercritical | The number of neutrons in one generation is greater than the number in the previous generation | Increasing neutron flux. |

### 4.1.2. Reactivity

Reactivity describes the departure from a critical reactor state. In terms of the effective neutron multiplication factor, reactivity is defined as:

$$\rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} \tag{23}$$

The units of reactivity are Δk/k or defined in consideration of the effective neutron multiplication factor. It is also common to express reactivity in percent or per cent mille (pcm) of Δk/k, since it is often a small number.

An alternative form of Eq. (23) can be found by solving for the effective neutron multiplication factor:

$$k_{\text{eff}} = \frac{1}{1 - \rho} \tag{24}$$

Reactivity can be used to describe the various reactor criticality states, as included in Table 4. The first three of these relations can be shown by inserting the state conditions for effective neutron multiplication factor, found in Table 4, into Eqs. (23) or (24).

In addition to describing reactor conditions, reactivity is also used to describe operator actions or component effects. For example, operators could ‘insert negative reactivity’ to lower the

overall core reactivity. Or similarly, a fuel assembly may have a 'reactivity worth' which contributes to the overall core reactivity. Reactivity insertions such as these are applied to the reactivity term in the previous equations. A positive reactivity insertion would reduce the denominator term of Eq. (24) by the inserted amount and increase the effective neutron multiplication factor. The following example shows a 0.150 Δk/k insertion to a subcritical ($k_{\text{eff}} = 0.70$) reactor:

$$\rho = \frac{0.70 - 1}{0.70} = -0.429\ \Delta\text{k/k} \tag{25}$$

$$k_{\text{eff}} = \frac{1}{1 - \left(-0.429\frac{\Delta\text{k}}{\text{k}} + 0.150\frac{\Delta\text{k}}{\text{k}}\right)} = 0.782 \tag{26}$$

Equation (25) shows the determination of the reactor's initial reactivity using Eq. (23) and Eq. (26) shows the resulting increased effective neutron multiplication factor. An important outcome of the definition of reactivity is the importance of recognizing that equal reactivity insertions do not correspond to equal changes in the effective neutron multiplication factor. This can be demonstrated by performing the same reactivity insertion on the output of the previous calculation.

$$\rho = \frac{0.782 - 1}{0.782} = -0.279\ \Delta\text{k/k} \tag{27}$$

$$k_{\text{eff}} = \frac{1}{1 - \left(-0.279\frac{\Delta\text{k}}{\text{k}} + 0.150\frac{\Delta\text{k}}{\text{k}}\right)} = 0.886 \tag{28}$$

Instead of increasing by 0.082 as in the previous calculation, a jump of 0.104 is observed.

### 4.1.3. Subcritical multiplication

Subcritical multiplication is a phenomenon which occurs during the startup process of a reactor. Prior to startup, the reactor is in a subcritical state. As part of normal startup procedure, reactivity insertions are performed to bring the reactor to a supercritical state and increase the neutron flux in the core. In the early stages of this process, where the reactor is still subcritical and despite the continued negative effect of subcriticality, the neutron flux increases. This is known as subcritical multiplication and occurs because, in addition to those neutrons produced from fission, neutrons are added to the system from other sources (spontaneous fission or interactions resulting in neutron production); these neutrons are known as source neutrons. In order for subcritical multiplication to occur, the number of source neutrons must compensate for those lost. This relationship is given by:

$$S_{\text{o}} = N(1 - k_{\text{eff}}) \tag{29}$$

where $S_{\text{o}}$ is the number of source neutrons and $N$ is the total number of neutrons, both source and fission neutrons. The difference of $k_{\text{eff}}$ from one is representative of the relative loss of

neutrons in each generation. Rearranging Eq. (29) can show that the neutron count at which the reactor stabilizes is:

$$N = \frac{S_{\mathrm{o}}}{1 - k_{\mathrm{eff}}} \tag{30}$$

This equation shows that, despite a subcritical reactor state, neutron count rises as the effective neutron multiplication factor approaches unity. Prior to reaching this stable level, the number of neutrons in the $n^{\mathrm{th}}$ generation is given by:

$$N_n = S_{\mathrm{o}} \frac{1 - k_{\mathrm{eff}}^n}{1 - k_{\mathrm{eff}}} \tag{31}$$

This can be calculated for given values of source neutron count and effective neutron multiplication factor:

Starting with a source neutron count of 100 introduced into each generation and an effective neutron multiplication factor of 0.8, the stable count can be found using Eq. (30):

$$N = \frac{100 \text{ neutrons}}{1 - 0.8} = 500 \text{ neutrons} \tag{32}$$

The approach to this value can be found iteratively using Eq. (31). The first 12 iterations are shown in Table 5. The total neutron count approaches the calculated stable level, however the net gain of neutrons between generations is observed to fall. With this particular model for neutron count per generation, the calculated stable level acts as a limit of the calculation and is only actually reached in rounding or at the infinitieth generation. However, an approximate value is reached in relatively few iterations.

TABLE 5. SUBCRITICAL MULTIPLICATION WITH $k_{\mathrm{eff}} = 0.8$

| GENERATION $n$ | NEUTRONS FROM FISSION $N_{f,n} = N_{n-1} \times k_{\mathrm{eff}}$ | SOURCE NEUTRONS $S_{\mathrm{o}}$ | TOTAL NEUTRONS $N_n = N_{f,n} + S_{\mathrm{o}}$ | NEUTRONS GAINED $N_{+,n} = N_n - N_{n-1}$ |
|---|---|---|---|---|
| 0 | 0 | 100 | 100 | — |
| 1 | 80 | 100 | 180 | 80 |
| 2 | 144 | 100 | 244 | 64 |
| 3 | 195 | 100 | 295 | 51 |
| 4 | 236 | 100 | 336 | 41 |
| 5 | 269 | 100 | 369 | 33 |
| 6 | 295 | 100 | 395 | 26 |
| 7 | 316 | 100 | 416 | 21 |
| 8 | 333 | 100 | 433 | 17 |
| 9 | 346 | 100 | 446 | 13 |
| 10 | 357 | 100 | 457 | 11 |
| 11 | 366 | 100 | 466 | 7 |
| 12 | 373 | 100 | 473 | 7 |
| 13 | 378 | 100 | 478 | 5 |
| 14 | 382 | 100 | 482 | 4 |
| 15 | 386 | 100 | 486 | 4 |

Because the source neutron level is constant, Eq. (29) can be set equal to itself for different neutron counts and multiplication factors, and the following relation for neutron counts across changes in the effective neutron multiplication factor is found:

$$N_1(1 - k_{\text{eff},1}) = N_2\left(1 - k_{\text{eff},2}\right) \tag{33}$$

From this relation, the changes to effective neutron multiplication factor can be determined based on changes to the neutron count. A simple outcome of this equation is that halving the distance to criticality doubles the neutron count. However, the same relation does not apply when relating to reactivity.

Figure 21 shows an example of three steps toward a critical state and corresponding subcritical multiplication.

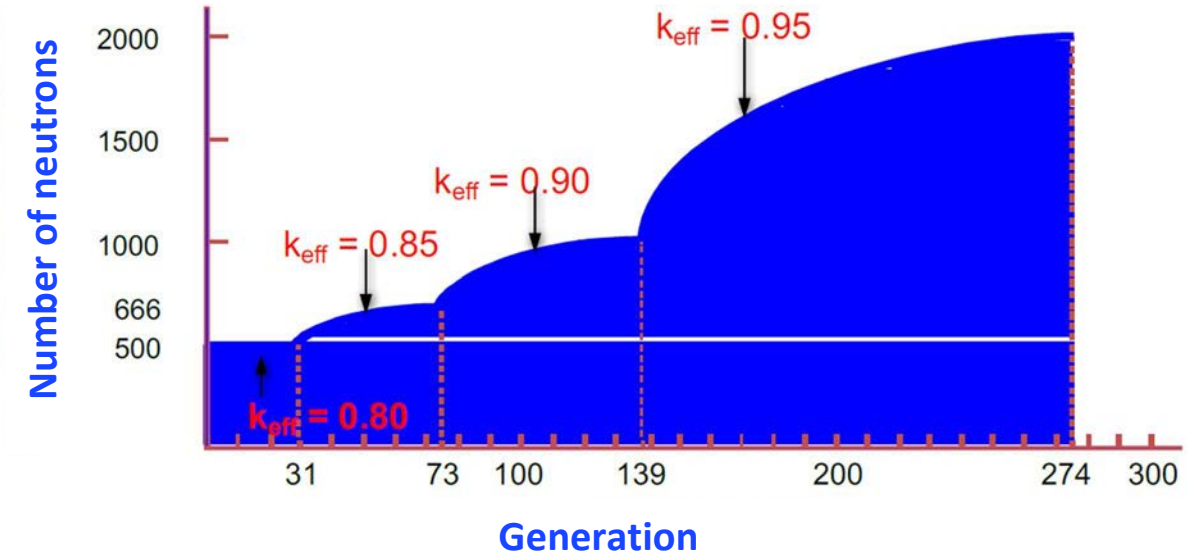


*FIG. 21. Subcritical multiplication in the rise to criticality.*

As mentioned earlier, reactivity and effective neutron multiplication factor changes are not equivalent. The insertion of reactivity necessary to double the neutron count is such that, if repeated, the reactor is then in a supercritical state. The following example demonstrates this effect for a subcritical ($k_{\text{eff}} = 0.80$) reactor:

$$\rho_1 = \frac{0.80 - 1}{0.80} = -0.250\ \Delta\text{k/k} \tag{34}$$

$$\rho_2 = \frac{0.90 - 1}{0.90} = -0.111\ \Delta\text{k/k} \tag{35}$$

Equation (34) shows the reactivity of the subcritical reactor prior to any reactivity insertions. Using the relation from Eq. (33), a doubling of the neutron count is observed when $k_{\text{eff}} = 0.90$, and the reactivity at this value is shown in Eq. (35). The difference between the two gives the reactivity insertion of 0.139 Δk/k. If this reactivity insertion is repeated, the reactor would be at a final reactivity of 0.028 Δk/k and be supercritical.

### 4.1.4. Neutron flux and power

The reactor power is a measure of the rate energy is produced by the reactor. This can refer to either the thermal power, which is the heat production rate, or the electric power, which is the amount of electrical power generated by harvesting the thermal energy. Thermal energy is produced by several mechanisms resulting from nuclear fission, such as the decay of fission products or the loss of kinetic energy by fission products and fission neutrons.

The neutron flux can be used to describe the neutron population, and in turn to describe the rate of fission and other reactions. The neutron flux describes the rate at which neutrons pass through a unit area and can be found by multiplying the neutron density by the neutron velocity as shown in Figure 22.

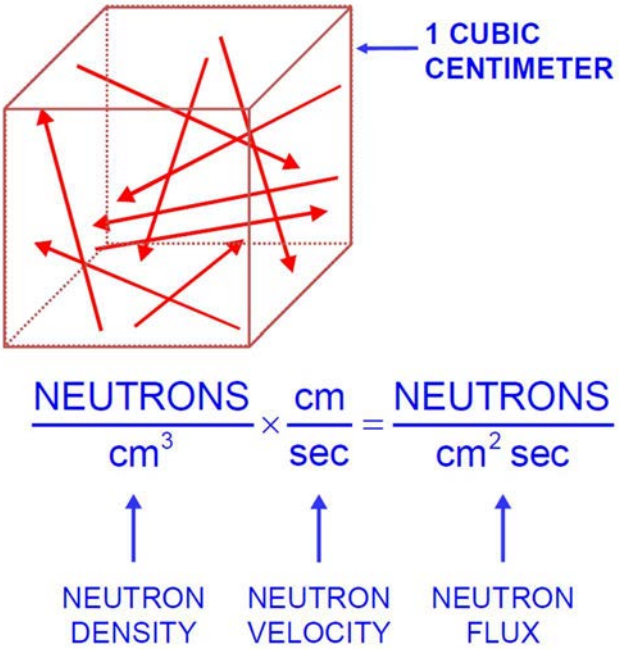


*FIG. 22. Illustration of neutron flux.*

Reaction rate per volume for each neutron interaction type can be calculated from the flux and neutron cross-section according to the following equation:

$$R = N\sigma\phi = \Sigma\phi \tag{36}$$

where $R$ is the reaction rate per volume, $N$ is the number density of target nuclei, $\sigma$ is the microscopic cross-section of the reaction, $\phi$ is the neutron flux and $\Sigma$ is the macroscopic cross-section of the reaction. From equations such as this, combined with the fission reaction equation, Eq. (16), it is possible to relate the neutron flux in a reactor to the reactor power. It is simple to solve this equation for a particular reaction with a single neutron energy, and specific target material. However, it is quite difficult to apply to reactor operation considering the number of different possible reactions, the spectrum-like distribution of neutron energies, and the complex material compositions and geometries present in the core, not to mention the time dependency of reactor operation.

Reactor power is derived from the thermal energy resulting from fission, such as the kinetic energy losses of fission products and neutrons. Because of this, it is closely related to the fission rate, as well as the neutron flux. In turn, it is related to the number of neutrons in the reactor core, and thus affected by the reactivity. In a critical state, the reactor power is stable and power rises or declines for supercritical and subcritical states, respectively. The effective neutron multiplication factor is the number of neutrons generated in one generation compared to those generated in the previous; therefore, the number of neutrons in the $n^{\text{th}}$ generation is given by:

$$N_n = N_0 k_{\text{eff}}^n \tag{37}$$

Because reactor power is so closely related and dependent on the number of fissions and, within reasonable approximation to the neutron population, the power after $n$ generations can similarly be described as:

$$P = P_0 k_{\text{eff}}^n \tag{38}$$

where $P_0$ is the initial power prior to an increase or decrease to the power $P$ after $n$ generations. Another form of Eq. (38) can be found, where power is dependent on time rather than generation number and effective neutron multiplication factor.

### 4.1.5. Reactor period

To find the time dependent equation for reactor power for given conditions, the generation number can be found by the number of mean neutron generation times, $\bar{l}$, that have passed at some time, *t*:

$$n = \frac{t}{\bar{l}} \tag{39}$$

The effective neutron multiplication factor can be set to a time dependent exponential with a new term $\tau$, the reactor period, and rearranged as follows:

$$e^{\frac{t}{\tau}} = k_{\text{eff}}^{\frac{t}{\bar{l}}}$$

$$\frac{t}{\tau} = \frac{t}{\bar{l}} \ln(k_{\text{eff}})$$

$$\ln(k_{\text{eff}}) = \frac{\bar{l}}{\tau} \tag{40}$$

For reactors, the effective neutron multiplication factor is generally close to that of a critical state and, because its value is near unity, the natural log can be approximated such that:

$$\ln(k_{\text{eff}}) \approx \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} = \rho \tag{41}$$

Using Eq. (41), the effective neutron multiplication factor can be removed from Eq. (40) in favour of the reactor period in terms of reactivity. The resultant reactor period is given by:

$$\tau = \frac{\bar{l}}{\rho} \tag{42}$$

By replacing the iterative effective neutron multiplication factor in Eq. (38) with the proposed form, the reactor power can then be expressed as a function of time:

$$P = P_0 \mathrm{e}^{\frac{t}{\tau}} \tag{43}$$

In this form, it is evident that the reactor period represents the amount of time it would take for the reactor power to increase by a factor of e.

The reactor period, however, depends on the mean neutron generation time and the reactivity. In order to find the mean neutron generation time, the effects of both prompt and delayed neutrons must be considered. Of practical use in this consideration is the effective delayed neutron fraction $\bar{\beta}_{\mathrm{eff}}$. In general, delayed neutrons are produced at lower energy than prompt neutrons. As a result, less thermalization must occur prior to thermal fission, and delayed neutrons are less likely to be absorbed or lost, but also less likely to induce fast fission. The effective delayed neutron fraction considers the relative number of delayed to total neutrons which proceed to fission and does not account for those lost in thermalization or leaked from the reactor core. In addition, the neutron release time for delayed neutrons results in a delayed rise in neutron population and power. This can be accounted for by using $\bar{\beta}_{\mathrm{eff}}$ as the delayed neutron fraction and $1 - \bar{\beta}_{\mathrm{eff}}$ for the prompt neutron fraction.

*4.1.5.1.Prompt critical reactor period*

Considering a reactor which is exactly critical by prompt neutrons alone, in a state called ‘prompt criticality’, the following would be true:

$$\left(1 - \bar{\beta}_{\mathrm{eff}}\right) k_{\mathrm{eff}} = 1 \tag{44}$$

However, by substituting Eq. (24) into Eq. (44) as $k_{\mathrm{eff}}$ and rearranging, the following results:

$$\bar{\beta}_{\mathrm{eff}} = \rho \tag{45}$$

Therefore, prompt criticality occurs when the reactor is supercritical by a positive reactivity equal to or greater than the effective delayed neutron fraction.

For a prompt critical reactor, which has no dependence on delayed neutrons, the mean neutron generation time is the prompt neutron generation time ($l^*$) and the reactor period from Eq. (42) becomes:

$$\tau = \frac{l^*}{\rho} \tag{46}$$

Because the prompt neutron generation time is on the order of $10^{-4}$ seconds, even small reactivity changes could result in a short reactor period and rapid rise in power. Such a condition is unstable and difficult to control.

*4.1.5.2.Delayed critical reactor period*

Power reactors typically maintain criticality and operate on a combination of delayed and prompt neutrons in a 'delayed critical' condition.

The mean neutron generation time for a delayed critical condition can be found by returning to Eq. (18), considering $\bar{\beta}_{\text{eff}}$ since the focus is on the impact on reactor operation and solving the summed term to get a weighted mean delayed neutron generation time. The resulting mean neutron generation time becomes:

$$\bar{l} = \left(1 - \bar{\beta}_{\text{eff}}\right) l^* + \bar{\beta}_{\text{eff}} l_{\text{d}} \tag{47}$$

where $l_{\text{d}}$ is the delayed neutron generation time. For reactivity changes, the response of prompt neutrons occurs more quickly than that of delayed neutrons because of the longer neutron generation time for delayed neutrons. This can be accounted for by adding the reactivity to the prompt neutron fraction and subtracting reactivity from the delayed neutron fraction, resulting in:

$$\bar{l} = \left(1 - \bar{\beta}_{\text{eff}} + \rho\right) l^* + \left(\bar{\beta}_{\text{eff}} - \rho\right) l_{\text{d}} \tag{48}$$

Because $\bar{\beta}_{\text{eff}}$ and $\rho$ for a delayed critical reactor, the following approximation can be made:

$$\left(1 - \bar{\beta}_{\text{eff}} + \rho\right) \approx 1$$

$$\bar{l} \approx l^* + (\bar{\beta}_{\text{eff}} - \rho) l_{\text{d}} \tag{49}$$

The delayed neutron generation time can also be replaced by the inverse of $\lambda_{\text{eff}}$, the effective neutron precursor decay constant. $\lambda_{\text{eff}}$ is approximately $0.1\ \text{sec}^{-1}$ for supercritical conditions, $0.08\ \text{sec}^{-1}$ for critical conditions and $0.05\ \text{sec}^{-1}$ for subcritical conditions [9]. By then substituting this into Eq. (42) the reactor period for a delayed critical reactor results:

$$\tau = \frac{l^*}{\rho} + \frac{\bar{\beta}_{\text{eff}} - \rho}{\rho \lambda_{\text{eff}}} \tag{50}$$

The time for a reactor with $\bar{\beta}_{\text{eff}} = 0.0065$ and kept at $k_{\text{eff}} = 1.0001$ to double in power could, for example, be solved first by using Eq. (23) to calculate reactivity:

$$\rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} = \frac{1.0001 - 1}{1.0001} \approx 0.0001\ \Delta\text{k/k} \tag{51}$$

This reactivity can then be used in Eq. (50) to calculate the reactor period (note that $\lambda_{\text{eff}}$ is approximately $0.1\ \text{sec}^{-1}$ for positive reactivity addition):

$$\tau = \frac{l^*}{\rho} + \frac{\bar{\beta}_{\text{eff}} - \rho}{\rho\lambda_{\text{eff}}} = \frac{10^{-4}\text{sec}}{0.001} + \frac{0.0065 - 0.0001}{(0.0001)(0.1\,\text{sec}^{-1})}$$

$$\tau = 0.1\text{ sec} + 640\text{ sec} \approx 640\text{ sec} \tag{52}$$

Note that in this example for a delayed critical reactor, the effect of the prompt neutron term on the reactor period is negligible in comparison to the delayed neutron term. The calculated reactor period can then be used in Eq. (43) to find the time for the reactor to double in power from the initial $P_0$ to $2P_0$:

$$2P_0 = P_0 \text{e}^{\frac{t}{\tau}}$$

$$2 = \text{e}^{\frac{t}{640\text{ sec}}}$$

$$\ln 2 = \frac{t}{640\text{ sec}}$$

$$t = (640\text{ sec})(\ln 2) \approx 443.6\text{ sec} \tag{53}$$

The result of this calculation is that the reactor will double in power every 443.6 sec.

While Eq. (50) does apply as a simple model dependent on reactivity, a more rigorous relation may be used to determine transient reactor periods:

$$\tau = \frac{l^*}{\rho} + \frac{\bar{\beta}_{\text{eff}} - \rho}{\rho\lambda_{\text{eff}} + \dot{\rho}} \tag{54}$$

where $\dot{\rho}$ is the reactivity addition rate. This equation could be used to predict the reactor period for a reactor during the time a reactivity addition is being made, for example during control rod withdrawal. The reason this may be necessary is because as reactivity changes, the generation of longer lived delayed neutron precursors is unable to catch up to the generation of prompt neutrons and shorter lived delayed neutron precursors.

*4.1.5.3.Prompt neutron response*

For the reactor period given in Eq. (50), the first term is generally much smaller than the second term and at a set reactivity would have only a minor contribution to the reactor period. However, the second term has a delayed response as delayed neutrons are not generated immediately following a reactivity change. This delay means that the first term dominates in the early stages following a reactivity change, with an accordingly small period and rapid power response. Because this term of the period equation is the result of prompt neutrons, these rapid responses are called ‘prompt jump’, in the case of a positive reactivity insertion, and ‘prompt drop’ for a negative reactivity insertion. The effect on reactivity and power for prompt jump, prompt drop, and prompt jump followed by drop are illustrated in Figures 23, 24, and 25, respectively.

The prompt responses of power can be described quantitatively as follows:

$$P_{\mathrm{f}} = P_{\mathrm{i}} \frac{\bar{\beta}_{\mathrm{eff}}}{\bar{\beta}_{\mathrm{eff}} - \rho} \tag{55}$$

where $P_{\mathrm{i}}$ and $P_{\mathrm{f}}$ represent the initial and final reactor power, before and after the prompt response, respectively.

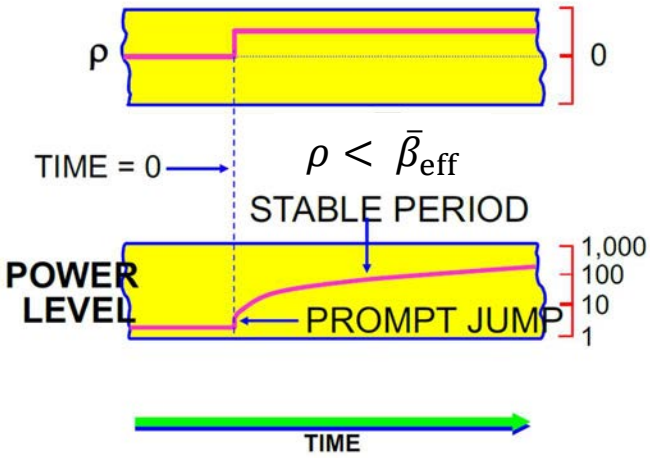


*FIG. 23. Prompt jump of positive reactivity insertion (% relative to final power).*

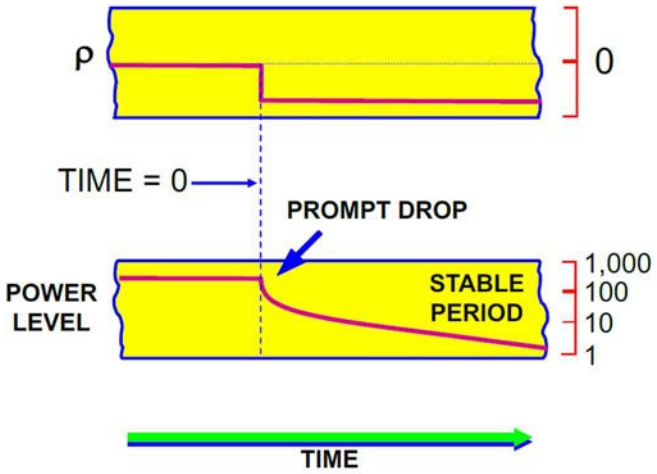


*FIG. 24. Prompt drop of negative reactivity insertion (% relative to initial power).*

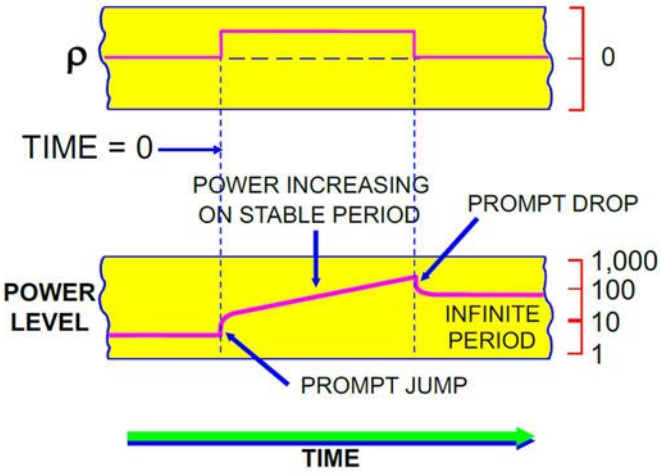


*FIG. 25. Prompt jump and drop of positive reactivity insertion followed by equal negative reactivity insertion (% relative to final power).*

*4.1.5.4.Delayed neutron response*

In addition to building up to a stable reactor period following small reactivity changes, the delayed neutron term of the reactor period equation results in a notable effect following large negative reactivity insertions. By delaying the response to a negative reactivity insertion, delayed neutrons can set a temporary minimum reactor period, or maximum power reduction rate.

Following the insertion of a large negative reactivity, the prompt neutron level falls very quickly. Over the course of a few seconds, the delayed neutron level drops because of fission products decaying away. However, some of these delayed neutron precursors have a substantial half-life, taking more time to decay and continuing the production of neutrons. As a result, the reactor period is dominated by the decay constant of the longest lived delayed neutron precursors.

Equation (54) for the reactor period is reduced, under the assumption that the negative reactivity is sufficient to approximate the removal of the effective delayed neutron fraction, as follows:

$$\tau = \frac{l^*}{\rho} + \frac{\bar{\beta}_{\text{eff}} - \rho}{\rho \lambda_{\text{eff}}} \approx -\frac{\rho}{\rho \lambda_{\text{longest}}} \approx -\frac{1}{\lambda_{\text{longest}}} \tag{56}$$

where $\lambda_{\text{longest}}$ is the decay constant of the longest lived neutron precursors. One of these longest lived delayed neutron precursors, $^{87}$Br has a half-life of 55.9 seconds which corresponds to a decay constant of approximately 0.0124 per second. The reactor period given in Eq. (56) can be solved using these values:

$$\tau = -\frac{1 \text{ second}}{0.0124} \approx -80 \text{ seconds} \tag{57}$$

This approximately negative 80 second period is observed following large negative reactivity insertions and limits the rate of power reduction, regardless of the amount of negative reactivity inserted. The approach to the negative 80 second reactor period for negative reactivity insertions is visualized in Figure 26.

As the longest lived delayed neutron precursors decay away, the reactor is eventually returned to a stable, subcritical state. The different reactor period response stages following a reactor ‘trip’, which is characterized by a large insertion of negative reactivity, are shown qualitatively by the relative changes in reactor power in Figure 27.

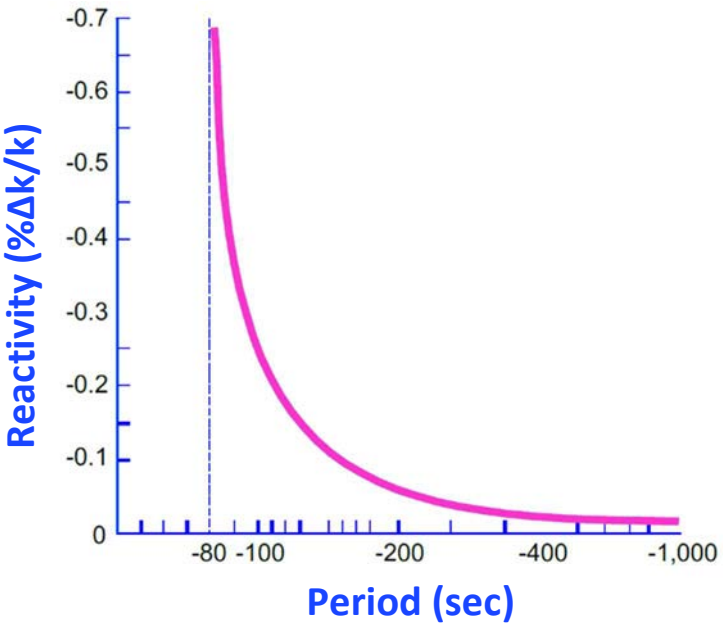


*FIG. 26. Negative 80 second reactor period boundary for power decreases due to delayed neutron precursors.*

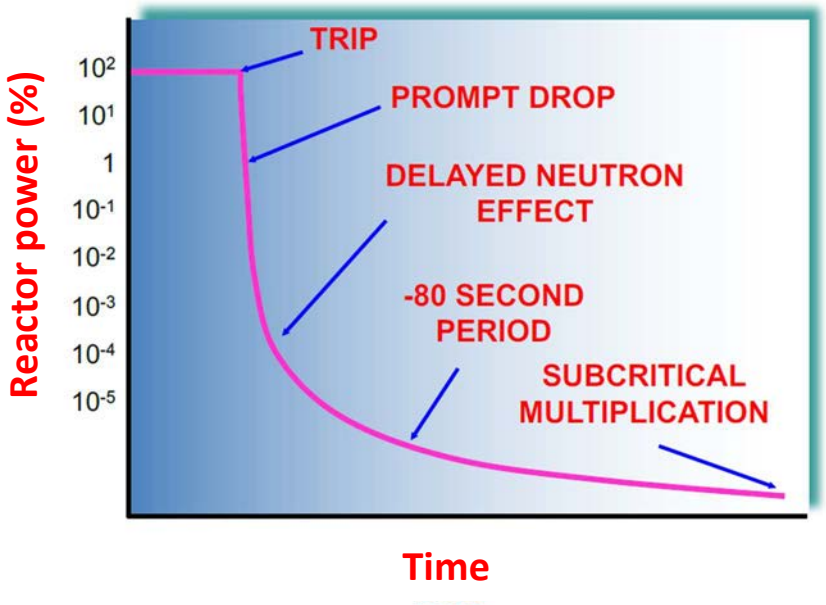


*FIG. 27. Reactor power change stages following a 'trip' or insertion of all control rods for rapid shutdown.*

## 4.2.CONTROL ROD WORTH

Control rods made of materials with high neutron absorption cross-sections are used to manipulate the reactivity in the reactor by reducing the number of neutrons allowed to fission in fuel. These materials, when inserted into the reactor, introduce negative reactivity. This negative reactivity insertion, however, is not constant and involves additional effects depending on the neutron flux distribution and core geometry.

### 4.2.1. Differential and integral control rod worth

The neutron flux in a reactor is not uniform and control rods must be inserted into the core starting from one end, typically from the top in PWRs and from the bottom in BWRs. As

inserted, control rods experience different neutron flux depending on position and impact the neutron flux in a non-uniform way as illustrated in Figure 28. This means that the reactivity inserted by the control rod, or control rod worth, depends on the height of the control rod. It is useful to describe the reactivity insertion of a control rod in two ways: differential control rod worth and integral control rod worth.

Differential control rod worth describes the change in the reactivity as it depends on control rod position, an example shown in Figure 29. In fact, the differential control rod worth can be described by accounting for how the change in reactivity is dependent on position.

$$\alpha_{\text{control rod}} = \frac{\Delta\rho}{\Delta x_{\text{control rod position}}} \tag{58}$$

where $\alpha_{\text{control rod}}$ is the differential control rod worth for control rod position, $\Delta\rho$ is the change in reactivity and $\Delta x_{\text{control rod position}}$ is the change in position in units of length. The units of the differential control rod worth could be pcm/cm, $(\Delta k/k)$ $cm^{-1}$ or other measure of reactivity (unitless) per unit of length.

By integrating the differential control rod worth across some distance of control rod insertion, the integral control rod worth is obtained. The integral control rod worth represents the total reactivity insertion to the reactor core caused by a control rod manipulation. An example of integral control rod worth for insertion into and withdrawal from the core is shown in Figure 30.

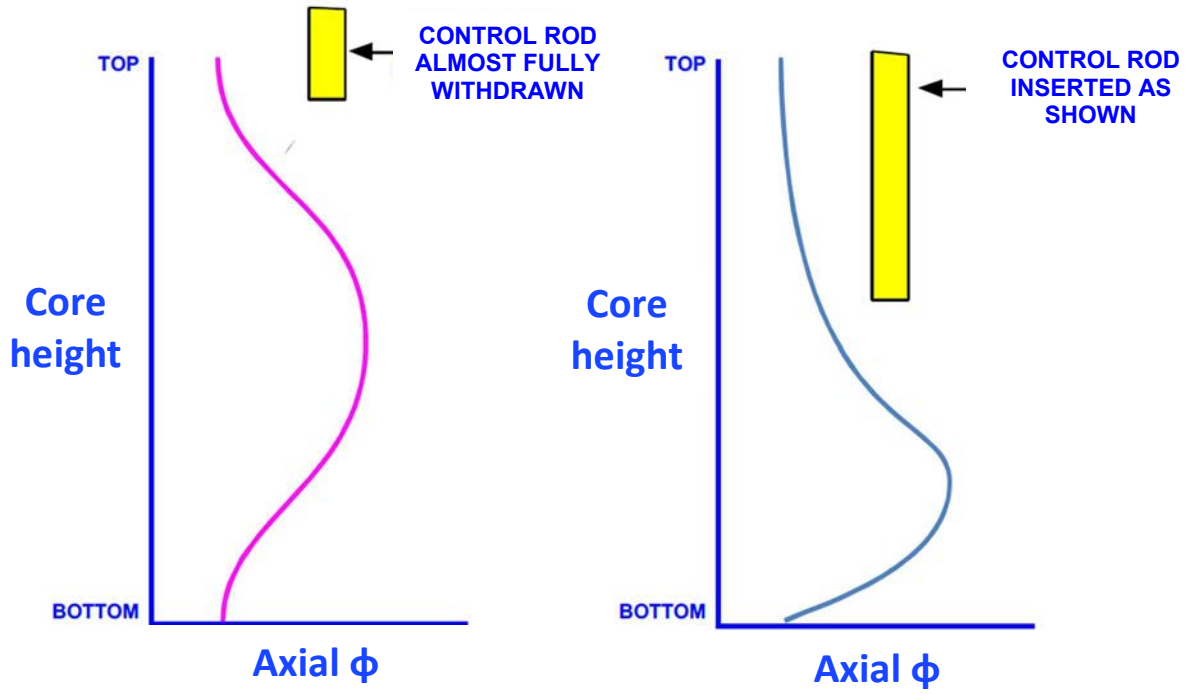


*FIG. 28. Axial neutron flux dependency of control rod position.*

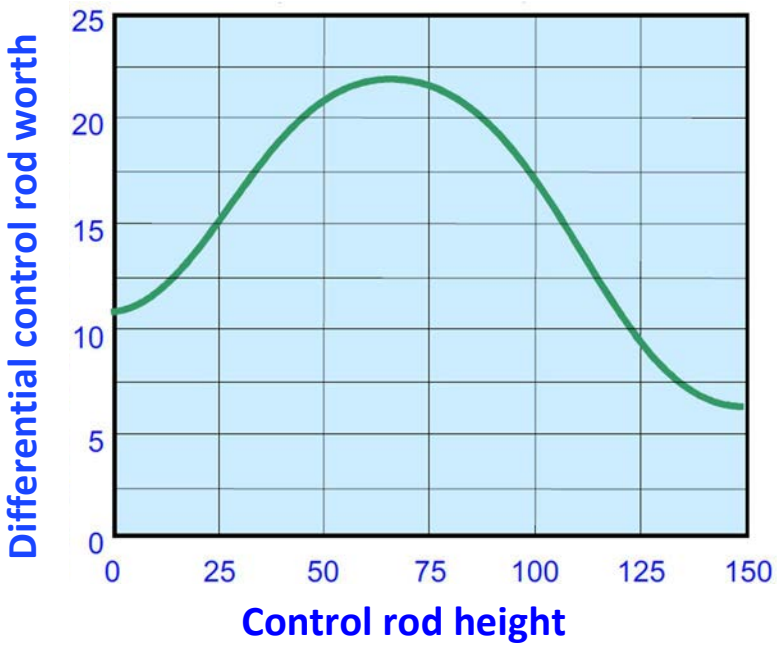


*FIG. 29. Position dependency of differential control rod worth.*

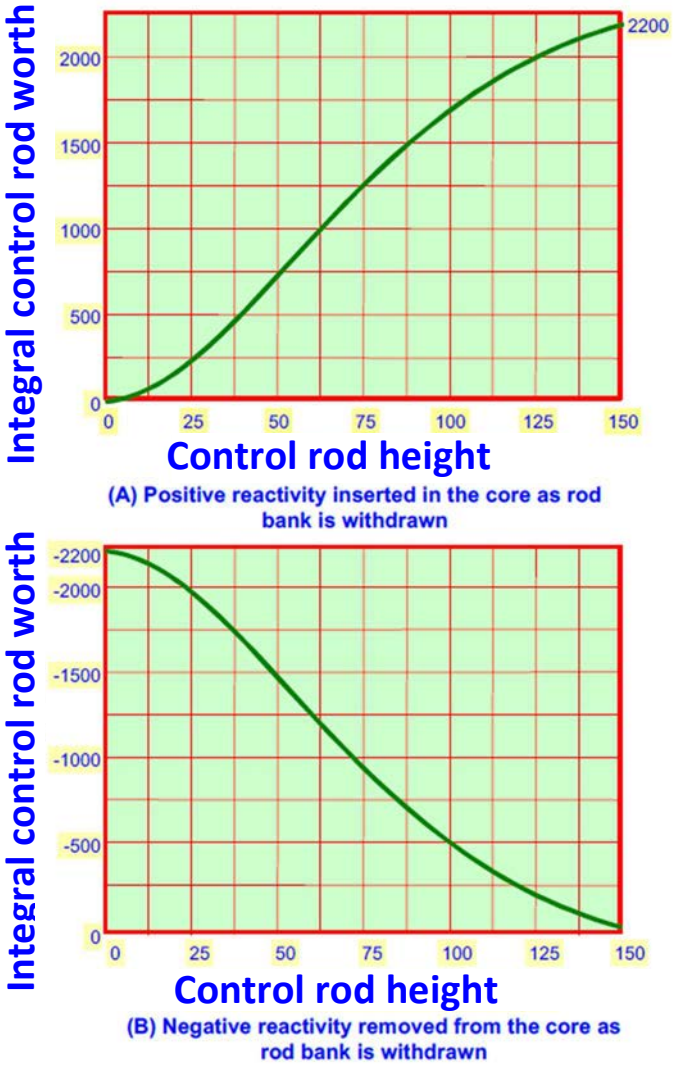


*FIG. 30. Integral control rod worth during (a) insertion into core (b) withdrawal from core.*

### 4.2.2. Control rod shadowing

Control rod shadowing occurs where the presence of control rods influences the reactivity worth of other control rods. When inserted into the reactor core, a control rod depresses the neutron flux nearby. To maintain consistent reactor power, despite the insertion of a control rod, the neutron flux must be compensated elsewhere in the core, leading to a rise in the flux away from the control rod. Therefore, in addition to reducing the neutron flux in an area around the control rod, neutron flux rises at a distance from it. In this way, three regions of neutron flux response are defined: the area near the control rod with decreased neutron flux, an area far from the control rod where neutron flux is increased, and a single point between the two areas in which the neutron flux is unaffected.

Because of the changes to neutron fluxes present in the core, a second control rod sees a different neutron flux than it would without the presence of the first, resulting in it having a different effect on overall reactivity and therefore having a different worth. If placed in the region near the first control rod, the worth of the second control rod is reduced from what it would be if the first were not there. If placed in the region far from the first, the rod worth is increased. However, the worth remains the same if placed at the exact point where neutron flux is unaffected by the first control rod. These conditions are illustrated in Figure 31.

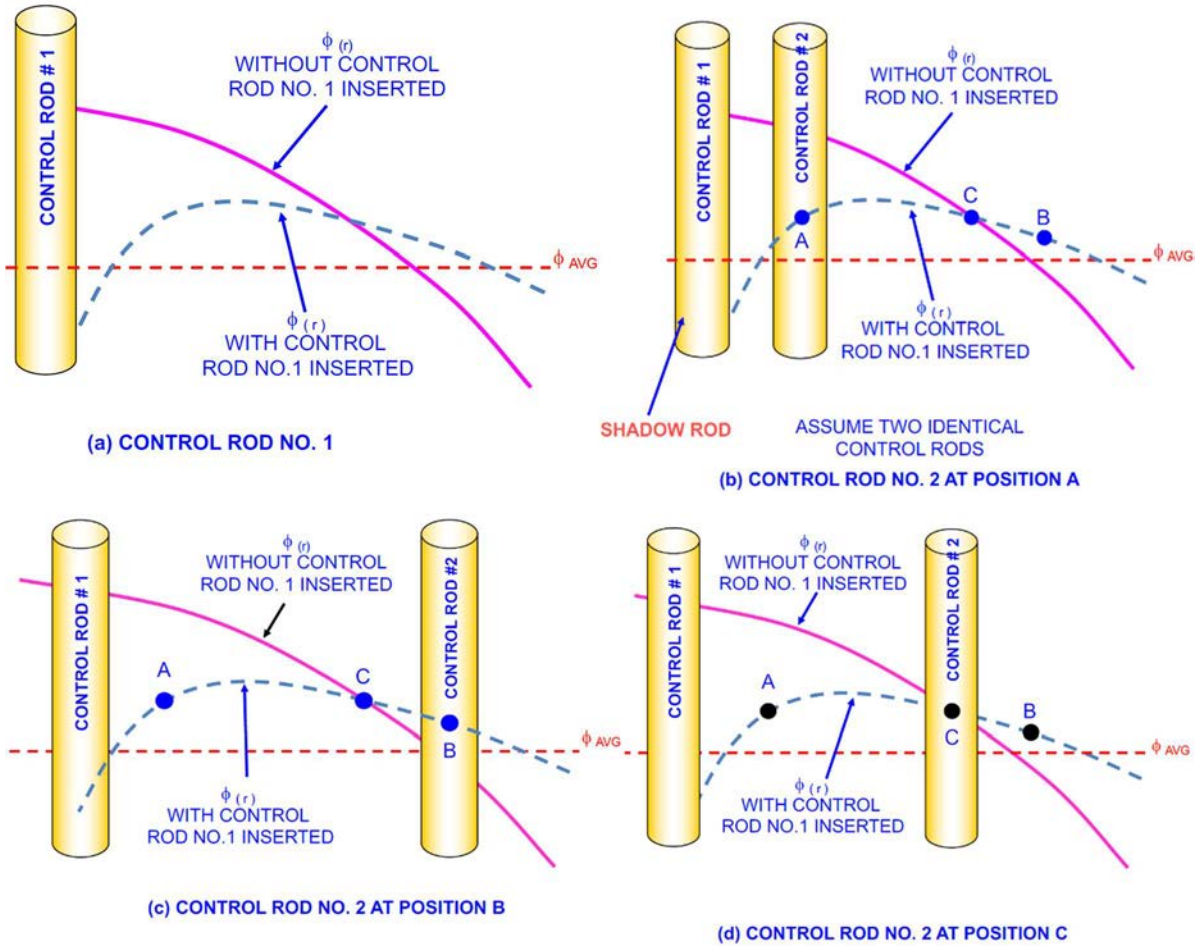


*FIG. 31. Effects of rod shadowing position such that a second control rod (a) is not inserted (b) worth is decreased (c) worth is increased (d) worth is unaffected.*

## 4.3.REACTIVITY COEFFICIENTS

Reactivity depends heavily on the thermodynamic state of the reactor system. Parameters which have an effect on reactivity can be described in terms of reactivity coefficients, including:

— Moderator temperature and moderator density coefficient;
— Void coefficient;
— Fuel temperature coefficient.

It is useful to quantify these parameters in terms of a coefficient $\alpha_x$, describing the reactivity change per unit change in the parameter:

$$\alpha_x = \frac{\Delta\rho}{\Delta x} \tag{59}$$

The direct meaning of this equation is that for an adjustment of some parameter *x*, a corresponding change in reactivity is seen equal to the reactivity coefficient multiplied by the change in *x*.

### 4.3.1. Moderator temperature and moderator density coefficients

Moderator temperature has an effect on reactivity due to thermal expansion of the moderator lowering moderator density. Depending on the moderator to fuel ratio, the effect on reactivity may be either positive or negative and is represented by:

$$\alpha_{\mathrm{T,moderator}} = \frac{\Delta\rho}{\Delta T_{\mathrm{moderator}}} \tag{60}$$

where $\alpha_{\mathrm{T,moderator}}$ is the moderator temperature reactivity coefficient and $\Delta T_{\mathrm{moderator}}$ is the change in temperature of the moderator. The units of this reactivity coefficient are then a measure of reactivity (unitless) per degree moderator temperature.

As moderator temperature increases, the moderator expands and density decreases. This is important because, with decreased moderator density, moderation becomes less effective. The change to density for changing temperature is not uniform; as temperature increases, temperature changes have an increased effect on the density, as shown in Figure 32.

With an increase in moderator density, more atoms of moderator are present between fuel plates or fuel rods. This results in more moderation in the region between fuel elements and increases the chances of neutrons to thermalize without experiencing resonance absorption, which mostly occurs within the fuel materials. The result is an increase in the resonance escape probability for increased moderator density.

Moderator density tends to decrease as moderator temperature increases. When this occurs, less moderation takes place in the core. In doing so, the path travelled by neutrons in the core is extended, increasing the likelihood they interact with non-fuel materials such as control rods.

As a result, control rod worth tends to increase for low density moderator, contributing to an overall decrease in the thermal utilization.

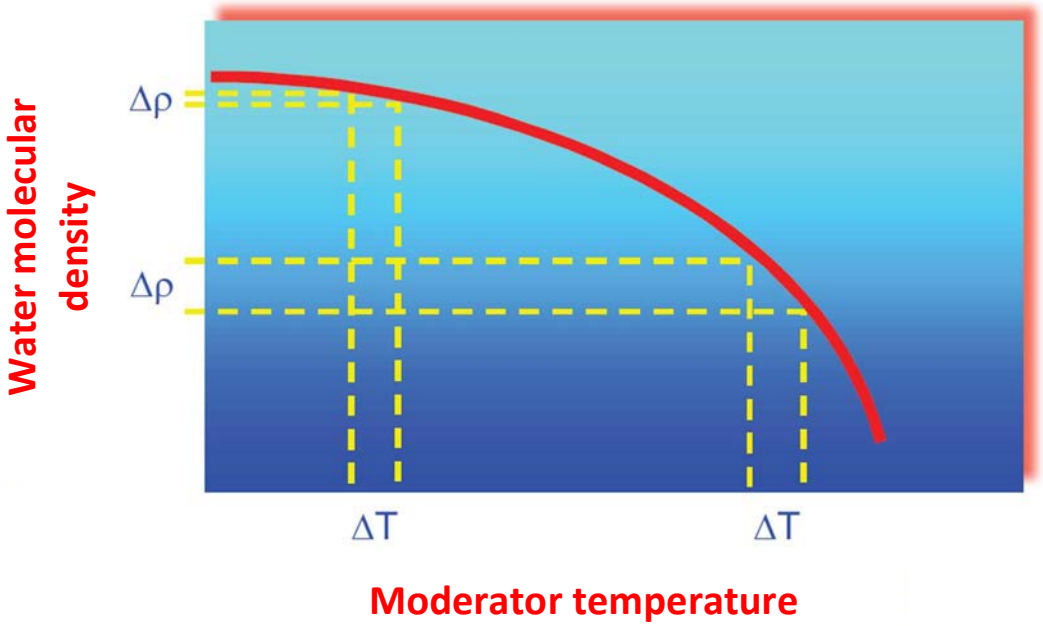


*FIG. 32. Relative moderator density changes for equal temperature changes.*

Because of the imbalanced reactivity contributions caused by changes to the resonance escape probability and the thermal utilization factor, it is useful to describe moderator density in terms of 'under' or 'over' moderated states, depending on the moderator to fuel ratio. These two states describe whether a moderator density change causes a net positive or net negative reactivity when accounting for the changes to the resonance escape probability and the thermal utilization factor, which have imbalanced reactivity dependency on moderator density. Decreasing density from an under moderated state causes a net decrease in reactivity, where a density decrease in an over moderated state causes a net increase in reactivity. These correspond to negative and positive moderator temperature reactivity coefficients, respectively. This is illustrated in Figure 33.

In addition to control rod position, moderator parameters enable another mode of operation which depends on the heat removal from fuel. By reducing the flow rate of the moderator (for example by changing the speed of reactor coolant pumps), water remains in the core region for a longer period of time. The water is then heated to a higher level in the core, leading to a decrease in moderator density. With reduced moderation (in an over moderated state), negative reactivity is inserted and power is reduced.

### 4.3.2. Void reactivity coefficient

The formation of steam bubbles in the moderator also influences reactivity. The effect on reactivity is described by a void reactivity coefficient and has similar effects to moderator density changes due to decreased moderator effectiveness:

$$\alpha_{\text{void}} = \frac{\Delta\rho}{\Delta void} \tag{61}$$

where $\alpha_{\text{void}}$ is the void reactivity coefficient and $\Delta void$ is the change in percentage of void volume. This reactivity coefficient could be considered in pcm/$\%_{\text{void}}$ or other unitless measure of reactivity and relative change in void volume.

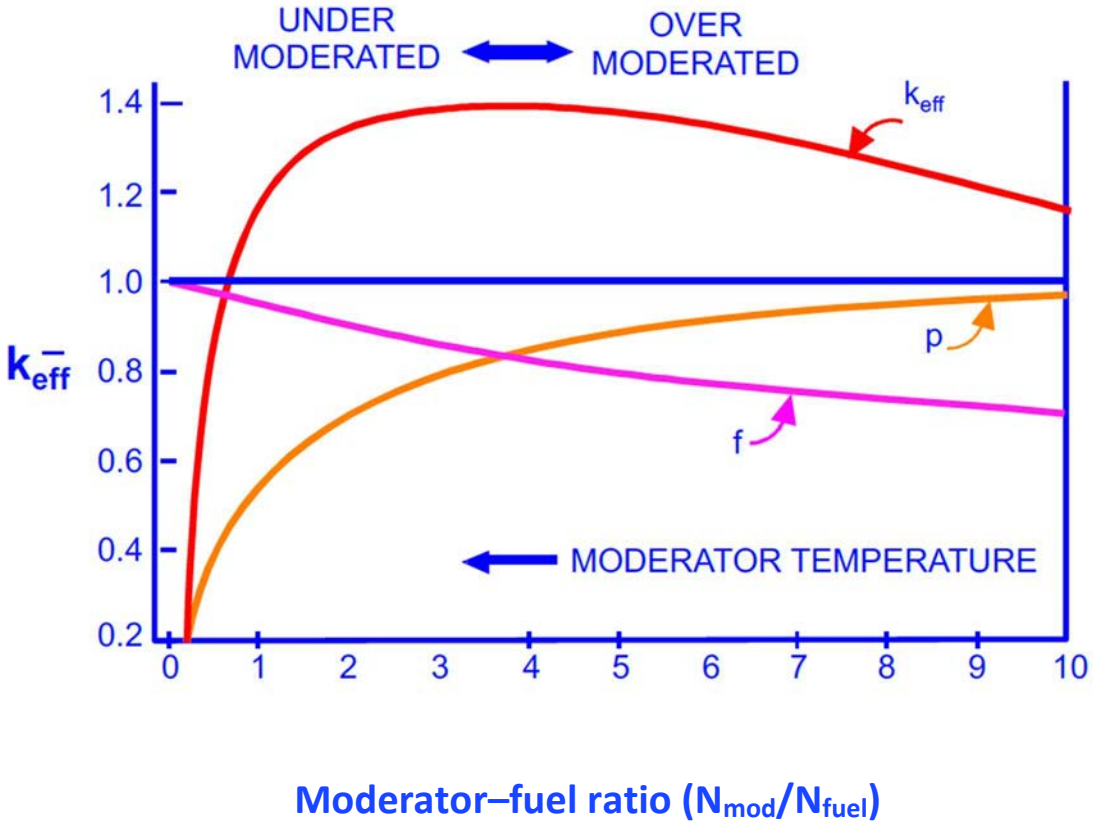


*FIG. 33. Moderation state dependency on moderator–fuel ratio.*

Much like a decreased moderator density, formation of void steam bubbles decreases moderator effectiveness and increase the neutron path length. The void reactivity coefficient is especially important for normal operation of BWRs, which have significant amounts of boiling near the fuel, and during certain accident scenarios in other WCRs.

### 4.3.3. Fuel temperature coefficient

Doppler broadening describes an increased chance for neutrons to be resonantly absorbed due to the higher temperature of material in the core. This phenomenon is related to nuclei vibration as temperature increases. By vibrating, depending on their direction of movement at a particular time, the relative energy of an incoming neutron may appear different. As a result, a single neutron may appear to be a few energies at once. These neutrons are more readily absorbed in the resonance region during the thermalization process since they may appear to have an energy corresponding to a resonance peak of a parasitic absorption of nuclei in a core.

One way of describing this is that the resonance peaks ‘broaden’ as the effective resonance peaks of capture cross-section are spread from their typical values. The result is a relative lower

cross-section where the resonance peak is located, and an increased cross-section at nearby energies. It is thereby possible that neutrons from energies outside of a resonance peak may still be absorbed. A representation of this phenomenon is illustrated in Figure 34 and the broadening of a resonance peak is shown in Figure 35.

In $^{235}$U fuelled reactors, the presence of $^{238}$U is responsible for a large amount of resonance absorption occurring in the fuel (see Figure 36). In thermal reactors, the effect of Doppler broadening has a negative effect on reactivity due to the fuel composition.

Unlike the process of moderator temperature increase and expansion, which do not occur until heat is transferred from the fuel, Doppler broadening effects occur immediately as a response to fuel temperature changes. In addition to its fast response, doppler broadening is a dominant contributor to the fuel temperature reactivity coefficient.

The resulting fuel temperature reactivity coefficient is given by:

$$\alpha_{\mathrm{T,fuel}} = \frac{\Delta\rho}{\Delta T_{\mathrm{fuel}}} \tag{62}$$

where $\alpha_{\mathrm{T,fuel}}$ is the fuel temperature reactivity coefficient and $\Delta T_{\mathrm{fuel}}$ is the change in temperature of the fuel. The units of this reactivity coefficient are then a measure of reactivity (unitless) per degree fuel temperature.

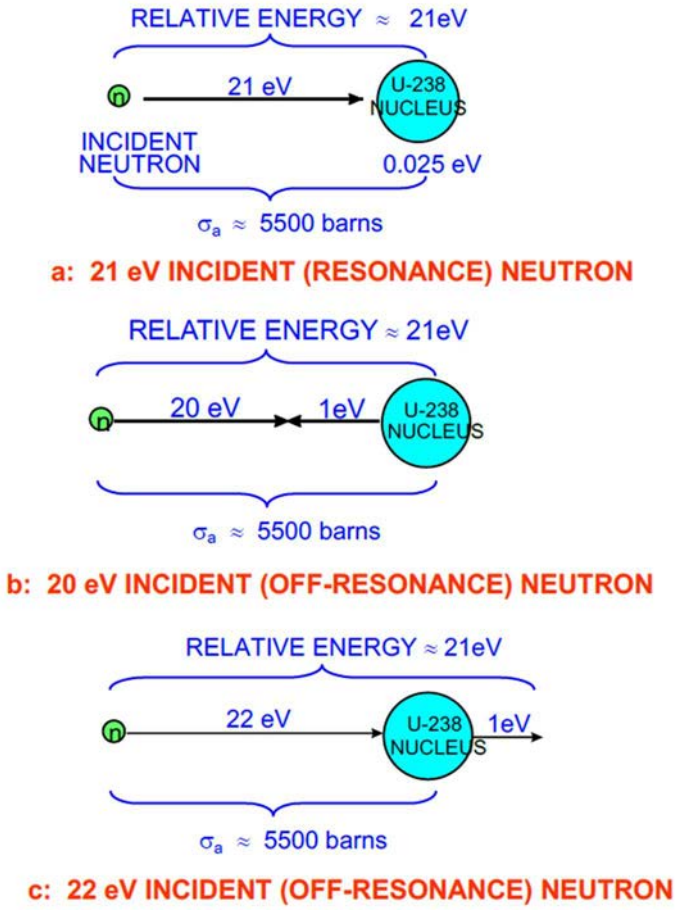


*FIG. 34. Doppler effect on neutron (a) at resonance (b) antiparallel off resonance (c) parallel off resonance.*

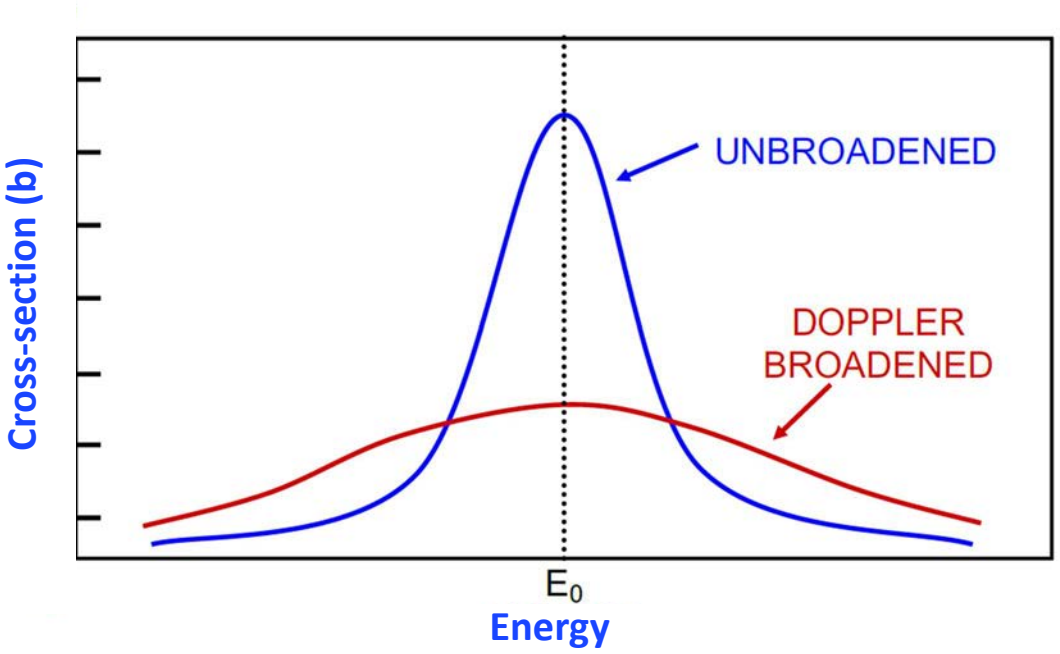


*FIG. 35. Doppler broadening of resonance peak in microscopic cross-section for absorption.*

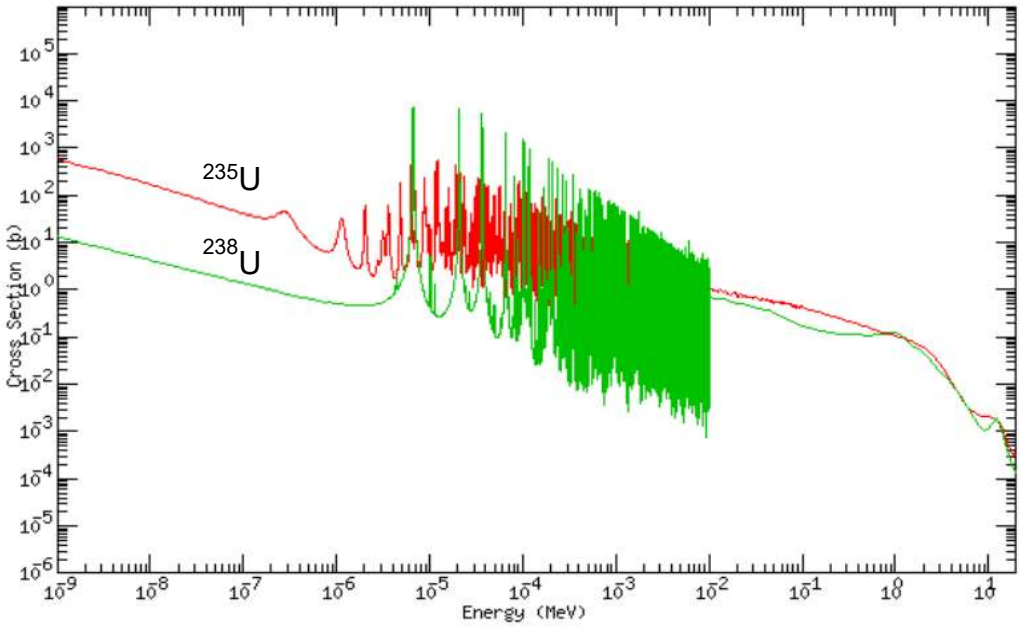


*FIG. 36. Microscopic cross-sections for radiative capture of $^{235}U$ and $^{238}U$ [5].*

## 4.4.NEUTRON POISONS

As explained in the previous section, fission products are generated from fission in the fuel. In addition to delayed neutron precursors, these fission products can influence the core reactivity by interacting with neutrons. Those with significantly high neutron absorption cross-sections, which have a large impact on the core reactivity, are known as neutron poisons. While there are several fission products with substantially high absorption cross-sections, those which are most significant to operation are $^{135}Xe$ and $^{149}Sm$ (see radiative capture cross-sections in Figure 37). The reactivity contribution of these poisons is closely related to their concentration in the

reactor. In addition to fission product poisons, control poisons may also be intentionally introduced to affect reactivity for extended periods of time.

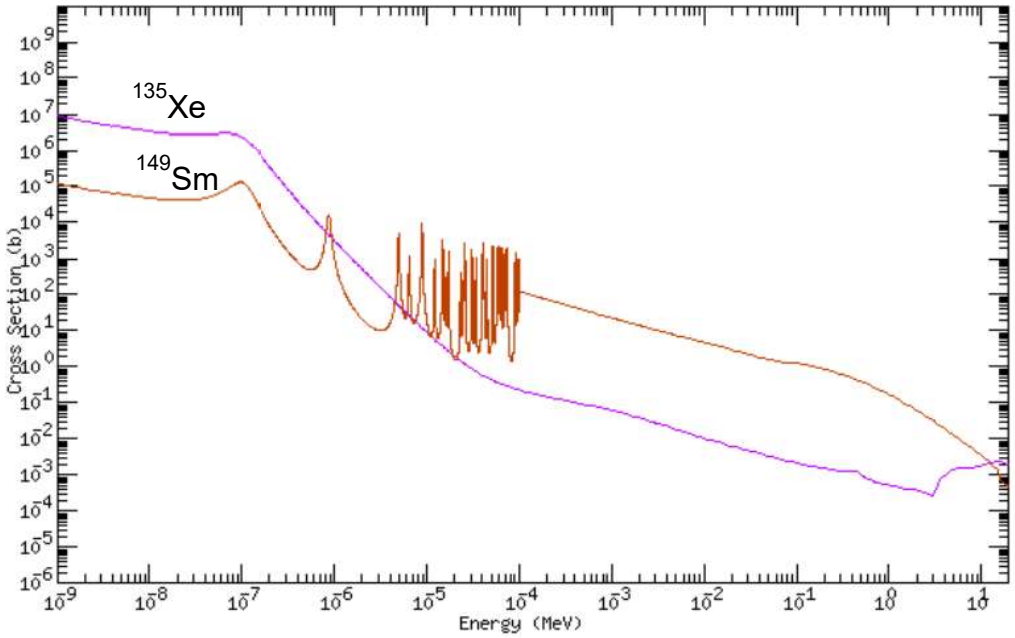


*FIG. 37. Microscopic cross-sections for radiative capture of $^{135}$Xe and $^{149}$Sm [5].*

### 4.4.1. Xenon concentration

$^{135}$Xe is produced in a reactor in two ways. The first is through direct production from fission, and the second is by the radioactive decay of other fission products. The decay chain which results in production of $^{135}$Xe is:

$$^{135}_{52}\mathrm{Te} \xrightarrow[19\ \text{seconds}]{\beta^-} {}^{135}_{53}\mathrm{I} \xrightarrow[6.6\ \text{hours}]{\beta^-} {}^{135}_{54}\mathrm{Xe} \tag{63}$$

Noting that the half-life of $^{135}$Te is short, the production of $^{135}$Xe by radioactive decay can be described as the decay of $^{135}$I. As a result, the total production rate of $^{135}$Xe, including both directly from fission and from the decay of $^{135}$I, is given by:

$$\left(\frac{dN_{\mathrm{Xe-135}}}{dt}\right)^{+} = \gamma_{\mathrm{Xe-135}}\phi\Sigma_{\mathrm{f}}^{\mathrm{fuel}} + \lambda_{\mathrm{I-135}}\,N_{\mathrm{I-135}} \tag{64}$$

where the left side of the equation is the rate of $^{135}$Xe production, $\gamma_{\mathrm{Xe-13}}$ is the fission product yield of $^{135}$Xe directly from fission of fuel materials, $\phi$ is the neutron flux, $\Sigma_{\mathrm{f}}^{\mathrm{fuel}}$ is the macroscopic cross-section for fission in the fuel materials, $\lambda_{\mathrm{I-135}}$ is the decay constant of $^{135}$I and $N_{\mathrm{I-135}}$ is the number density of $^{135}$I nuclei. At a stable reactor power, the production from fuel fission accounts for only about 5% of the $^{135}$Xe produced, whereas $^{135}$I radioactive decay makes up 95% of production.

$^{135}$Xe is a strong neutron poison; however, in the process of absorbing neutrons, it is depleted. Additionally, it is a radioactive nuclide itself and undergoes radioactive decay. The removal of $^{135}$Xe from the reactor is then given by:

$$\left(\frac{dN_{\mathrm{Xe-135}}}{dt}\right)^{-} = \phi\sigma_{\mathrm{a}}^{\mathrm{Xe-135}}N_{\mathrm{Xe-135}} + \lambda_{\mathrm{Xe-135}}N_{\mathrm{Xe-135}} \tag{65}$$

where the left side of the equation is the rate of $^{135}$Xe removal, $\sigma_{\mathrm{a}}^{\mathrm{Xe-13}}$ is the microscopic-cross-section for absorption of $^{135}$Xe, $N_{\mathrm{Xe-13}}$ is the number density of $^{135}$Xe nuclei and $\lambda_{\mathrm{Xe-13}}$ is the decay constant of $^{135}$Xe. The absorption cross-section for $^{135}$Xe is approximately 2.6 × $10^6$ barns for thermal energy neutrons (see Figure 37) and has a half-life of 9.1 hours; at a stable reactor power, the depletion of $^{135}$Xe by absorption of neutrons accounts for approximately 80% of $^{135}$Xe loss, compared to 20% through decay.

The amount of $^{135}$I in the reactor stabilizes when the production from fission and decay of $^{135}$I are equal, and the stable number density of nuclei can be found as follows:

$$\gamma_{\mathrm{I-135}}\phi\Sigma_{\mathrm{f}}^{\mathrm{fuel}} = \lambda_{\mathrm{I-135}}N_{\mathrm{I-135}}$$

$$N_{\mathrm{I-1}} = \frac{\gamma_{\mathrm{I-1}}\ \phi\Sigma_{\mathrm{f}}^{\mathrm{fuel}}}{\lambda_{\mathrm{I-135}}} \tag{66}$$

In this equation, the production yield of $^{135}$I includes the production of $^{135}$Te. When this condition is met, and neutron flux is stabilized, the amount of $^{135}$Xe in the reactor may also begin to stabilize. This can similarly be described by setting the production and loss terms equal:

$$\gamma_{\mathrm{Xe-135}}\phi\Sigma_{\mathrm{f}}^{\mathrm{fuel}} + \lambda_{\mathrm{I-135}}\,N_{\mathrm{I-135}} = \phi\sigma_{\mathrm{a}}^{\mathrm{Xe-135}}N_{\mathrm{Xe-135}} + \lambda_{\mathrm{Xe-135}}N_{\mathrm{Xe-135}} \tag{67}$$

By substituting the equilibrium number density of $^{135}$I atoms, given in Eq. (66), into Eq. (67), the number density of $^{135}$Xe atoms at equilibrium at a stable reactor power is found to be:

$$N_{\mathrm{Xe-135}} = \frac{\phi\Sigma_{\mathrm{f}}^{\mathrm{fuel}}(\gamma_{\mathrm{Xe-135}} + \gamma_{\mathrm{I-135}})}{\sigma_{\mathrm{a}}^{\mathrm{Xe-135}}\phi + \lambda_{\mathrm{Xe-135}}} \tag{68}$$

The $^{135}$Xe concentration reaches an equilibrium value that is dependent on the neutron flux and in turn the reactor power. However, the concentration, and corresponding reactivity, of $^{135}$Xe is not directly related (i.e. double power implies neither double $^{135}$Xe concentration nor double negative reactivity contribution), as shown in Figure 38.

In an initial rise to power, the 135Xe concentration increases to the equilibrium value. However, it builds up with a delay from the power change because 135I must accumulate and decay prior to stabilization. During this time, the neutron flux has also increased, resulting in a disproportionate amount of depletion by absorption until reaching the equilibrium value. Similarly, any adjustments to reactor power immediately change the absorption depletion rate with the production through decay lagging.

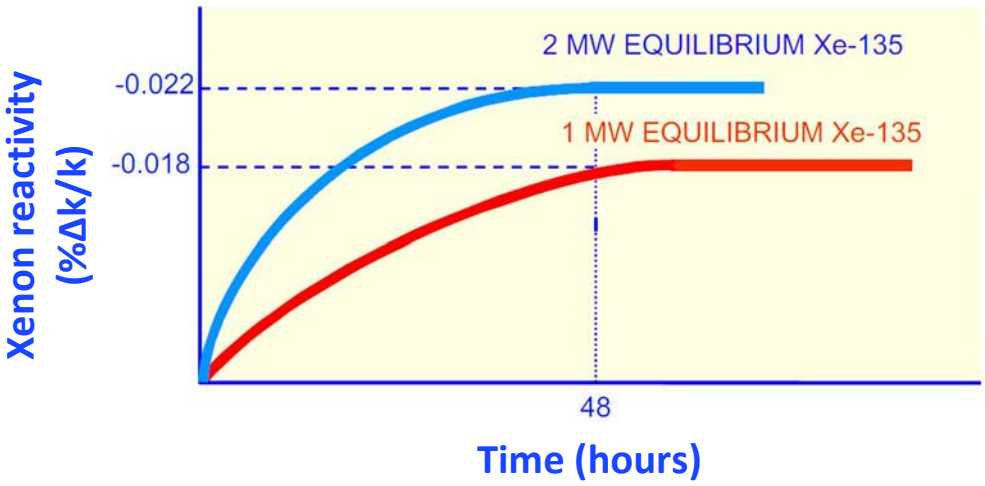


*FIG. 38. $^{135}$Xe reactivity for different power levels.*

When a reactor is shutdown, the $^{135}$Xe production through fission and depletion through absorption immediately stop. However, there remains an inventory of $^{135}$I that continues to decay; for this reason, the $^{135}$Xe concentration continues to rise for several hours following a shutdown or other negative power change. Only after a significant inventory of $^{135}$I has been depleted does the decay of $^{135}$Xe dominate the balance and $^{135}$Xe concentration begin to fall. The progression of $^{135}$Xe reactivity, related to its concentration in the core, for an initial rise to power of 2 MW followed by shutdown, is shown in Figure 39.

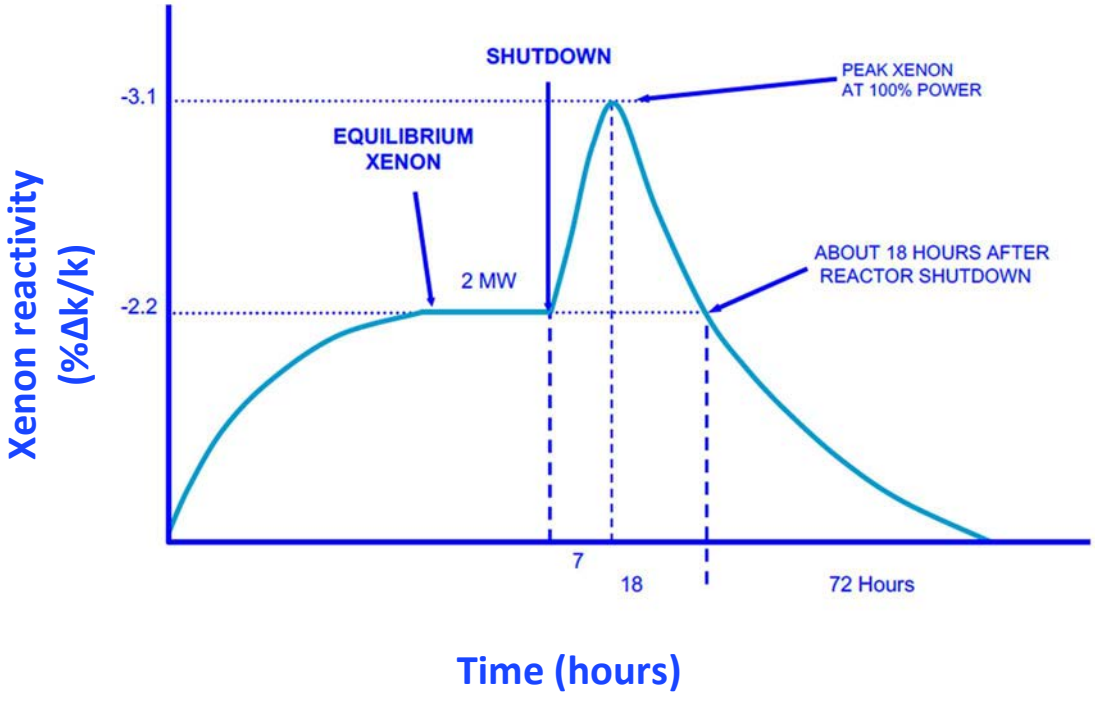


*FIG. 39. $^{135}$Xe reactivity for startup to 2 MW followed by shutdown.*

If a positive power change is made from a stable power level, the depletion through absorption rises and the concentration of $^{135}$I takes time to accumulate and produce $^{135}$Xe. This can result in the opposite effect of a negative power change, resulting in a temporarily lowered $^{135}$Xe concentration whenever positive power changes are made. Figure 40 shows the initial rise to

power, decline to half power, and return to full power and the corresponding effects on $^{135}Xe$ including the temporary drop in concentration for a rise in power.

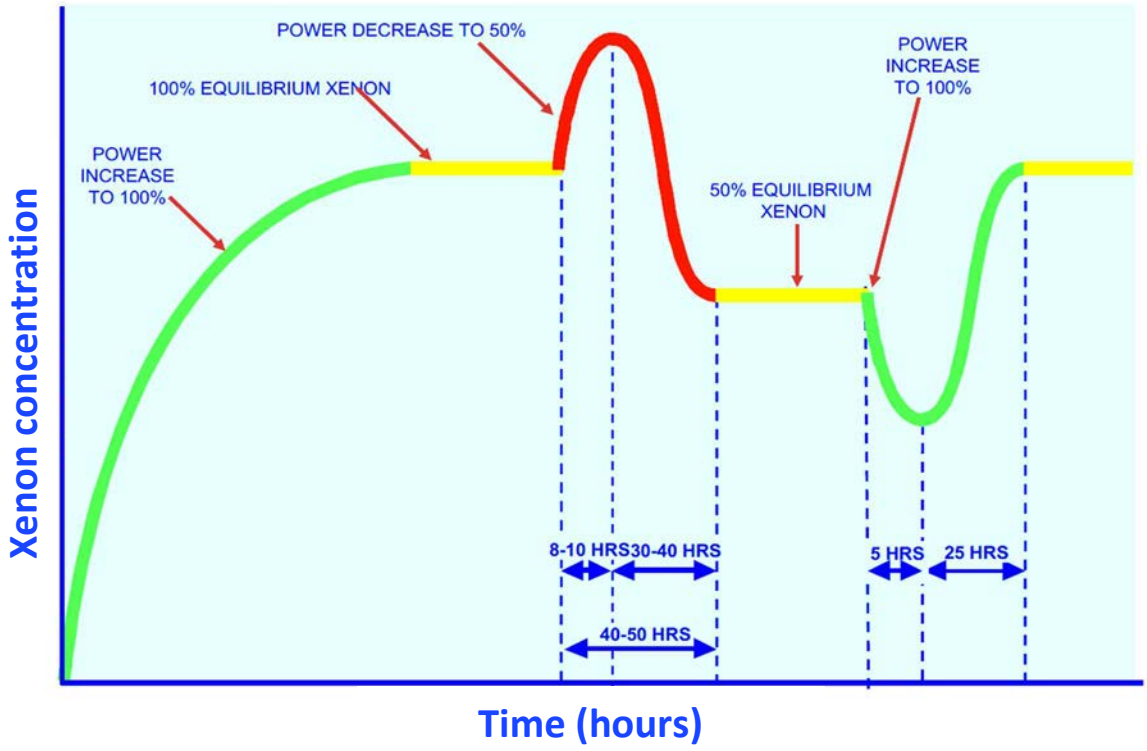


*FIG. 40. Illustration of $^{135}Xe$ concentration rise to power followed by reduction and return to power.*

The fluctuation of $^{135}Xe$ concentrations in the reactor can greatly impact reactor operation and even cause a phenomenon known as 'xenon precluded startup' to occur, in which a reactor startup is inhibited by $^{135}Xe$ remaining from the previous shutdown. This occurs because, following a shutdown, $^{135}Xe$ concentration remains at a risen level for several hours. If a reactor startup is attempted during this time, as shown in Figure 41, the negative reactivity effects of the risen $^{135}Xe$ concentration can slow or even prevent the startup.

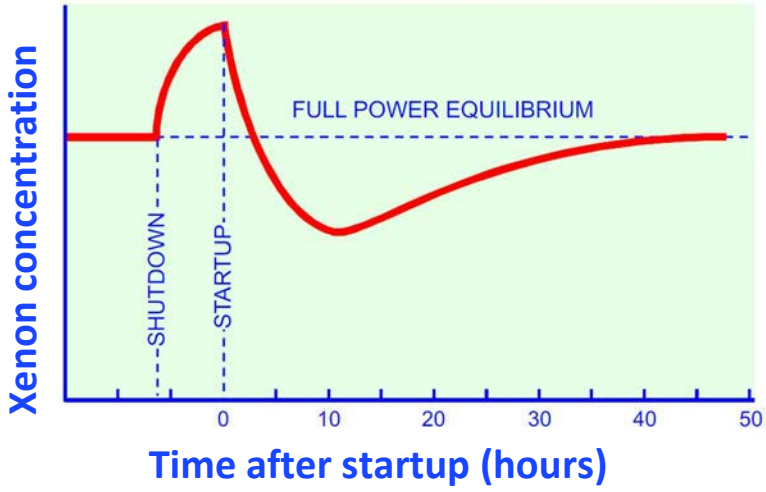


*FIG. 41. Illustration of $^{135}Xe$ concentration for a shutdown followed by startup.*

### 4.4.2. Samarium concentration

$^{149}$Sm does not result from fission of $^{235}$U fuel and is only produced by radioactive decay of direct fission products. The radioactive decay pathway for $^{149}$Sm production is:

$$^{149}_{60}\mathrm{Nd} \xrightarrow[1.7\ \text{hours}]{\beta^-} {}^{149}_{61}\mathrm{Pm} \xrightarrow[53\ \text{hours}]{\beta^-} {}^{149}_{62}\mathrm{Sm} \tag{69}$$

Though $^{149}$Nd has a half-life on the scale of hours, it is still short in comparison to the half-life of $^{149}$Pm. As a result, this decay chain can be collapsed, similar to ignoring $^{135}$Te in the $^{135}$Xe calculation, and the production of $^{149}$Sm can be described as the decay of $^{149}$Pm. At a stable reactor power, the production and decay of $^{149}$Pm reaches a stable value given by:

$$\gamma_{\mathrm{Pm-149}}\phi\Sigma_{\mathrm{f}}^{\mathrm{fuel}} = \lambda_{\mathrm{Pm-149}}N_{\mathrm{Pm-149}}$$

$$N_{\mathrm{Pm-149}} = \frac{\gamma_{\mathrm{Pm-}}\ \ \phi\Sigma_{\mathrm{f}}^{\mathrm{fuel}}}{\lambda_{\mathrm{Pm-149}}} \tag{70}$$

where $\gamma_{\mathrm{Pm-1}}$ is the fission product yield of $^{149}$Pm directly from fission of fuel materials, $\phi$ is the neutron flux, $\Sigma_{\mathrm{f}}^{\mathrm{fuel}}$ is the macroscopic cross-section for fission in the fuel materials, $\lambda_{\mathrm{Pm-}}$ is the decay constant of $^{149}$Pm and $N_{\mathrm{Pm-1}}$ is the number density of $^{149}$Pm nuclei. Unlike $^{135}$Xe, $^{149}$Sm has a long half-life of approximately $2 \times 10^{15}$ years and decay losses are negligible. Therefore, the only significant mode for removal is through neutron absorption and an equilibrium is reached when the production by decay of $^{149}$Pm and neutron absorption of $^{149}$Sm are equal:

$$\lambda_{\mathrm{Pm-149}}N_{\mathrm{Pm-149}} = \phi\sigma_{\mathrm{a}}^{\mathrm{Sm-149}}N_{\mathrm{Sm-149}} \tag{71}$$

where $\sigma_{\mathrm{a}}^{\mathrm{Sm-149}}$ is the microscopic cross-section for absorption in $^{149}$Sm and $N_{\mathrm{Sm-149}}$ is the number density of $^{149}$Sm nuclei. Substituting the equilibrium number density of $^{149}$Pm atoms, given in Eq. (70), into Eq. (71) gives the equilibrium number density of $^{149}$Sm atoms:

$$\gamma_{\mathrm{Pm-149}}\phi\Sigma_{\mathrm{f}}^{\mathrm{fuel}} = \phi\sigma_{\mathrm{a}}^{\mathrm{Sm-149}}N_{\mathrm{Sm-149}}$$

$$N_{\mathrm{Sm-}} \quad = \frac{\gamma_{\mathrm{Pm-149}}\Sigma_{\mathrm{f}}^{\mathrm{fuel}}}{\sigma_{\mathrm{a}}^{\mathrm{Sm-1}}} \tag{72}$$

Because $^{149}$Sm only has one mode of production and one significant mode of depletion, the concentration behaves very differently from $^{135}$Xe. Unlike the equilibrium number density of $^{135}$Xe atoms, the equilibrium $^{149}$Sm is not dependent on the neutron flux. This means that it reaches a single equilibrium concentration in the reactor, regardless of power, as illustrated in Figure 42.

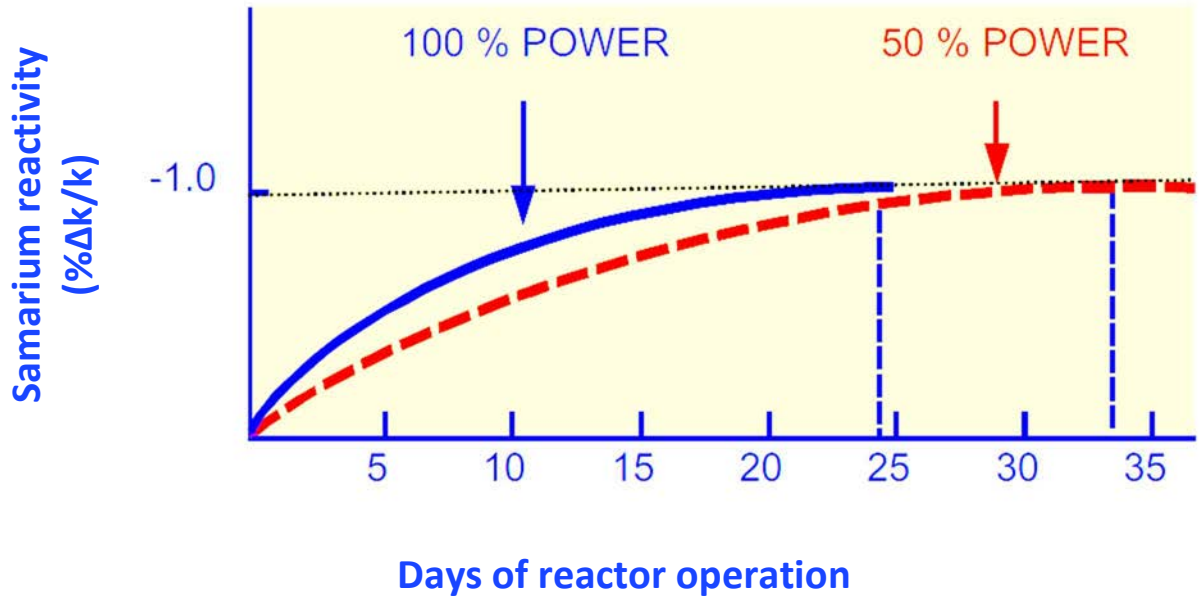


*FIG. 42. $^{149}$Sm reactivity for different power levels.*

Because the only significant mode for $^{149}$Sm depletion is through neutron absorption, the removal of $^{149}$Sm from a reactor effectively stops following shutdown. This means that, with the exception of the initial startup from a clean core, there is always a concentration of $^{149}$Sm. Similar to with $^{135}$Xe, $^{149}$Sm production continues after shutdown as there remains an inventory of $^{149}$Pm, which continues to decay. As a result, $^{149}$Sm concentration rises until $^{149}$Pm is depleted. This is shown in Figure 43.

Because the shutdown equilibrium concentration of $^{149}$Sm is higher than the operating equilibrium concentration, the reactivity effect of $^{149}$Sm is maximum at startup (restart) as shown in Figure 44.

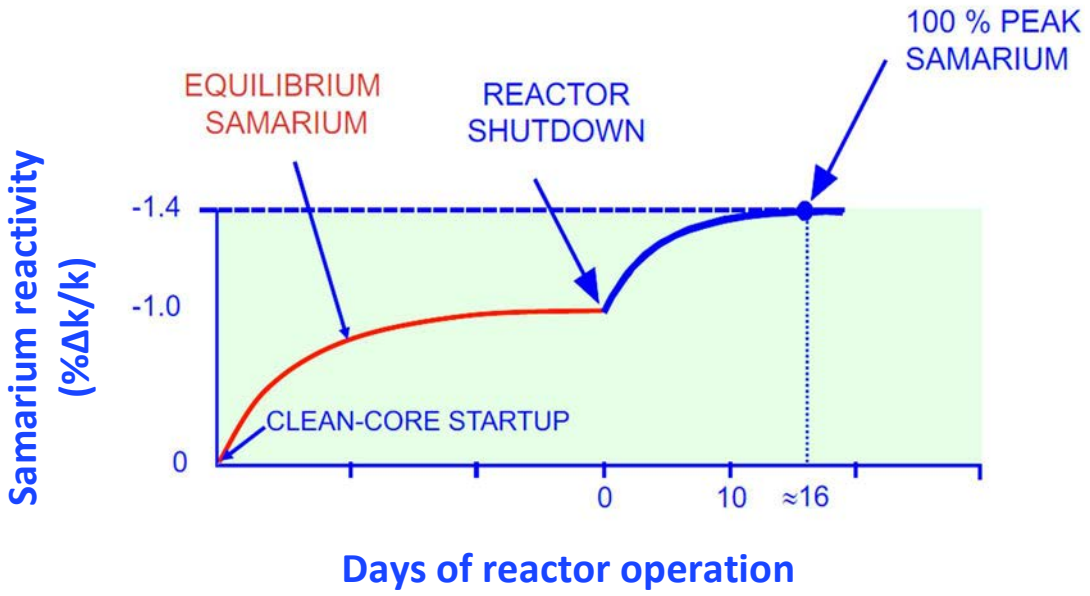


*FIG. 43. $^{149}$Sm reactivity for clean core startup and shutdown.*

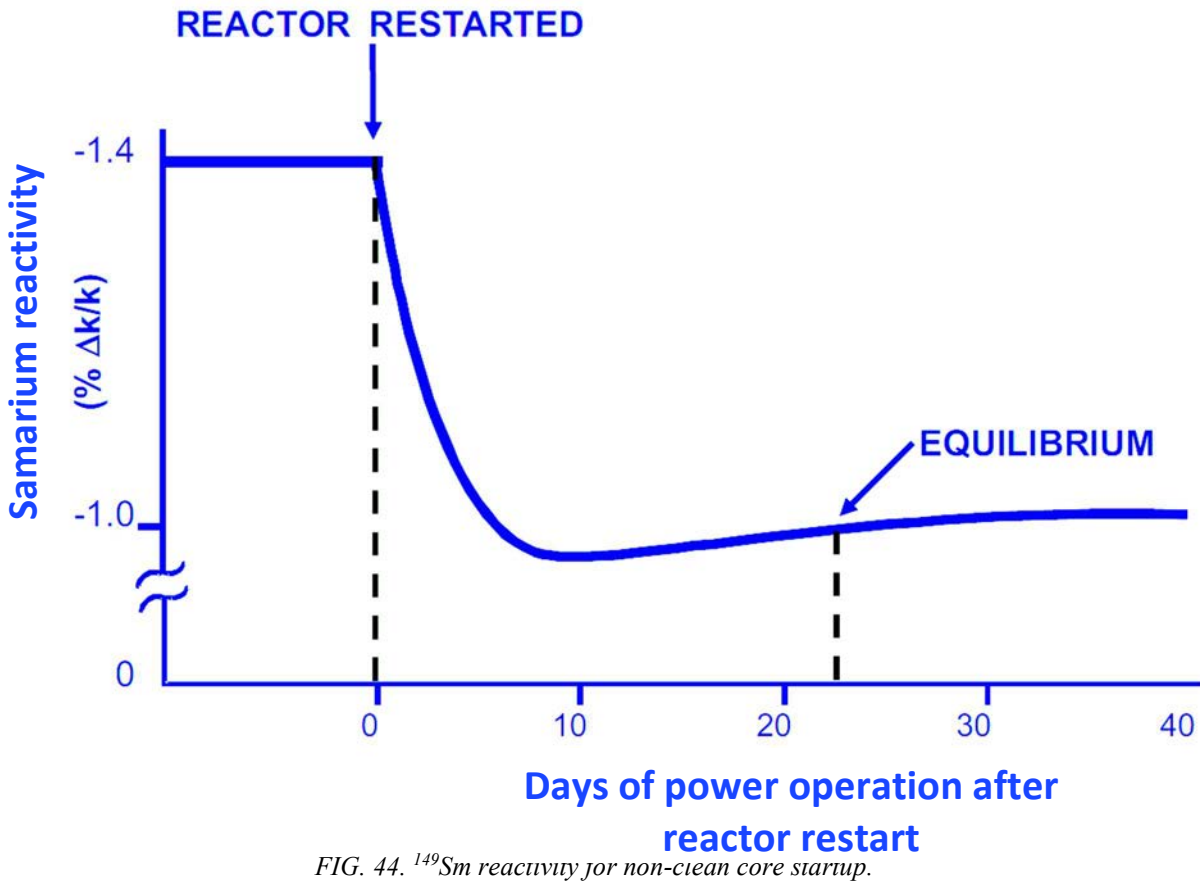


*FIG. 44. $^{149}Sm$ reactivity for non-clean core startup.*

### 4.4.3. Control poisons

In addition to fission product poisons, other neutron poisons may be introduced to the reactor to control the reactivity over the reactor lifetime. Two common examples are burnable absorbers and chemical shim.

Burnable absorbers consist of high neutron absorption cross-section materials which are introduced to the core either in fuel materials or as separate core components. The main idea of burnable absorbers is that, as neutrons are absorbed, the material is converted into a lower cross-section nuclide. $^{10}B$ is a common absorber material due to its large cross-section for the interaction given in Eq. (4), shown in Figure 45.

In comparison to 10B, the products of the interaction have small cross-sections for absorption interactions. This means that the magnitude of the negative reactivity effect of burnable absorbers decreases over time. The purpose of these burnable absorbers is to decrease reactivity during the early lifetime of the core when there are still large amounts of fuel. As fuel is depleted and corresponding neutron population decreased, the absorber is burned away so that the overall reactivity stays near the same value over the core lifetime.

In PWRs, chemical shim is soluble boric acid dissolved in the coolant or moderator of a reactor. Because it contains boron, which has a large neutron absorption cross-section, chemical shim introduces negative reactivity by reducing the neutrons proceeding to fission and lowering the

thermal utilization. Unlike other forms of reactivity control, which are localized to inserted rods or stationary components, chemical shim is homogeneously distributed throughout the core.

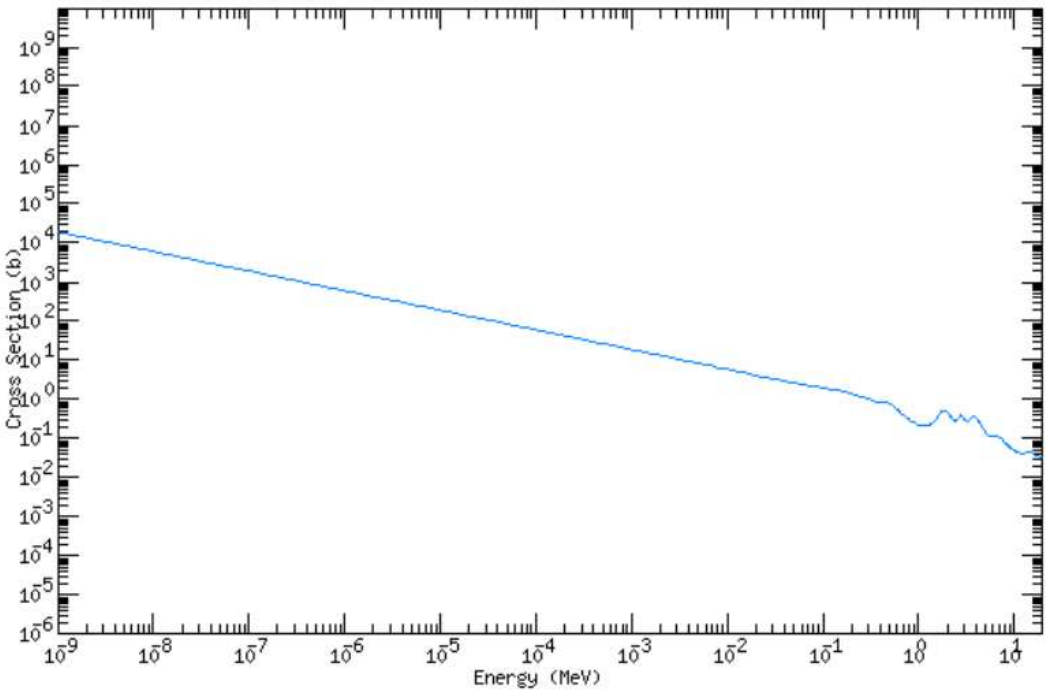


*FIG. 45. Microscopic cross-section for (n,α) interaction of $^{10}B$ [5].*

Chemical shim concentration is also used to control the long term reactivity in a similar manner to burnable absorbers. During the early core life, the coolant or moderator has high concentrations of shim and over the lifetime this concentration is reduced.

During normal operation, the reactivity influence of chemical shim is relatively slow in comparison to control rod manipulation. High boron concentration shim is also used in several safety systems to inject large negative reactivity to assist in shutdown of reactors following an accident.

## 4.5.CORE LIFETIME

Over the course of reactor operation, several changes take place in the reactor core. This is the result of many material changes, which take place as a result of neutron interactions over extended periods of time. Fuel is slowly depleted as $^{235}U$ or other fuel undergoes fission and converts to fission products. This long term process is known as burnup and measured in units such as gigawatt–days per metric ton of heavy metal (GWd/t), or the energy produced per unit of fuel material mass.

In addition to describing the depletion of fuel, burnup is also related to the production of various isotopes such as fissile $^{239}Pu$ generated by neutron capture followed by two beta decays in $^{238}U$, accumulation of fission products, and depletion of absorbing materials. These material changes result in differences in the neutron flux profile and subsequently affect the power and temperature distributions within the core. Relevant MPSL depletion analysis exercises, which visualize some of the effects of burnup, are provided in Section 5.1.

Several considerations must be made in the design and operation of the reactor to account for heterogeneous conditions and changing core material composition. These considerations ensure safe, long term, and economic operation of the reactor by optimizing the rate of burnup within the reactor core.

### 4.5.1. Core excess reactivity

$^{235}$U fuelled nuclear reactors are depleted as a result of fission interactions. A generic equation for the reaction rate of fission per volume is given by:

$$R_{\mathrm{f}} = \phi \Sigma_{\mathrm{f}}^{\mathrm{fuel}} \tag{73}$$

Notably, the rate of burnup depends on the neutron flux, or power, and the amount of fuel remaining.

Another useful way to describe the amount of fuel remaining, as well as other long term contributors to reactivity, is in terms of the excess neutron multiplication factor and excess reactivity. The excess reactivity refers to the core reactivity in the absence of control rods and describes how much fuel there is in excess of the absolute minimum to maintain a stable effective neutron multiplication factor. Similarly, the excess neutron multiplication factor is the difference between the effective neutron multiplication factor and criticality, such that:

$$k_{\mathrm{excess}} = k_{\mathrm{eff(max)}} - 1 \tag{74}$$

Core excess reactivity can then be given by:

$$\rho_{\mathrm{excess}} = \frac{\left(k_{\mathrm{eff(max)}} - 1\right)}{k_{\mathrm{eff(max)}}} = \frac{k_{\mathrm{excess}}}{k_{\mathrm{eff(max)}}} \tag{75}$$

In addition to fuel burnup, the core excess terms account for the influence of fission product poisons and burnable absorbers (Section 4.4). In the early stages of operation, fission product poisons accumulate to equilibrium concentrations, decreasing the excess reactivity as shown by the decrease in $k_{\mathrm{excess}}$ between points A and B in Figure 46. In the absence of other factors, fuel depletion would continue to lower the excess reactivity. However, burnable poisons present in the core are depleted faster than fuel, such that the excess reactivity temporarily increases as shown by the increase in $k_{\mathrm{excess}}$ between points B and C in Figure 46. Once enough of the burnable absorber is burned away, fuel depletion becomes the dominant effect and excess falls as shown by the decrease in $k_{\mathrm{excess}}$ between points C and D in Figure 46. Once the excess reactivity reaches zero, corresponding to $k_{\mathrm{excess}} = 1$ at point D in Figure 46, the reactor can no longer operate and must be refuelled. The goal of this design is to optimize the operating time before refuelling by accounting for fission product poisons and introducing burnable poisons to the system to balance with the depletion of fuel.

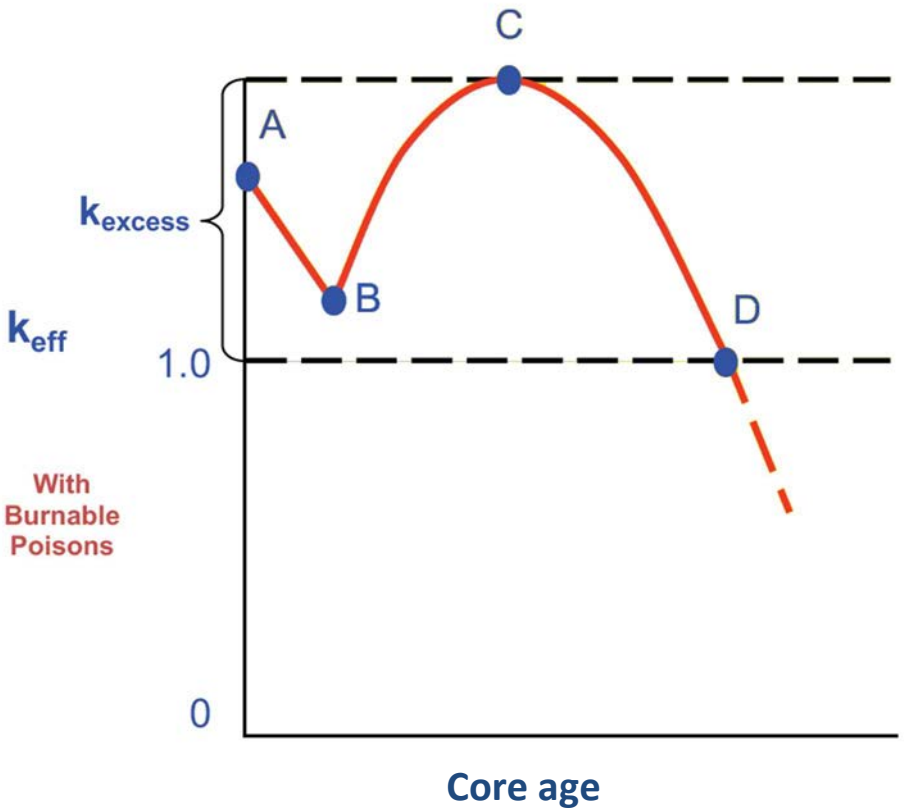


*FIG. 46. $k_{excess}$ change over operating fuel cycle.*

### 4.5.2. Neutron flux profiles

For most of the explanations in this publication, reactor cores have been treated under a point kinetics model. However, there are several notable considerations which arise from the reality of a heterogeneous core and non-uniform flux.

#### *4.5.2.1.Heterogeneous core flux*

Because the reactor core consists of a variety of finite components and materials, the neutron flux profile does not consistently decrease away from the core centre. Instead, the neutron flux consists of a series of local variations corresponding to the different core materials. The flux increases and decreases due to the presence of different materials. Figure 47 provides a simplified illustration of the flux radial profile where only fuel rods and moderator materials are considered. In this scenario, the thermal neutron flux in the moderator is increased compared to the homogeneous core, due to the thermalization of neutrons in this region. Thermal neutron flux within fuel materials, however, is dipped due to absorption of neutrons. In reality, further complexity is introduced as there are also the different materials such as structural components or control rods, as well as discrete regions of a fuel rod: fuel, gap, and cladding. This leads to different neutron flux depending not only on radial position in the core, but also on proximity to sources (fuel), moderator and absorbers (see 'relative power' plots in Section 5.1).

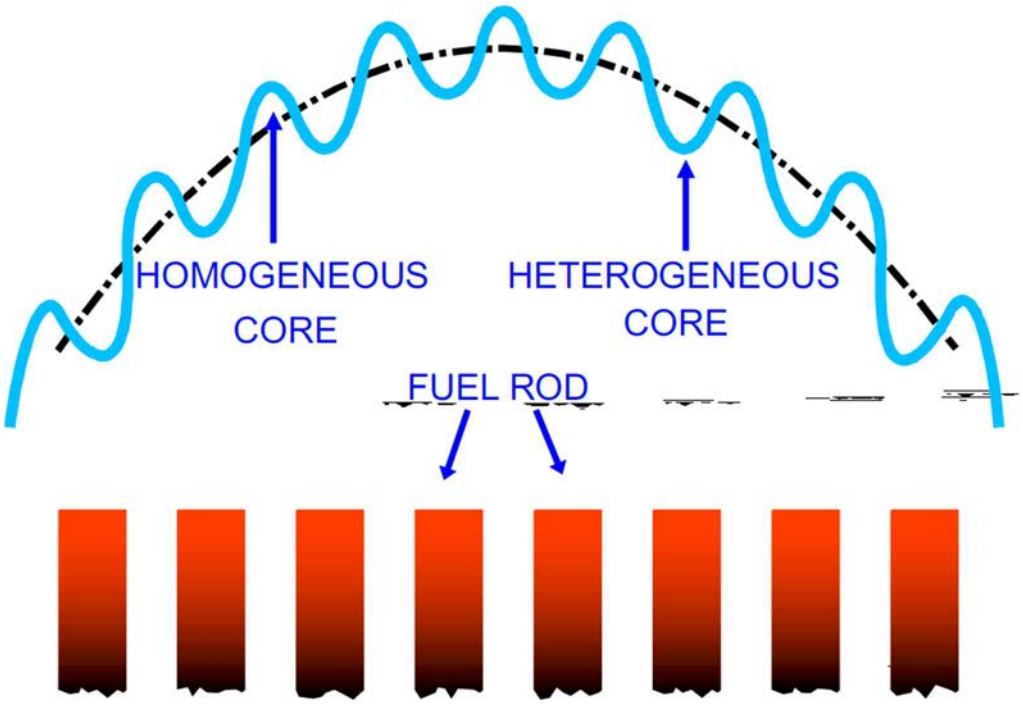


*FIG. 47. Homogeneous and heterogeneous thermal neutron flux profiles.*

*4.5.2.2.Core lifetime neutron flux profile*

Fuel assemblies are typically arranged in cylindrical patterns to form a reactor core. Neutrons produced from fission in the centre of the core are less likely to escape than neutrons produced in assemblies far from the centre. This results in an increased neutron flux in the centre of the core, which decreases towards the edges of the core.

This increased flux, however, increases the rate of burnup and fuel depletion in the centre of the core over time. The beginning of life (BOL) neutron flux profile is then significantly different from the end of life (EOL) profile because of the differences in burnup. Because fuel in the centre is depleted faster, the flux profile is depressed in the centre over time, as shown in Figure 48.

Note that similar behaviour is observed for the BOL and EOL axial neutron flux profiles, as the axial centre of the fuel also has a higher neutron flux than axial ends. The neutron flux profile also corresponds closely to relative power and temperature profiles of the core, as the number of fissions occurring is dependent on the flux (see BOL and EOL 'relative power' plots in Section 5.2).

*4.5.2.3.Neutron reflectors*

Neutron reflectors are materials with high elastic scattering cross-sections such as graphite or water used to reflect neutrons back to the core. In addition, they may be used to minimize the BOL and EOL neutron flux discrepancies by flattening the overall neutron flux profile. These materials are placed around the outside of the core to reflect neutrons which would otherwise leak from the core. These reflected neutrons contribute to fission in fuel near the edges of the

core, increasing the neutron flux. The resulting flattened neutron flux profiles reduce variations in power, burnup, and temperature throughout the core, as shown in Figure 49.

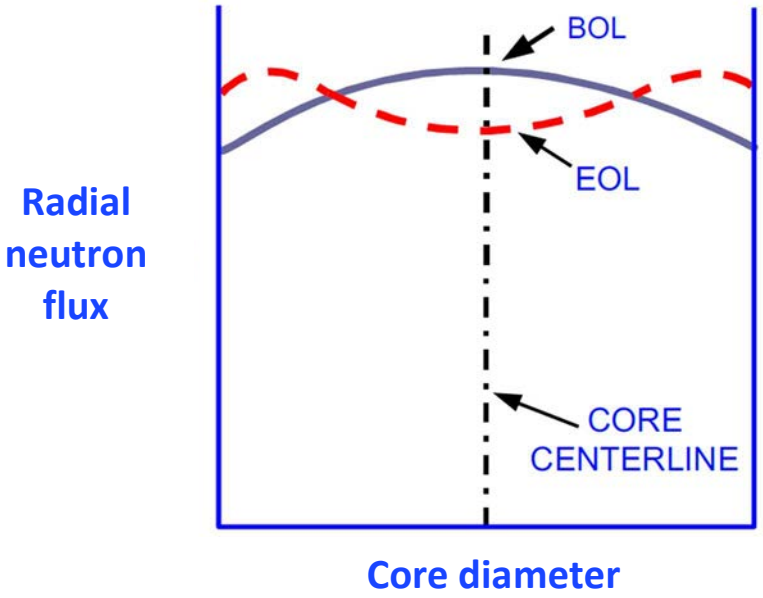


*FIG. 48. Radial neutron flux profile at BOL and EOL.*

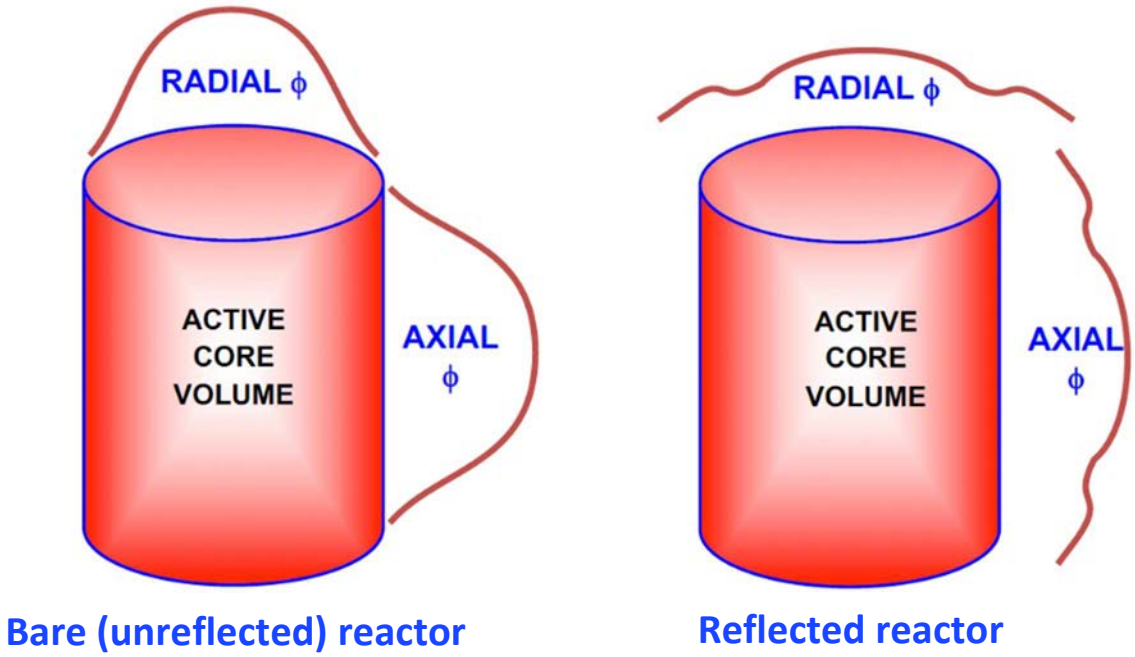


*FIG. 49. Effect of reflection on neutron flux profiles.*

*4.5.2.4.Fuel loading patterns*

Another method to flatten the neutron flux profile to efficiently burn fuel is to load differently burned fuel into the core in specific patterns. A simple example consideration for fuel loading is that, when refuelling the reactor at the EOL, low burnup fuel can be reused by moving it from the outer region of the core toward the centre. The high burnup fuel, from the centre of the core, is removed and fresh fuel is placed in the outer regions of the core. This means the final core

has partially burned fuel in the centre and unburned fuel toward the outside at the BOL, resulting in a flattened neutron flux profile throughout core life. This concept is shown in Figure 50.

Real fuel loading patterns consider flux distributions far more rigorously than the simple example described here and many additional considerations are taken into account; however, the basic principle remains the same. For example, more effective loading patterns may have high burnup fuel around the outside of the core to reduce the neutrons leakage or may have alternating layers of high burnup and low burnup fuel to minimize local neutron flux variations.

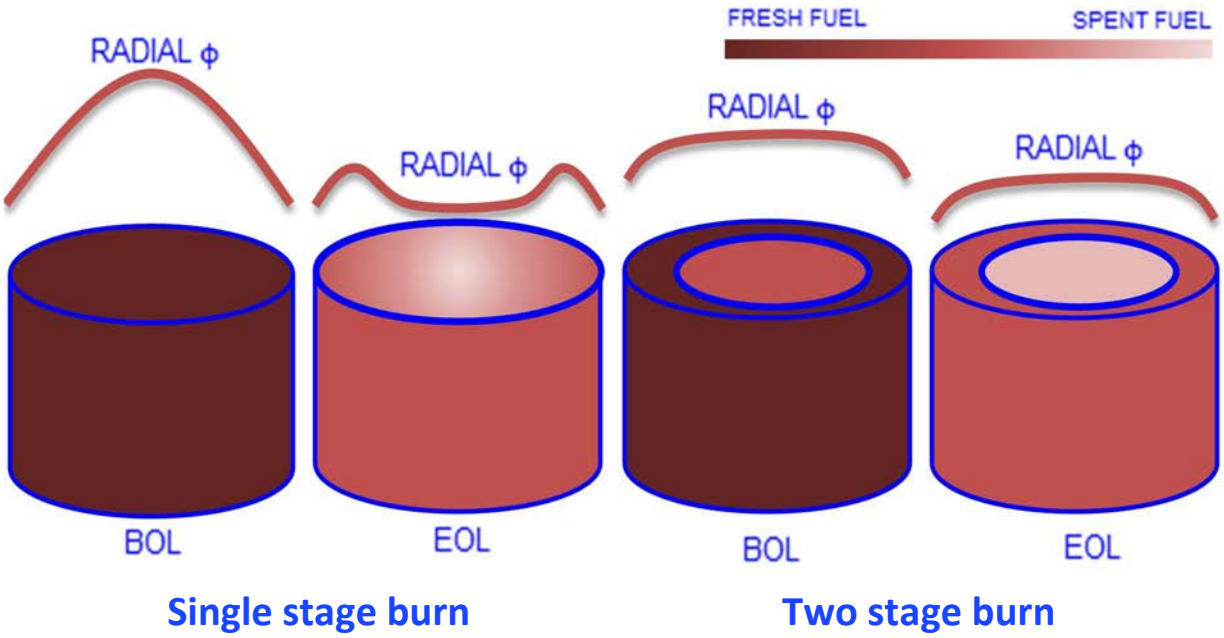


*FIG. 50. Simplified two stage burn fuel loading to flatten neutron flux profile.*

*4.5.2.5.Coolant temperature*

Temperature distribution is also not uniform in a reactor, as coolant must be flowing in a specified direction, from the bottom to the top of the core in PWRs as shown in Figure 51. As coolant temperature increases, it becomes less effective in removing heat from the core. This results in cooler regions of the core toward the coolant entrance and warmer regions toward the coolant exit. This in turn leads to variations in both fuel and moderator temperatures, non-uniformly affecting thermodynamic reactivity coefficients, and in turn leading to variations in neutron flux, power, and burnup in the axial direction (see 'moderator temperature distribution' plots in Section 5.1).

## 4.6.TRANSIENTS

Transients are characterized by changes to operating parameters, temperatures and pressures, in response to either operator action or accident conditions. As such, they often exhibit significant and simultaneous changes to many reactivity contributing parameters. The explanation of various transients in relation to the affected parameters is useful for understanding the complexity of reactor system operation and design as well as in understanding the functioning principles of reactor systems.

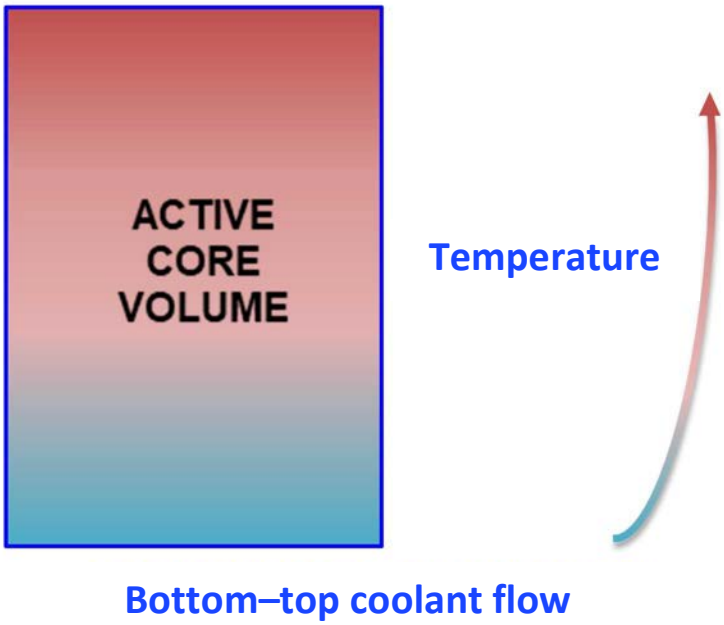


*FIG. 51. Illustration of the bottom–top coolant flow temperature profile in PWRs.*

There is a large variance between, and a large number of, possible transients which may take place in a reactor. Therefore, a few simple and representative transient scenarios are selected for explanation in this section: power manoeuvring, reactor scram, and loss of flow. Power manoeuvring is included as a simple, basic representation of normal operation. The reactor scram transient is important because the effects of a scram are part of many accident transients as an important accident prevention and response mechanism. Loss of flow is included because it is a symptom of several possible failures, the effects of which, like those of the scram transient, may be seen in a variety of accident transients. Each of these transients is described in terms of their effect on each of the various parameters explained previously in this section. Relevant MPSL transient exercises, including for the transients presented here, are included in Section 5.2.

It is assumed, for simplicity, that reactor safety systems remain operational for all the following explained transients. However, safety system failure or inadequacy may lead to extreme and undesirable accident conditions. The detailed behaviour of operating parameters in the most severe of these accidents is often not well known and difficult to model; the basic descriptions previously included in this publication may not be sufficient to explain all the phenomena of these accident transients.

### 4.6.1. Power manoeuvring

Power manoeuvring is a change in reactor power demand. The following analysis is relevant to a PWR operating and 100% power load which is then reduced to 50% power. After several days at this new power level, the reactor is returned to its nominal 100% power. A relevant transient is considered in an MPSL exercise, Section 5.2.1.

Control rods are inserted to reduce the power, giving the core negative reactivity (Section 4.2). The negative reactivity gives the reactor power a negative period, and power reduces over time

to the 50% level (Section 4.1.5). This power change is brought on by increased absorbance of neutrons in the control rods. In terms of the six factor formula Eq. (19), the thermal utilization factor Eq. (21), or the ratio of neutrons which are absorbed in fuel to those absorbed by non-fuel nuclei, is reduced. In general, for power manoeuvring, the insertion of control rods is done slowly. As a result, fast phenomena such as a prompt drop in the neutron level are not significant and reactor parameters change slowly. As power is reduced, fewer fissions occur and the fuel and moderator temperatures both fall:

— The moderator temperature coefficient is dominated by water density changes. As the temperature falls, moderator density increases. Most reactors are designed to be in an under moderated state to protect the reactor by inserting negative reactivity in response to rising temperatures. Therefore, this system has a negative moderator temperature coefficient. This, however, leads to a positive reactivity insertion as the temperature falls (Section 4.3.1).
— The fuel temperature coefficient is dominated by the effect of Doppler broadening that results in a negative fuel temperature coefficient. This means that reactivity is added as the fuel temperature falls (Section 4.3.3).

The positive reactivity insertion caused by fuel and moderator temperature changes counters the negative reactivity insertion of control rods. Once control rods are inserted to the 50% power position, temperatures decrease so that the overall core reactivity is zero and power stabilizes. Additional adjustments to either control rod positions or chemical shim may counter the effect of slow processes, such as fission products decaying to equilibrate to the new power.

$^{135}$Xe and $^{149}$Sm begin at the equilibrium levels for 100% power. When the reactor power is lowered to 50%, these concentrations start to change. $^{135}$Xe continues to be produced by the decay of the 100% power $^{135}$I inventory and concentration increases for several hours to its maximum value (Section 4.4.1). Once the $^{135}$Xe level begins to decrease, it falls over a couple of days to the new equilibrium value that corresponds to 50% power, which remains greater than half that at the full power. $^{149}$Sm concentration increases for several days following the power reduction due to its continued production by decay of $^{149}$Pm (Section 4.4.2). If left long enough, the $^{149}$Sm concentration then falls to reach an equilibrium at the same level as at 100% power.

In addition to $^{135}$Xe and $^{149}$Sm, other fission products also decay to new equilibrium values. The decay of these fission products results in continued production of decay heat, at higher levels than the equilibrium for 50% power. Delayed neutron precursor inventory continues to decay and produce delayed neutrons at an increased rate prior to reaching the 50% equilibrium concentration (Section 4.1.5.4). Reaching new equilibriums for decay heat and delayed neutron precursor concentration is a relatively slow process. Reactivity adjustments may need to be made for the slight reduction in production of heat and delayed neutrons.

Once all conditions are fully stabilized, the control rods are slowly removed to return to 100% power. The effects of this operation are largely opposite to that of the power reduction. A

positive reactivity insertion caused by control rod withdrawal results in an increased thermal utilization factor and a positive reactor power period.

Moderator and fuel temperatures rise due to the increased number of fissions. Because the moderator and fuel temperature coefficients are both negative, negative reactivity is inserted as the power rises. This slows the rise in power, countering the positive reactivity insertion of control rod withdrawal. After some time, the reactor is stabilized by balancing reactivity at zero. As with the power reduction, some operations may be made to account for slow processes. As other fission products accumulate to new equilibrium values, the heat production by decay of fission products and the production of delayed neutrons by delayed neutron precursors rise.

On returning to 100% power, $^{135}$Xe and $^{149}$Sm are burned faster than they are being produced. $^{135}$Xe concentration falls for a few hours due to the increased neutron flux until decay of $^{135}$I from the new power level catches up. Following this, the concentration rises to the new, greater equilibrium value for 100% power. $^{149}$Sm concentration initially falls due to the increased neutron flux but returns to the original equilibrium value over a few weeks.

### 4.6.2. Reactor scram

Within accident conditions or when reactor parameter limits are reached, the reactor responds with insertion of all control rods to prevent, or mitigate, an accident (Sections 2.3 and 2.4). This sudden insertion of all control rods is known as a reactor scram. This explanation refers to an example of a PWR in which a manual scram is performed from reactor full power. A relevant transient is considered in a MPSL exercise, Section 5.2.2.

Control rod insertion creates a large negative reactivity (Section 4.2). This results in rapid decrease in power and neutron flux. Because heat is no longer produced from fission, the moderator and fuel temperatures decrease as well. Despite the reduced number of fissions and resulting loss of heat production, there is still both a neutron source and a heat source in the core following a scram:

— The initial inventory of radioactive fission products and activated materials continues to undergo radioactive decay for extended periods following the shutdown, producing heat. This is known as decay heat, and accounts for the continued production of approximately 6% of the heat of full power. If, for some reason, heat removal capabilities are lost, this could lead to rising fuel and moderator temperatures.
— Neutrons continue to be generated from the inventory of delayed neutron precursors produced at full power. This leads the negative period of the reactor power to stabilize at approximately negative 80 seconds following the rapid drop of prompt neutron levels (Section 4.1.5.4).

The prompt neutron response to the negative reactivity insertion is very fast (Section 4.1.5.3). In comparison, moderator and fuel temperature decrease slowly. As a result, the negative reactivity insertion results in a sudden low reactivity followed by the reactivity changes caused by moderator and fuel temperature change expressed as moderator and fuel temperature

coefficients (Sections 4.3.1 and 4.3.3). These two effects lead to a slight increase in reactivity following the initial decrease:

— The moderator temperature coefficient is dominated by water density changes. As the temperature decreases, moderator density increases. Most reactors are designed to be in an under moderated state to protect the reactor by inserting negative reactivity in response to increased temperatures. Therefore, this system has a negative moderator temperature coefficient. This, however, leads to a positive reactivity insertion as the temperature decreases.
— The fuel temperature coefficient is dominated by the effect of Doppler broadening. Doppler broadening results in a negative fuel temperature coefficient. This means that reactivity is added as the fuel temperature decreases.

Following the scram, $^{135}$Xe continues to be produced for several hours thereby inserting negative reactivity during this period of time (Section 4.4.1). Once radioactive decay becomes the dominant term in $^{135}$Xe production–loss, it decays away over the course of a few days and the negative reactivity effect wanes. $^{149}$Sm continues to be produced following the scram as well, and approaches the shutdown equilibrium concentration, and stable negative reactivity insertion, over several days (Section 4.4.2).

### 4.6.3. Loss of flow

Loss of flow refers to the lack of flow in the coolant loop. This explanation considers a PWR with loss of flow in the primary coolant loop during full power operation. Coolant flow for this example decreases linearly from 100% to 50% over a few seconds. A relevant transient is considered in a MPSL exercise, Section 5.2.3.

As coolant flow rate decreases, the water remains in the core region for a longer time. This leads to an increase in the coolant and moderator temperatures. Because the reactor is normally under moderated, and therefore has a negative moderator temperature coefficient, negative reactivity is inserted as the water temperature is increased (Section 4.3.1). The lowered reactivity results in decreased reactor power. However, as the reactor power decreases, so does the fuel temperature. Fuel temperature has a negative reactivity coefficient due to Doppler broadening, and decreased fuel temperate inserts positive reactivity (Section 4.3.3).

The reactor will reach a stable condition, at lower than full power, once moderator temperature increase and fuel temperature decrease reactivity effects balance. In the following hours, $^{135}$Xe concentration increases before decreasing to the new equilibrium level (Section 4.4.1). Temperatures will slightly adjust over this time to counter $^{135}$Xe concentration changes.

# 5. MICRO-PHYSICS SIMULATOR LITE EXERCISES

This section includes descriptions of example simulations with the Micro-Physics Simulator Lite. These descriptions are inclusive of all default cases (included by default without user definition) and one user defined case, and includes two modes of operation:

- **Depletion analysis.** In depletion analysis, parameters are presented as a function of core burnup. This allows for demonstration and understanding of long term effects in the core such as a changing neutron/power profile or accumulation of fission product poisons over core life.
- **Transient analysis.** In transient analysis, parameters are presented as a function of time and user defined steps. This allows for demonstration and understanding of the effects of power changes and control rod movement or a large variety of accident conditions on various core parameters.

Refer to the Micro-Physics Nuclear Reactor Simulator™ Lite Edition User Manual, Nuclear Engineering Ltd., for an overview of the simulator [10]. This user manual describes the simulated reactor system, instructs on the installation and operation of the simulator, and provides detailed descriptions of the models used for calculations and verification benchmarks performed.

## 5.1.DEPLETION ANALYSIS

The following examples are descriptions of simulation results for the Micro-Physics Simulator Lite depletion analysis mode:

- Depletion analysis cycle 1;
- Depletion analysis cycle 2.

### 5.1.1. Depletion analysis cycle 1

Depletion analysis cycle 1 is the case of a new core with all unburned fuel of a variety of fuel enrichments (Section 4.5.2.4). The average exposure of fuel assemblies at the beginning and end of cycle is 0.0 GWd/t and 13.0 GWd/t burnup respectively. These are shown in Figures 52 and 53.

Initially, the soluble boron concentration is high to compensate for the positive reactivity due to fresh fuel. As fuel is used, fission products accumulate (Sections 4.4.1 and 4.4.2). To compensate for negative reactivity due to accumulation of fission products such as $^{135}$Xe and $^{149}$Sm, and fuel depletion, soluble boron is diluted to keep the core critical (Section 4.4.3). Figure 54. is an example of a trend graph showing an increase in core averaged $^{135}$Xe and $^{149}$Sm, and decrease in critical boron concentration as a function of fuel burnup.

Figures 55 and 56 show relative core power and $^{135}$Xe distribution at the BOL and EOL, respectively. Comparison of these two figures provides an indication of the changes in core

parameters as a function of burnup. In particular, the right hand side of Figure 56 shows that $^{135}$Xe density increases in a core at the EOL and shows the distribution of the fission product.

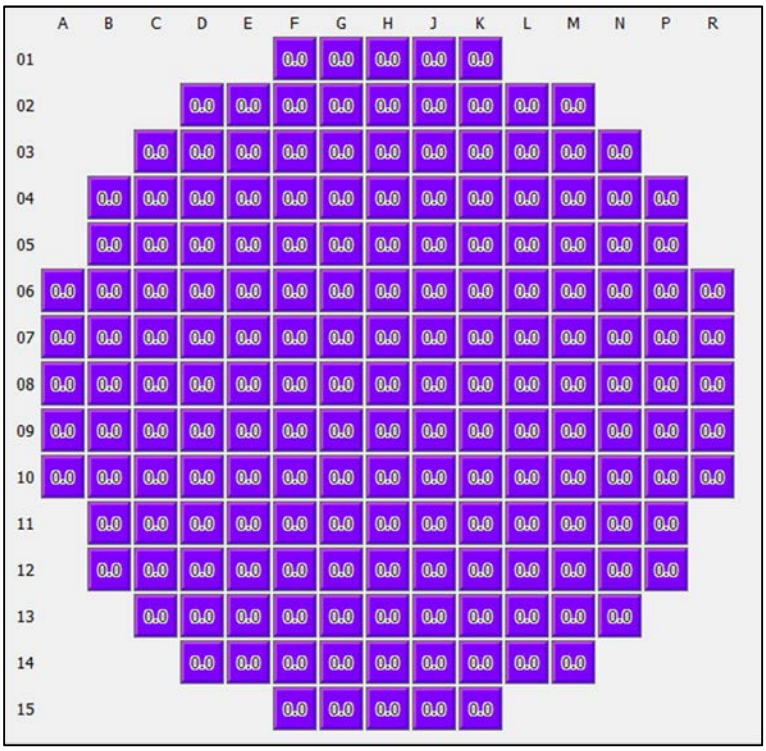


*FIG. 52. Assembly exposure for cycle 1 core burnup of 0 GWd/t.*

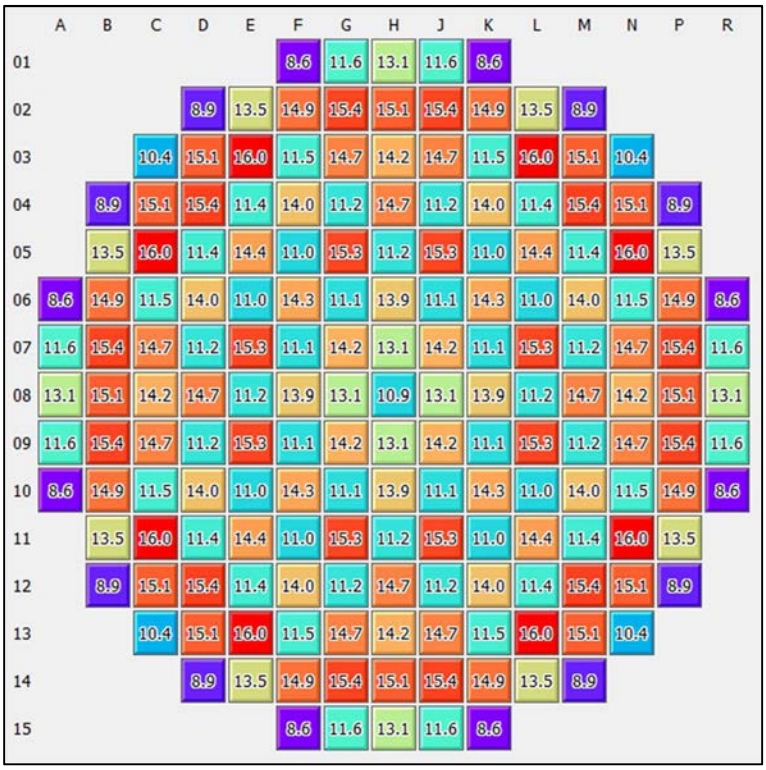


*FIG. 53. Assembly exposure for cycle 1 core burnup of 13 GWd/t.*

Radial-averaged and axial-averaged relative power distributions in the core at the BOL and EOL are shown in Figures 57 and 58, respectively (Section 4.5.2.2). Relative power axial distribution, shown in Figure 57, indicates that the core centre (in the axial direction) is a high power region due to the large number of fissions being generated in the core centre. As a result, this region is depleted faster than the upper and lower areas of the core, so the relative axial power flattens as the fuel is depleted and becomes effectively flat by the EOL, as Figure 58 shows.

As coolant/moderator water passes through the core region, it is heated at a rate corresponding to the relative power at each height (Section 4.5.2.5). The result of this is a moderator temperature curve as shown in Figure 59. As the relative power profile flattens, so too does the effect on heating of coolant/moderator. This can be seen by the near linear increase in moderator temperature shown in Figure 60.

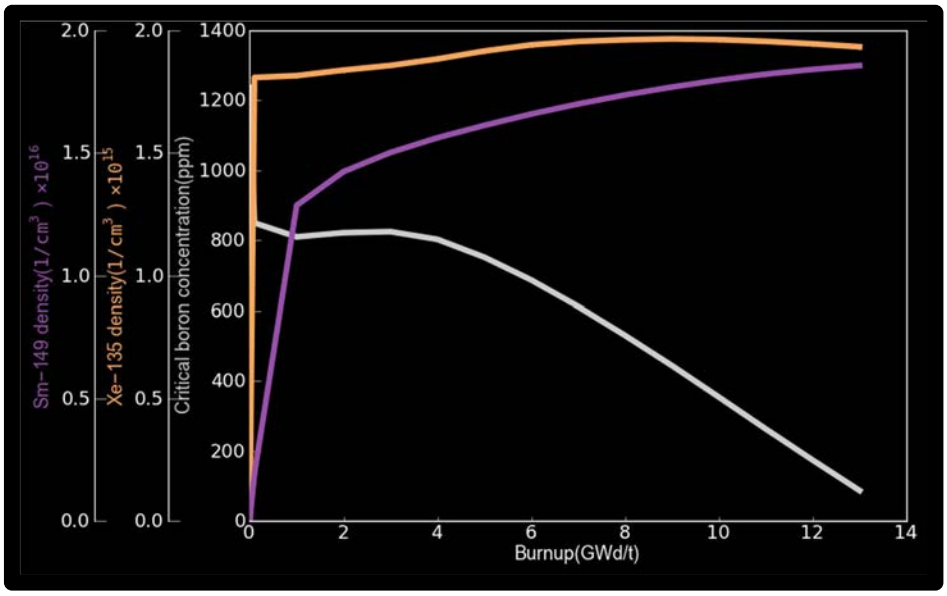


*FIG. 54. Critical boron, $^{135}Xe$ concentration and $^{149}Sm$ concentration over fuel burnup.*

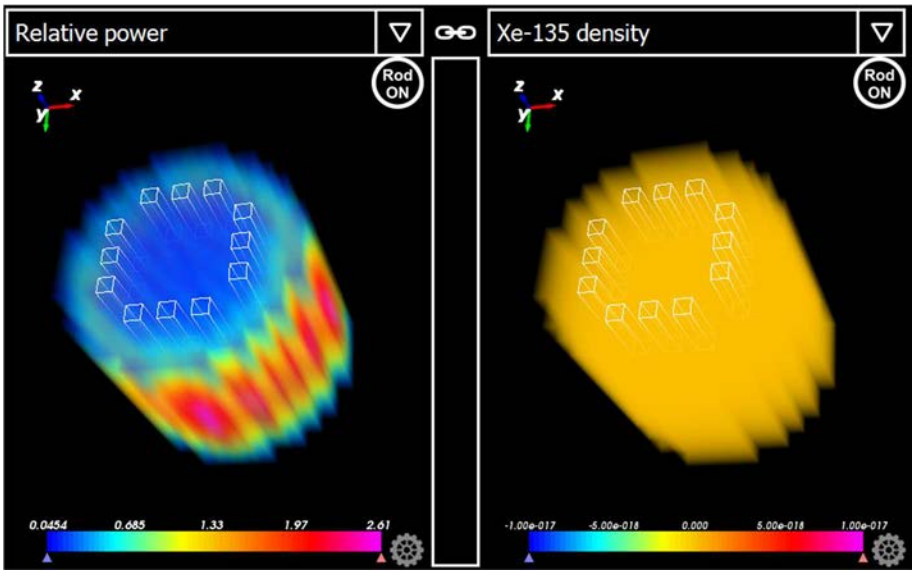


*FIG. 55. Relative power and $^{135}Xe$ distribution at BOL.*

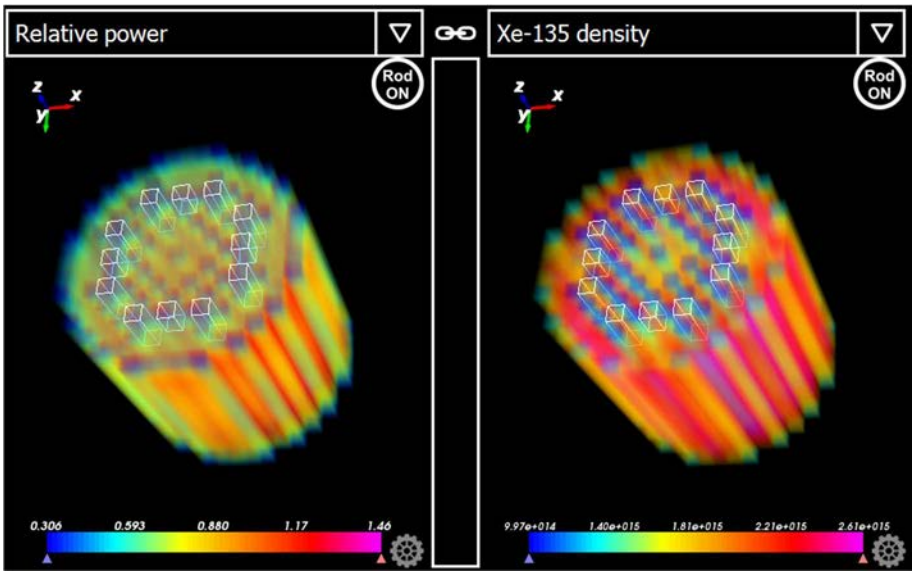


*FIG. 56. Relative power and $^{135}Xe$ distribution at EOL.*

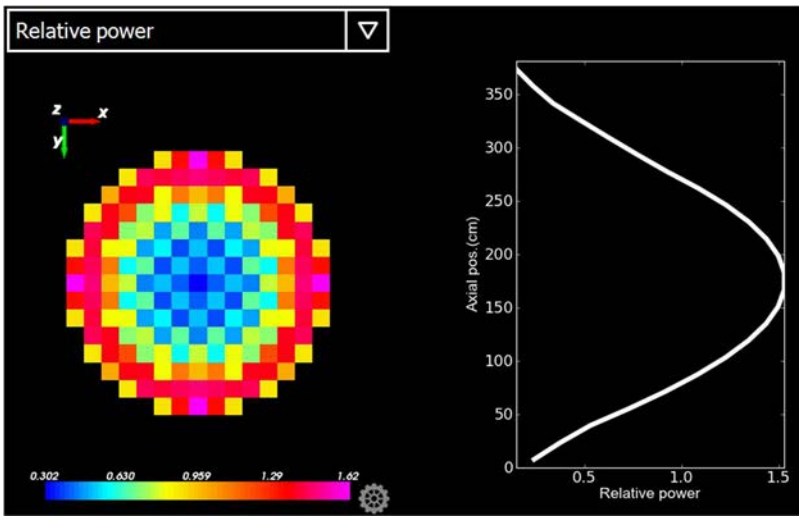


*FIG. 57. Radial averaged and axial averaged relative power distributions at BOL.*

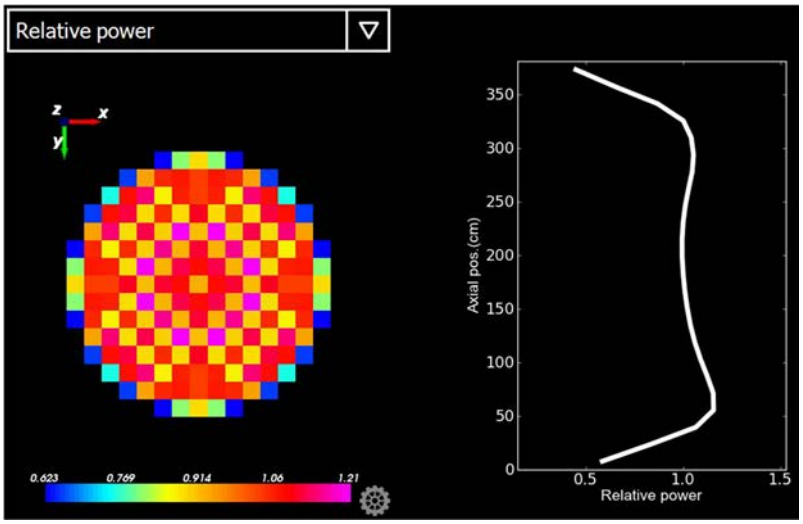


*FIG. 58. Radial averaged and axial averaged relative power distributions at EOL.*

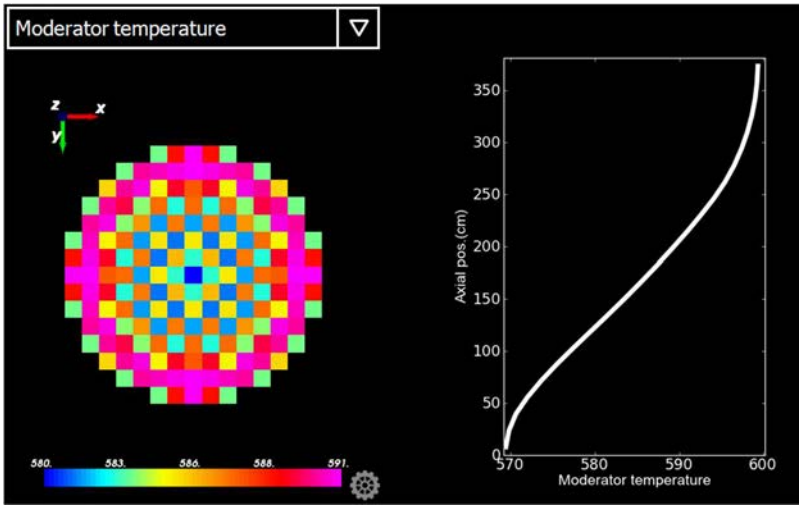


*FIG. 59. Radial averaged and axial averaged moderator temperature distributions at BOL.*

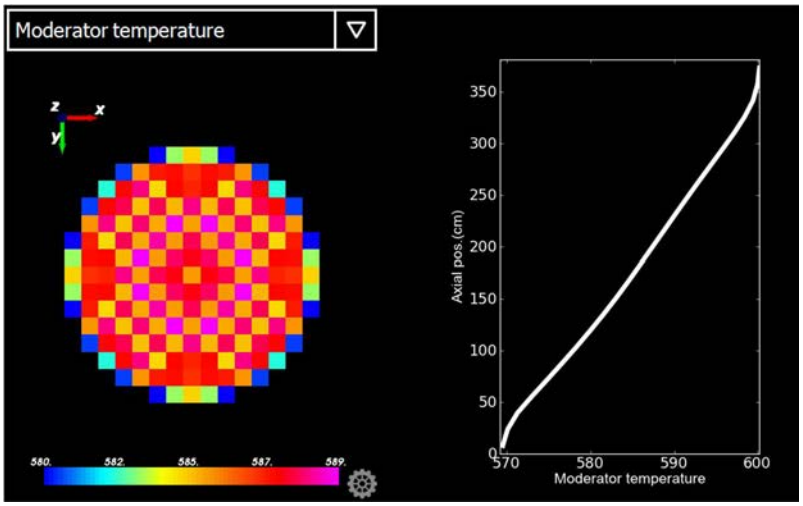


*FIG. 60. Radial averaged and axial averaged moderator temperature distributions at EOL.*

### 5.1.2. Depletion analysis cycle 2

Depletion analysis cycle 2 is the case of a core loaded with a mix of partially burned and fresh fuel of various fuel enrichments (Section 4.5.2.4). The average exposure of fuel assemblies at the beginning and end of cycle, 0.0 GWd/t and 13.0 GWd/t burnup respectively, are shown in Figures 61 and 62.

Depletion analysis cycle 2 can be used to show many of the same concepts as demonstrated by depletion analysis cycle 1; however, there are some noticeable differences. One of the most notable differences is the effect of reuse of fuel on neutron poison concentrations (Section 4.3). Due to some of the fuel having been previously burned, initial fission product poisons (most significantly Sm, since Xe can decay) exist which affect the quantity of soluble boron necessary to maintain criticality (Section 4.4.3). The resultant poison concentrations are shown in Figure 63.

Another effect of reusing fuel in this depletion cycle can be observed in the relative power of the core. As shown in Figure 64, the radial and axial profiles at the BOL are flattened when

compared with all fresh fuel (Section 4.5.2.2). This is because the partially burned fuel was burned most significantly in the highest power regions of the core during the previous cycle. The relative power profiles at the EOL, shown in Figure 65, show only a small amount of flattening compared to the BOL profiles.

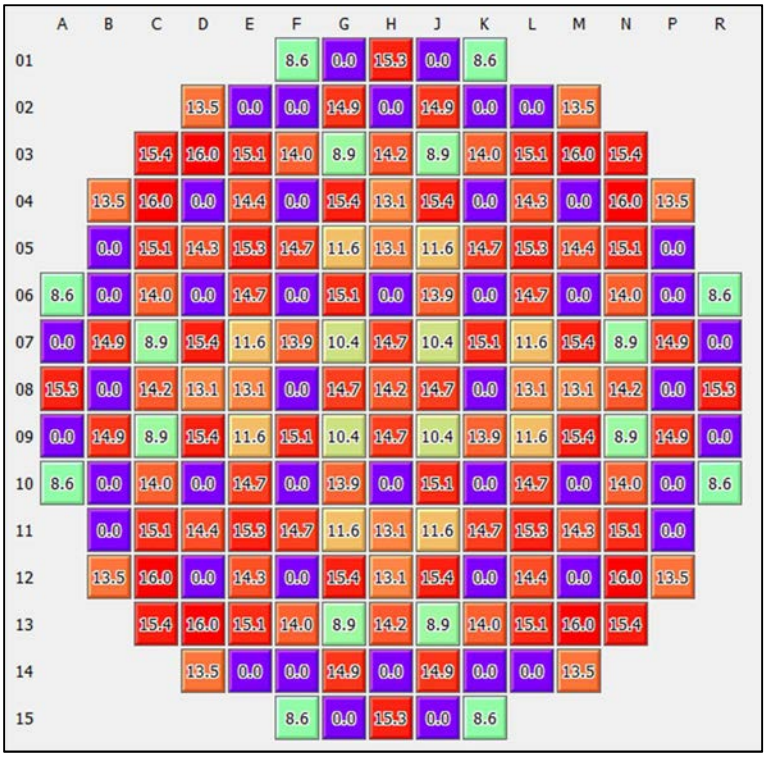


*FIG. 61. Assembly exposure for cycle 2 core burnup of 0.0 GWd/t.*

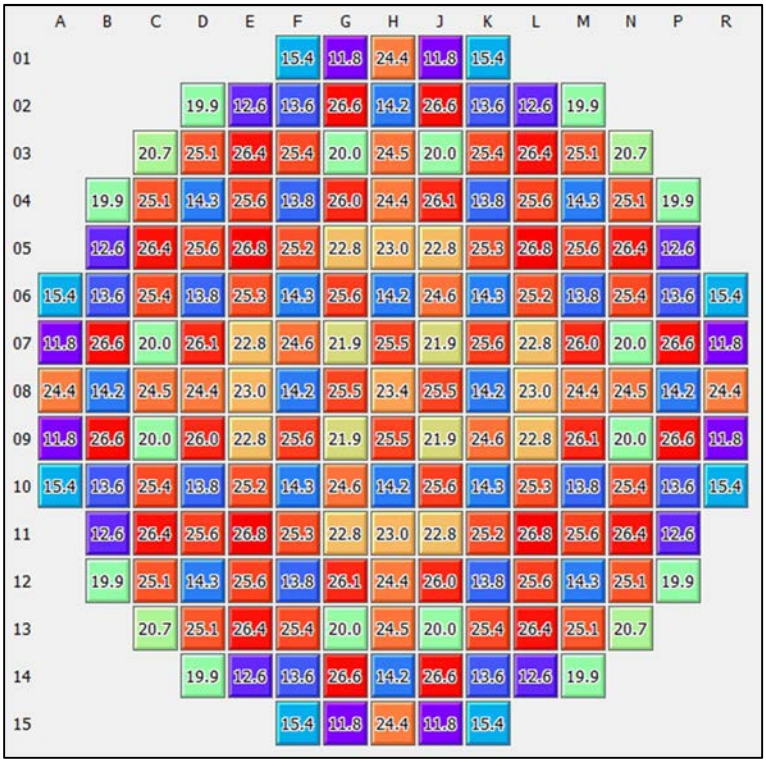


*FIG. 62. Assembly exposure for cycle 2 core burnup of 13.0 GWd/t (right).*

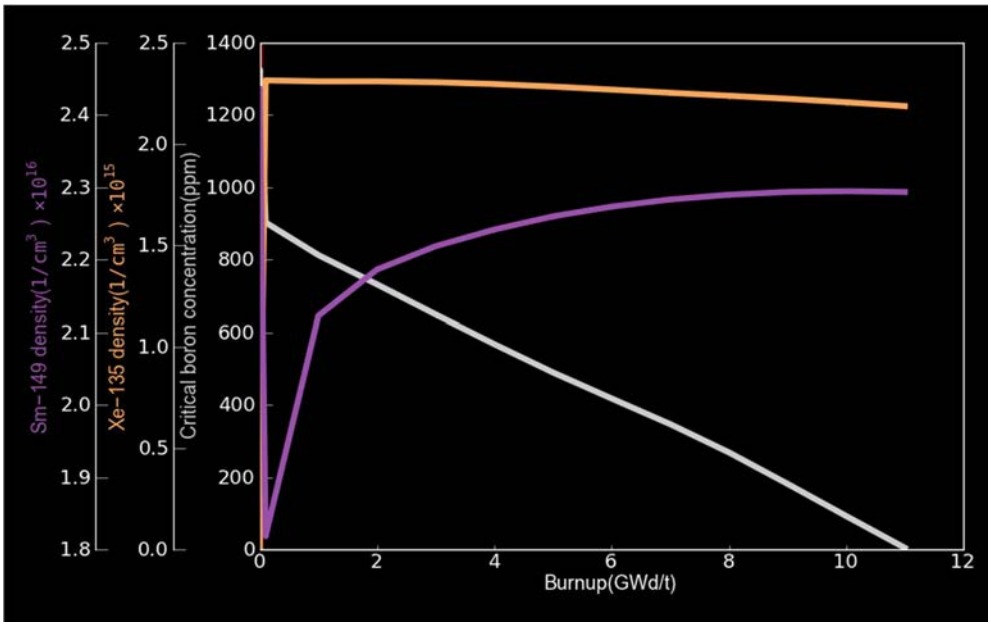


*FIG. 63. Critical boron, $^{135}Xe$ concentration and $^{149}Sm$ concentration over fuel burnup.*

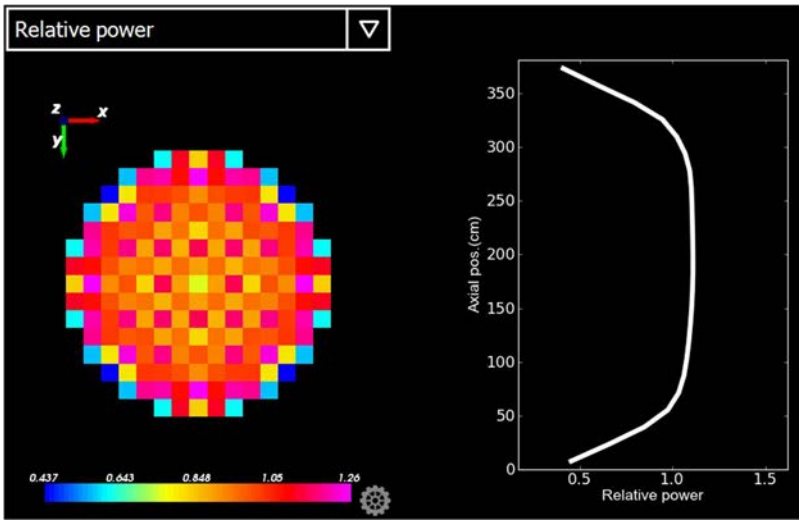


*FIG. 64. Radial averaged and axial averaged relative power distributions at EOL.*

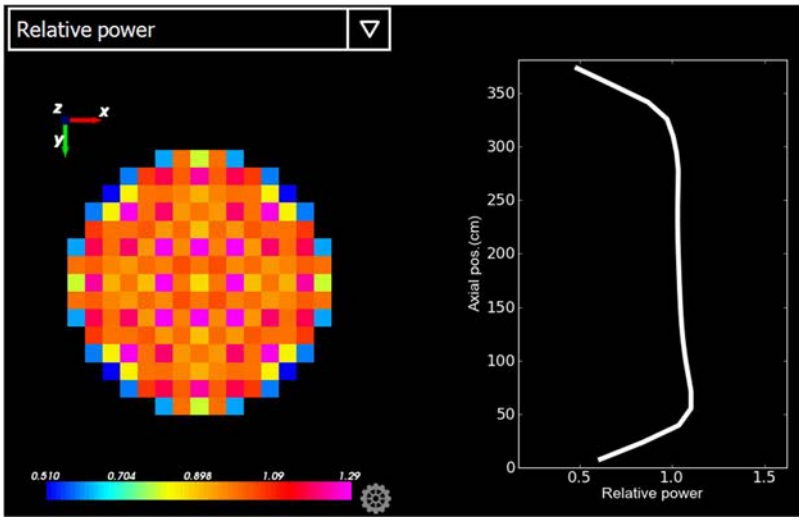


*FIG. 65. Radial averaged and axial averaged relative power distributions at EOL.*

5.2.TRANSIENT ANALYSIS

The examples included in this section are descriptions of simulation results for the following transients in the Micro-Physics Simulator Lite transient analysis mode:

— Power manoeuvring;
— Reactor scram;
— Loss of flow;
— Main steam line break;
— Hot zero power condition: abnormal withdrawal of a control rod bank and dilution of boron in the coolant;
— Hot full power condition: withdrawal of a control rod bank;
— Hot full power condition: abnormal dilution of boron.

### 5.2.1. Power manoeuvring

The reactor is initially at hot full power condition (100% core power operation at time = 0 seconds) with a cycle burnup of 13.0 GWd/t. The coolant mass flow is lowered linearly to reduce the reactor power to just over 80% at approximately time = 320 seconds. At approximately time = 430 seconds, coolant mass flow is raised linearly until reactor power returns to 100%. A similar transient was considered in Section 4.6.1.

Figure 66 shows the reactor power, moderator density, coolant mass flow rate and coolant temperature over the course of the power manoeuvre from 100% power to 80% power and back. Once coolant mass flow rate is lowered, the coolant remains in the core region for a longer period of time. This leads to the temperature rise in the coolant, which in turn reduces the moderator density. PWRs have a negative moderator temperature coefficient, so negative reactivity is inserted as the coolant water temperature rises. This is due to reduced moderation, and a resultant decrease in the resonance escape probability (Section 4.3.1).

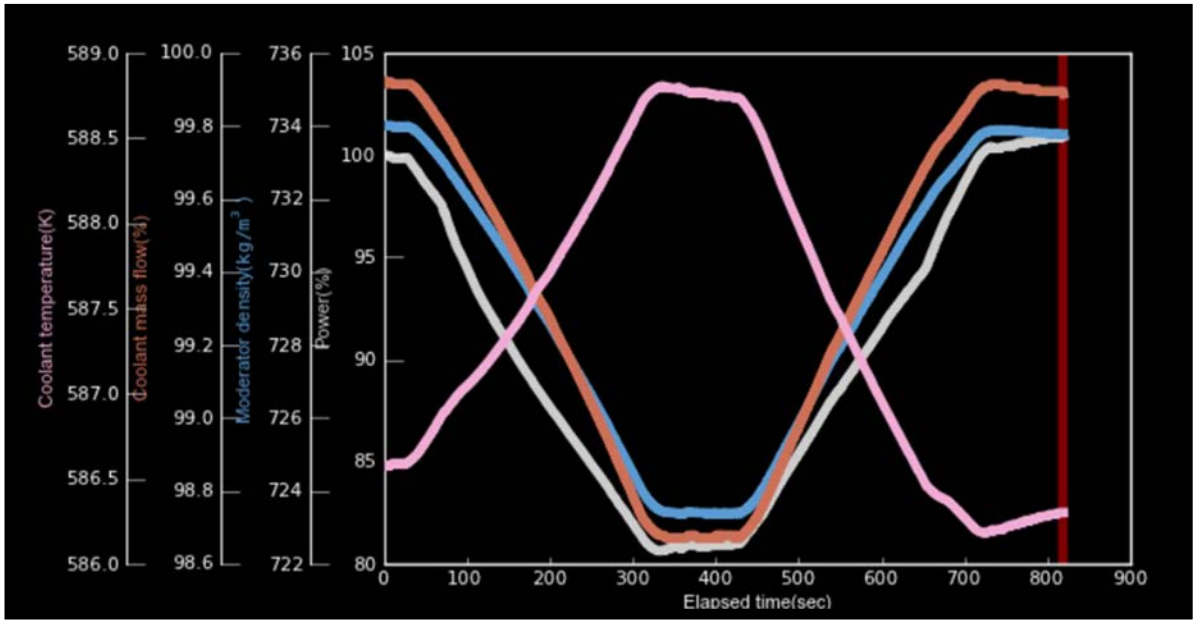


*FIG. 66. Power, moderator density, coolant mass flow and coolant temperature during power manoeuvre.*

In the return to 100% power, the opposite effects are seen. Coolant mass flow rate is increased, reducing coolant temperature and increasing moderator density. As a result of the negative moderator temperature coefficient, positive reactivity is inserted until the reactor power stabilizes at 100%.

### 5.2.2. Reactor scram

The reactor is initially at hot full power condition (100% core power operation at time = 0 seconds) with a cycle burnup of 13.0 GWd/t. A scram is initiated from time = 5 seconds after the initial time, and it takes 4 seconds for the control rods to be fully inserted (fully inserted at time = 9 seconds). A similar transient was considered in Section 4.6.2.

Figure 67 shows the reactor power, reactivity, and fuel and coolant temperatures. These variables are constant until time = 5 seconds. After the scram was initiated at time = 5 seconds, the power, fuel temperature and coolant temperature decrease. The reactivity also decreases after the scram due to the insertion of control rods. However, after this insertion is completed at time = 9 seconds, a positive reactivity insertion is observed. This positive reactivity insertion is because the reactor has a negative moderator temperature coefficient and negative fuel temperature coefficient (Sections 4.3.1 and 4.3.3). As seen in Figure 67, fuel and coolant temperature decrease, giving rise to the positive reactivity insertion.

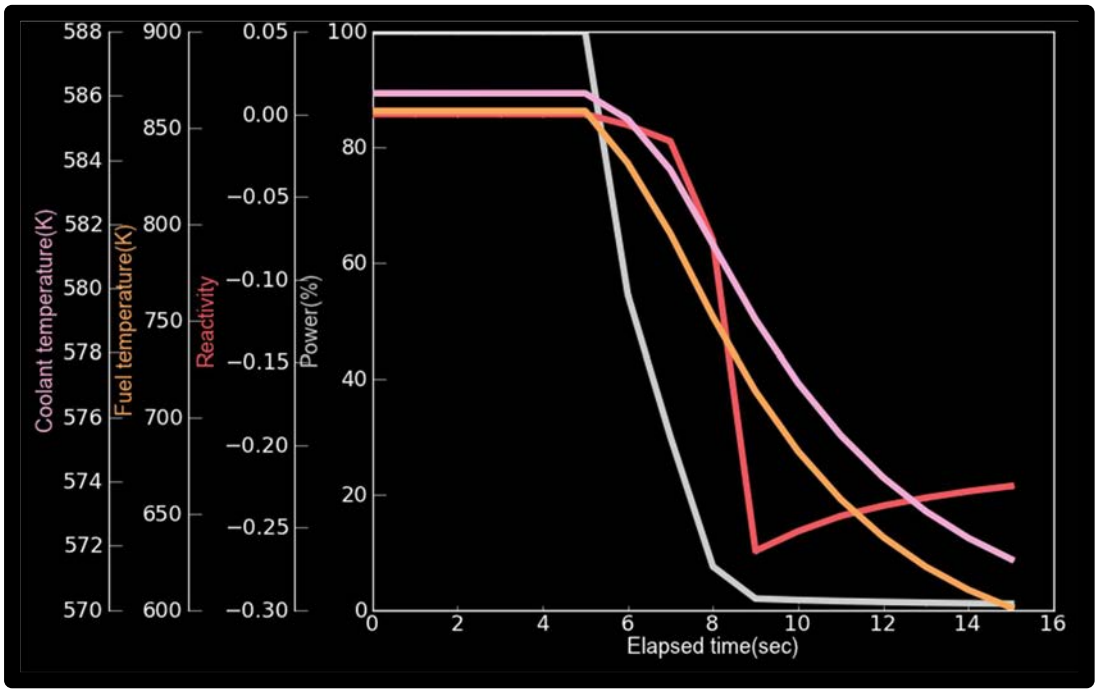


*FIG. 67. Power, reactivity, fuel temperature and coolant temperature during the reactor scram.*

### 5.2.3. Loss of flow

The reactor is initially at hot full power condition (100% core power operation at time = 0 seconds) with a cycle burnup of 13.0 GWd/t. The coolant flow decreases from 100% to 50% after 10 seconds. A similar transient was considered in Section 4.6.3.

Figure 68 shows relative power, reactivity, fuel temperature and coolant temperature as a function of transient time. Since there is less coolant flow to remove the heat produced in the fuel, and coolant remains within the core region longer, the coolant temperature increases. A gradual loss of coolant flow causes a reduction of neutron moderation (Section 4.3.1), which results in a decrease in reactivity, power and fuel temperature. The coolant temperature reaches a peak at approximately time = 9 seconds. At approximately time = 7.5 seconds, reactivity begins to increase because of the negative fuel temperature coefficient; the fuel temperature decreases (Section 4.3.3).

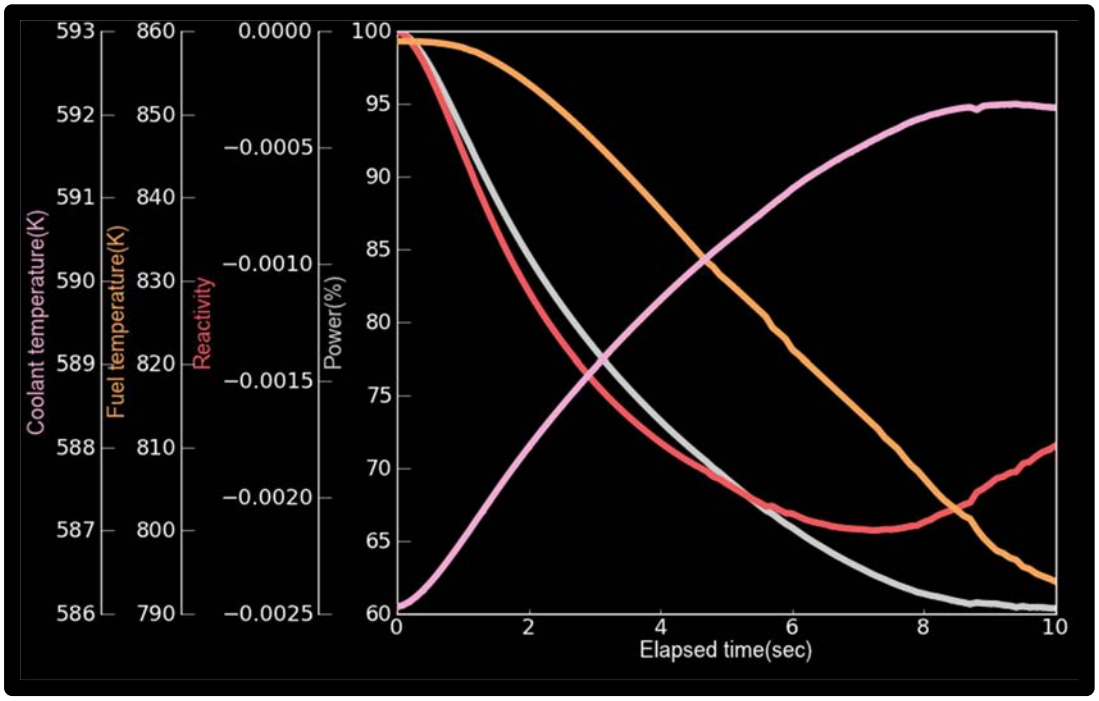


*FIG. 68. Power, reactivity, fuel temperature and coolant temperature during loss of flow transient.*

### 5.2.4. Main steam line break

The reactor is initially at hot full power condition (100% core power operation at time = 0 seconds) with a cycle burnup of 13.0 GWd/t. A main steam line break (MSLB) occurs at 10 seconds from the initial time (break in steam generator tubes, see Section 2.1).

Figure 69 shows average outlet coolant pressure, power, and fuel and coolant temperatures. A reactor scram is programmed at 10 seconds after the MSLB (time = 20 seconds); the scram may be initiated in response to high power or low coolant flow (Sections 2.4 and 4.6.2). In an MSLB, when coolant from the ruptured loop enters the core, power increases quickly and continues until reactor scram. The fuel temperature follows a similar trend to reactor power. The pressure is constant before the transient, and when the MSLB occurs, the pressure decreases as the loop with the break cools down. The coolant temperature also decreases after the MSLB for the same reason, and the decreasing trend becomes steeper following the scram as less heat is generated in the core region.

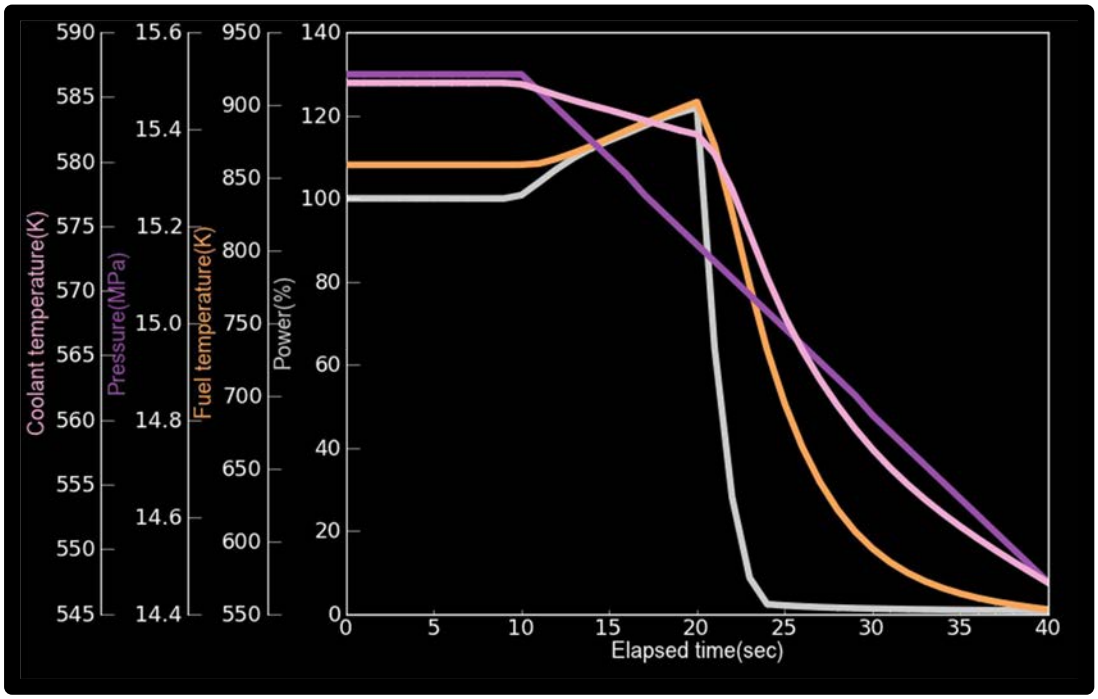


*FIG. 69. Power, pressure, fuel temperature and coolant temperature during the MSLB.*

### 5.2.5. Hot zero power condition: abnormal withdrawal of a control rod bank and dilution of boron in the coolant

The reactor is initially at hot zero power condition (0.0% core power operation at time = 0 seconds) at the BOL, cycle burnup of 0 GWd/t. Unlike typical design basis accidents with abnormal withdrawal of a control rod bank, where boron concentration is constant, the transient is simulated by steadily and simultaneously withdrawing control rods and reducing boron concentration until the bank is fully withdrawn at time = 107 seconds and continuing the boron concentration reduction until time = 180 seconds in order to make larger reactivity changes and demonstrate feedback effects.

Figure 70 illustrates the behaviour of four parameters: reactivity, power, fuel temperature and coolant temperature. Since the reactor is at the beginning of a fuel cycle, there is no significant concentration of fission products in the fuel.

Positive reactivity is introduced into the core due to the withdrawal of control rods and the reduction of boron concentration (Sections 4.2 and 4.4.3). Accordingly, this figure shows a steady increase in reactivity until the fuel temperature (and moderator temperature) increases, with the consequent addition of negative reactivity due to the negative moderator temperature and fuel temperature coefficients in a PWR (Sections 4.3.1 and 4.3.3). The reactivity decreases rapidly from about time = 170 seconds due to this negative fuel temperature coefficient, and boron concentration reduction ending at time = 180 seconds. Due to the insertion of positive reactivity, the reactor power increases substantially from about time = 160 seconds, and the fuel temperature closely follows the behaviour of the reactor power as a result of the heat generated by the core. The coolant temperature roughly follows the behaviour of the fuel temperature for the same reason.

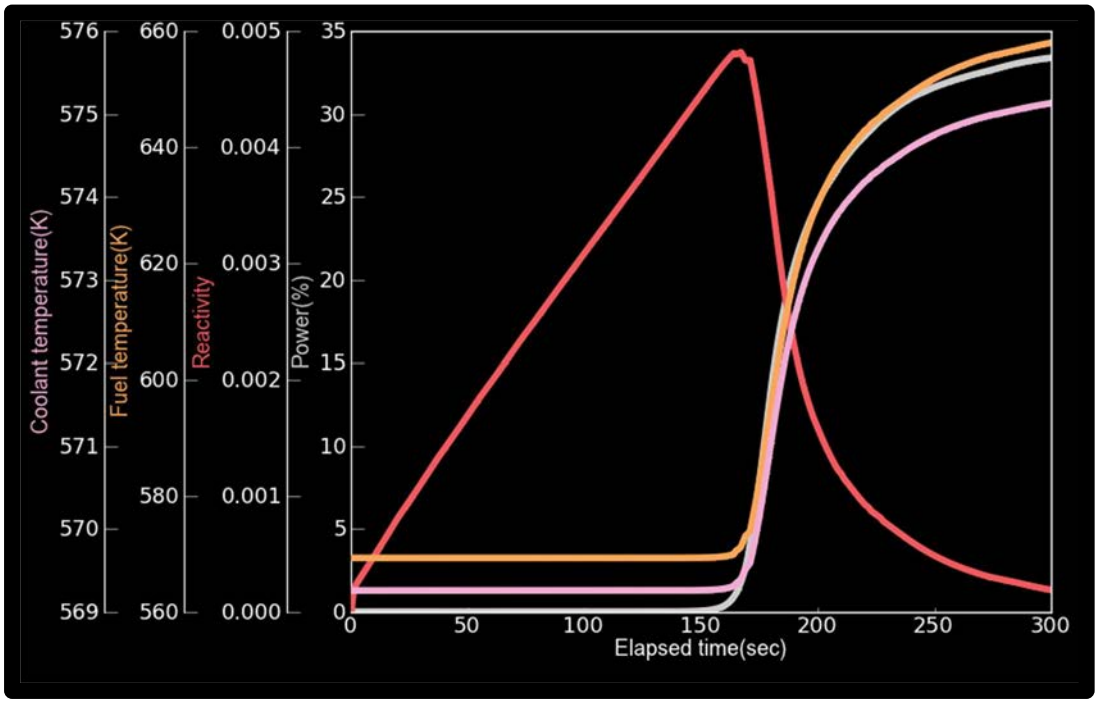


*FIG. 70. Hot zero power condition: power, reactivity, fuel temperature and coolant temperature during the abnormal withdrawal of control rods.*

### 5.2.6. Hot full power condition: withdrawal of a control rod bank

The reactor is initially at hot full power condition (100% core power operation at time = 0 seconds) with a cycle burnup of 13.0 GWd/t. The transient is simulated by linearly withdrawing control rods over 30 seconds.

Figure 71 illustrates a trend of four core parameters: reactivity, core power, fuel temperature and reactor period. Since the reactor is at power and has a cycle burnup of 13.0 GWd/t, which may be considered the EOL, there are significant concentrations of fission products with large cross-sections for thermal neutron absorption, notably $^{135}$Xe and $^{149}$Sm (Sections 4.4.1 and 4.4.2). As a result, this transient demonstrates the effect on reactivity of several factors including positive reactivity due to withdrawal of a control rods (Section 4.2), negative reactivity due to the fission product poisons and two negative temperature coefficients as the fuel and moderator temperatures increase (Sections 4.3.1 and 4.3.3).

The overall effect of the control rod withdrawal is an increase in positive reactivity that is non-linear. The reactor period is inversely proportional to the reactivity as in Eq. (42); therefore, an increase in reactivity means that reactor period will decrease, and neutron flux will change at a faster rate. The reactor power increases in a roughly linear way as a result of the net addition of positive reactivity and the fuel temperature closely follows this trend due to the heat generated in the core.

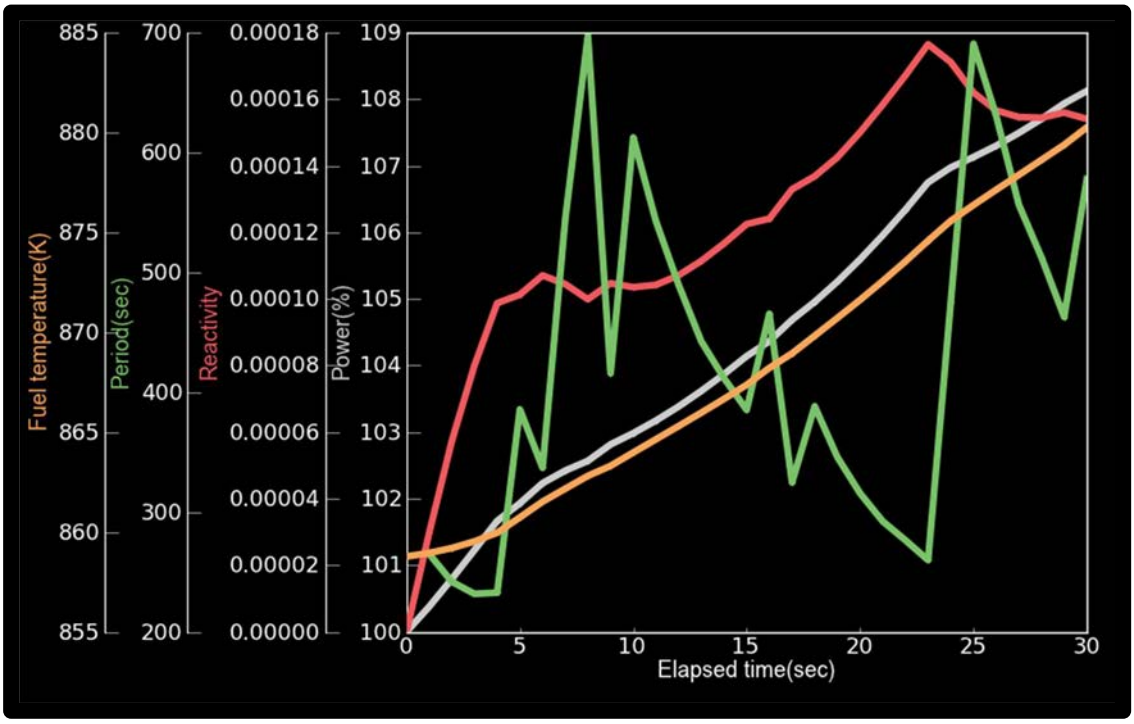


*FIG. 71. Hot full power condition: reactivity, core power, fuel temperature and reactor period during the withdrawal of control rods.*

### 5.2.7. Hot full power condition: abnormal dilution of boron

**This example is user defined.** It is based on the initial conditions for hot full power, provided by the previous MPSL example, Section 5.2.6. Instead of withdrawing a control rod (Section 4.2), however, this example adjusts boron concentration (Section 4.4.3) to simulate an abnormal boron dilution in which a slug of water with reduced boron concentration passes through the reactor core. The initial conditions for this case are provided in Table 6.

TABLE 6. INITIAL CONDITIONS FOR EXAMPLE 5.2.7

| PARAMETER | VALUE |
|---|---|
| Cycle burnup (GWd/t) | 13.0 |
| Initial core power (%) | 100.0 |
| A bank | 381.0 |
| B bank | 381.0 |
| R0 bank | 381.0 |
| R1 bank | 381.0 |
| R2 bank | 381.0 |
| R3 bank | 381.0 |
| R4 bank | 381.0 |
| R5 bank | 343.0 |
| S bank | 381.0 |
| P bank | 305.0 |
| Stuck rod bank | 381.0 |
| Coolant temperature (K) | 569.261 |
| Coolant mass flow rate (%) | 100.000 |
| Boron concentration (ppm) | 87.000 |
| Average outlet coolant pressure (MPa) | 15.513 |

'Elapsed time (sec)' is input with a timestep of 1 second from 0 to 60 seconds. The boron concentration remains stable at 87.000 ppm until time = 15 seconds. At time = 15 seconds, it follows Eq. (76) before returning to a stable value of 87.000 ppm on the timestep corresponding to time = 28 seconds. Eq. (76) is meant to roughly mimic the behaviour of a reduced boron concentration slug of water entering and passing through the core. Note that this is not a formally constructed model of such behaviour, but rather an arbitrary equation used to demonstrate the resultant phenomena. Figure 72 shows the boron concentration data produced by this equation on the timestep of 1 second. All other parameters cells are left blank after the initial step.

Prior to the boron dilution, the reactor is at a stable power of 100%. At time = 15 seconds, boron dilution begins, removing the negative reactivity contribution of the chemical shim. As a result, reactivity rises, causing an increase in power reaching a peak value of about 175% at approximately time = 20 seconds. At this higher power, fuel and coolant temperatures also rise. The combined effects of boron returning to its starting concentration and the negative reactivity insertions due to the negative fuel and moderator temperature reactivity coefficients (Sections 4.3.1 and 4.3.3) cause the net reactivity to become negative, causing the power to drop below 100%, reaching a minimum of about 80% at approximately time = 28 seconds. As temperatures fall to the new, lessened power level, reactivity becomes positive and returns the reactor to a stable 100% power at approximately time = 35 seconds. These effects are shown in Figure 73. In a reactor, a rise in power such as that shown in Figure 73 could raise the power above the operational limit (Section 2.4) and cause the reactor protection system to initiate a scram (Section 4.6.2).

$$n_{\text{boron}}(\text{ppm}) = 47 - 40 * \cos\left(\frac{t-15}{2} - \pi\right) \quad \text{for } 15 < t < 28 \tag{76}$$

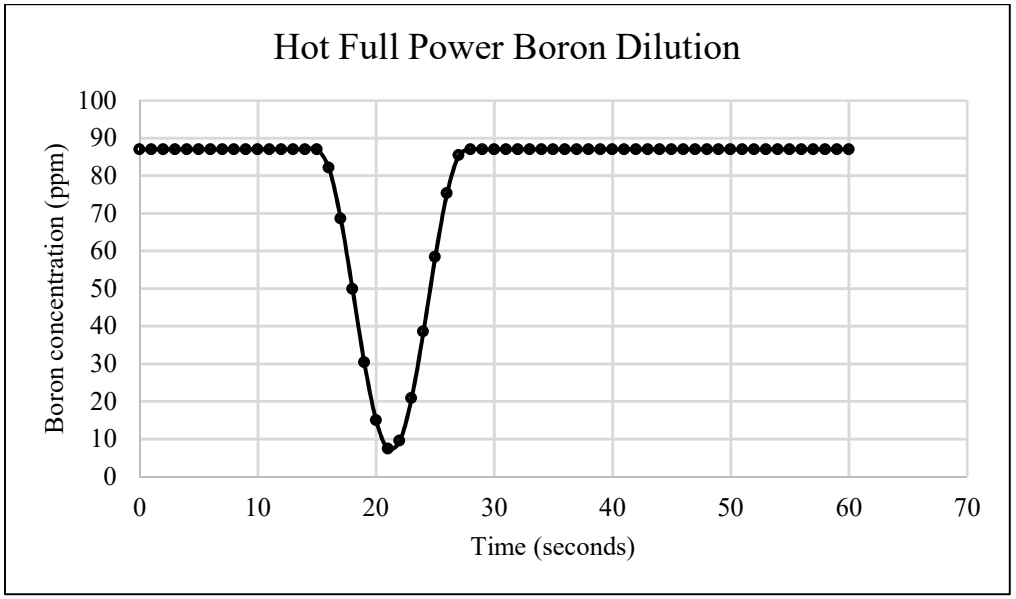


*FIG. 72. Hot full power boron dilution example, boron concentration as a function of time.*

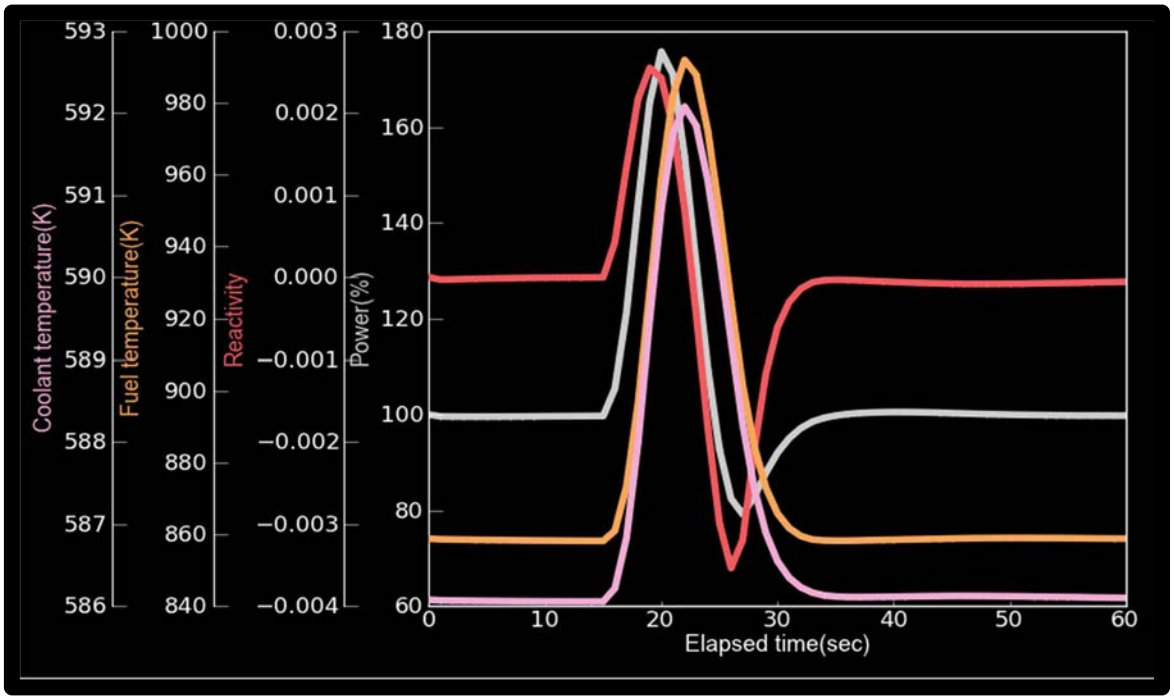


*FIG. 73. Power, reactivity, fuel temperature and coolant temperature as a function of time during a hot full power condition abnormal boron dilution.*

# 6. CONCLUDING REMARKS

Nuclear reactors are complex systems and their successful design and operation must take account of a large number of dynamic and interrelated parameters. Understanding how these parameters change, in relation to each other and over time, is fundamental to understanding reactor operation.

The lessons presented in this publication are introductory in nature, but sufficient to apply to the Micro-Physics Simulator Lite exercises in Section 5. Each discussed topic can be further explored in great detail and many of the models presented are highly simplified; however, this publication provides a basic overview of the ‘big picture’, major parameters and essential considerations for water cooled reactor operation.

One specific application of learned knowledge is to assist in the use of educational basic principle reactor simulators, such as the Micro-Physics Simulator Lite and others available at the request of Member States to the IAEA, which may be used to reinforce concepts in a hands-on and interactive way. These simulators apply the basic taught concepts to full reactor systems, and many of the explained parameters can be observed in simulator responses to transient conditions.

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# GLOSSARY

*This glossary compiles relevant reference terminology from the IAEA Safety Glossary (2016 Revision) and the IAEA Radioactive Waste Management Glossary (2003 Edition).*

**activity.** The quantity $A$ for an amount of radionuclide in a given energy state at a given time, defined as: $A(t) = \mathrm{d}N/\mathrm{d}t$ where d$N$ is the expectation value of the number of spontaneous nuclear transformations from the given energy state in the interval d$t$.

**burnable absorber.** Neutron absorbing material, used to manage reactivity, with the particular capability of being depleted by neutron absorption.

**cladding.** Typically, the tube of material that houses nuclear fuel pellets and provides the containment (means of confinement) of radionuclides produced during fission.

**core components.** The elements of a reactor core, other than fuel assemblies, that are used to provide structural support of the core construction, or the tools, devices or other items that are inserted into the reactor core for core monitoring, flow control or other technological purposes and are treated as core elements. Examples of core components are reactivity control devices or shutdown devices, neutron sources, dummy fuel, fuel channels, instrumentation, flow restrictors and burnable absorbers.

**critical.** Having a reactivity of zero.

**criticality.** The state of a nuclear chain reacting medium when the chain reaction is just self-sustaining (or critical), i.e. when the reactivity is zero. Often used, slightly more loosely, to refer to states in which the reactivity is greater than zero.

**decay constant.** For a radionuclide in a particular energy state, the quotient of d$P$ by d$t$, where d$P$ is the likelihood for a single nucleus of undergoing a spontaneous nuclear transition from that energy state in the time interval d$t$. $\lambda = \mathrm{d}P/\mathrm{d}t = -1/N\ \mathrm{d}N/\mathrm{d}t = A/N$ where $N$ is the number of nuclei of concern existing at time $t$ and $A$ is the activity.

**equilibrium, radioactive.** The state of a radioactive decay chain (or part thereof) where the activity of each radionuclide in the chain (or part of the chain) is the same).

**fissile material.** Material containing any fissile nuclides.

**fissile nuclide.** Nuclides, in particular uranium-233, uranium-235, plutonium-239 and plutonium-241, that are able to support a self-sustaining nuclear chain reaction with neutrons of all energies, but predominantly with slow neutrons.

**fission fragment.** A nucleus resulting from nuclear fission carrying kinetic energy from that fission.

**fission product.** A radionuclide produced by nuclear fission.

**fissionable material.** Material containing any fissionable nuclides.

**fissionable nuclide.** Nuclides such as uranium-238 that are capable of supporting a self-sustaining nuclear chain reaction, including a self-sustaining nuclear chain reaction with fast neutrons.

**fresh fuel.** New fuel or unirradiated fuel, including fuel fabricated from fissionable material recovered by reprocessing previously irradiated fuel.

**fuel assembly.** A set of fuel elements and associated components which are loaded into and subsequently removed from a reactor core as a single unit.

**fuel element.** A rod of nuclear fuel, its cladding and any associated components necessary to form a structural entity. Commonly referred to as fuel rod in light water reactors.

**half-life.** For a radionuclide, the time required for the activity to decrease, by a radioactive decay process, by half. The time taken for the quantity of a specified material (e.g. a radionuclide) in a specified place to decrease by half as a result of any specified process or processes that follow similar exponential patterns to radioactive decay.

**mixed oxide fuel**. Nuclear reactor fuel which contains more than one type of fissile nuclide, both or all being in the form of oxides. Most commonly refers to fuel containing both uranium oxide and plutonium oxide.

**nuclear fuel.** Fissionable nuclear material in the form of fabricated elements for loading into the reactor core of a civil nuclear power plant or research reactor. Fissionable and fertile material used in a nuclear reactor for the purpose of generating energy.

**operational limits and conditions.** A set of rules setting forth parameter limits, the functional capability and the performance levels of equipment and personnel approved by the regulatory body for safe operation of an authorized facility.

**poison.** A substance used to reduce reactivity (typically in a reactor core), by virtue of its high neutron absorption cross-section.

**radioactive.** Exhibiting radioactivity; emitting or relating to the emission of ionizing radiation or particles.

**radioactivity.** The phenomenon whereby atoms undergo spontaneous random disintegration, usually accompanied by the emission of radiation.

**reactivity.** For a nuclear chain reacting medium: $\rho = 1 - 1/k\_\text{eff}$ where $k_{\text{eff}}$ is the ratio between the number of fissions in two succeeding generation (later to earlier) of the chain reaction.

**residual heat.** The sum of the heat originating from radioactive decay and shutdown fission and the heat stored in the reactor related structures and in heat transport media.

**safety function.** A specific purpose that must be accomplished for safety for a facility or activity to prevent or to mitigate radiological consequences of normal operation, anticipated operational occurrences and accident conditions.

**safety limits.** Limits on operational parameters within which an authorized facility has been shown to be safe.

**safety system.** A system important to safety, provided to ensure the safe shutdown of the reactor or the residual heat removal from the reactor core, or to limit the consequence of anticipated operation occurrences and design basis accidents.

**safety system settings.** Settings for levels at which safety systems are automatically actuated in the event of anticipated operational occurrences or design basis accidents, to prevent safety limits from being exceeded.

**scram.** A rapid shutdown of a nuclear reactor in an emergency.

**shutdown.** The cessation of operation of a facility.

**spent fuel.** Nuclear fuel removed from a reactor following irradiation that is no longer usable in its present form because of depletion of fissile material, poison buildup or radiation damage.

## ABBREVIATIONS

| | |
|---|---|
| BOL | Beginning of life |
| BWR | Boiling water reactor |
| EOL | End of life |
| IAEA | International Atomic Energy Agency |
| LWGR | Light water cooled graphite moderated reactor |
| MOX | Mixed oxide |
| MPSL | Micro-Physics Nuclear Reactor Simulator™ Lite Edition |
| MSLB | Main steam line break |
| NEL | Nuclear Engineering, Ltd. |
| NPP | Nuclear power plant |
| OLC | Operating limits and conditions |
| PHWR | Pressurized heavy water reactor |
| PWR | Pressurized water reactor |
| WCR | Water cooled reactor |

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