Accelerator Beams for X-Ray-Gamma Lasers

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Abstract. The relativistic charged particles' accelerator beams coherent interaction with strong laser fields in different resonant schemes is of special interest in view of generation of intense shortwave coherent radiation in so-called hybrid systems of the conventional Quantum Generators and Free Electron Lasers. Specifically, as such systems for X-ray–Gamma-ray lasers, the high brightness fast ion beams or channelled in the crystals ultrarelativistic electron beams (due to the existence of quantum bound states) may be proposed. In the present work the SASE (Self-Amplified Spontaneous Emission) regime of semiclassical X-ray laser on the channeling in a crystal of a high brightness ultrarelativistic electron beam, in which the initial shot noise on the electron beam is amplified over the course of propagation through a crystal channel, is investigated.

The creation of X-ray lasers on the relativistic charged particles' accelerator beams is a subject of extreme interest at the present time and numerous projects around the world are focused on the construction and development of a so-called fourth generation light sources. The current state-of-the-art in light sources is referred to as third generation light sources that are large-scale synchrotron facilities whose output is incoherent synchrotron radiation. The requests on fourth generation sources are much higher brightness and peak photon density. Hence, any concepts that can be used to make a coherent fourth generation source are of prime importance. Regarding the problem of X-ray lasers there is an actual design currently in progress based on the Free Electron Laser (FEL) in a magnetic undulator [1, 2]. It is expected that X-ray FEL will operate in the so-called SASE (Self-Amplified Spontaneous Emission) regime [3] providing coherent radiation of ultrarelativistic high brightness electron beams in the powerful wigglers. Two international projects TESLA and LCLS are being currently implemented for this purpose [4]. The recent experimental advance in accelerator and electromagnetic field technologies evidences the feasibility of construction of such facilities [5]. Nevertheless, because there are no drivers or mirrors operable at X-ray wavelengths, these FEL systems operating in SASE regime, in which the initial shot noise on the electron beam is amplified over the course of propagation through a long wiggler, require the lengths in the order of several ten-to hundred meters. An incomparable more effective gain of FEL we can expect for nonlinear schemes of amplification/generation in the single-pass regime in the induced processes of charged particles' accelerator beams and strong laser fields coherent interaction, providing incomparable shorter generation lengths [6]. Moreover, these mechanisms practically may appear more reasonable for X-ray FEL due to smaller set up requirements (particularly, using electron beams of considerably lower energies).

Besides, the classical SASE amplifiers have one drawback with regard to its application as a useful source of short wavelength coherent light. This is connected with the fact that the SASE amplifiers are essentially noise amplifiers having spiky and fluctuating outputs and, consequently, wide radiation spectrum. Several schemes have been proposed in order to avoid the large spectral width associated with SASE. These involve superradiance of pre-modulated electron beam, self-amplification of coherent spontaneous emission, etc [7].

As the photon wavelength moves into the deep UV and X-ray regions the interaction becomes quantum mechanical, i.e. quantum recoil becomes comparable to or larger than the gain bandwidth and quantum effects play essential role. In particular, such quantum investigations for Compton X-ray FEL have been made by us in the regime where the quantum recoil is much larger than the electron beam energy spread and discrete-resonant transitions take place, excluding the concurrent process of radiation absorption as well [8]. The operation of this quantum regime is similar to conventional atomic lasers: two momentum states are involved in the interaction process and one can expect strong narrowing of the output radiation spectrum in the quantum SASE regime.

The quantum effects are also essential if one considers the FEL versions where one or two degrees of freedom of the charged particles are quantized and the resonant enhancement of electron-photon interaction cross section holds. This takes place for the X-ray laser schemes based on the electron/positron beam channeling radiation in the crystals [9]. The smallness of free electron-photon interaction cross section in comparison with the photon-atom one can also be compensated and the quality of the output X-ray radiation can be enhanced in the hybrid schemes of FEL and conventional atomic laser due to the existence of bound electron states. It can be achieved by means of fast high-density ion beam interaction with a strong counter-propagating pump laser field [10], or with a crystal periodic potential [11]. To achieve the efficient output intensity of coherent X-ray on amplification lengths rather smaller than the lengths of the current classical FEL facilities in wigglers, the nonlinear quantum schemes of generation with high brightness ultrarelativistic electron beams were proposed and investigated in high gain and SASE regimes [12].

The existence of quantized-discrete electronic levels substantially enhances the electron-coupling field interaction coefficient and, consequently, the lasing gain on rather small coherent lengths due to the resonant excitation of such semiclassical FEL system by strong pump fields. Hence, it is of certain interest the induced radiation by relativistic charged particle beams with discrete 1D or 2D transversal energy levels (one or two degrees of freedom of the beam are quantized, e.g., at the planar or axial channeling in a crystal), as a potential source of semiclassical FEL in the x-ray-gamma-ray domains.

In the present work the SASE regime of semiclassical X-ray FEL start up from spontaneous channeling radiation of a high brightness ultrarelativistic electron beam in a crystal, in which the initial shot noise on the electron beam is amplified over the course of propagation through a crystal channel, is investigated. Apart from a significant factor –ultrashort amplification lengths in the crystal channel ("micro-undulator"), the spectral intensity of spontaneous radiation by the channelled ultrarelativistic electrons/positrons well exceeds the intensities of all other type radiation processes at high frequencies. Thus, we study the amplification of ultrarelativistic electron beam channeling radiation in a crystal, in the SASE regime, stimulated by strong counterpropagating pump laser field. Beside the resonant enhancement of lasing gain due to the 1D or 2D resonant transitions of electrons at the channeling, one can expect purification of the output radiation spectrum in this semiclassical SASE amplifier as well, compared to the X-ray Compton FEL in SASE regime [13] which has spiky and fluctuating output.

The consideration is based on the self-consistent set of the Maxwell and relativistic quantum kinetic equations. In considering scheme the pump wave (optical or strong infrared laser radiation) due to the Doppler up-shifting resonantly couples two transverse electronic levels in the crystal channel, and the necessity of the initial inverse population of energy levels for lasing (which is obligatory for conventional quantum generators on atoms) vanishes.

To achieve maximal Doppler shift and optimal conditions of amplification, we will assume a scheme with a channeled particle beam and counterpropagating pump electromagnetic (EM) wave (as well as with a co-propagating probe EM wave). We will consider a linearly polarized (along OX) pump EM wave with the frequency ω and wave vector k that is described by the vector potential

$$\mathbf{A} = \widehat{\mathbf{x}}\frac{A_0}{2}\left[e^{i(\omega t+kz)} + \text{c.c.}\right]. \tag{1}$$

We assume the probe wave to be linearly polarized along the same direction as the pump wave, with the carrier frequency ω', wave vector k', and vector potential

$$\mathbf{A}_e = \widehat{\mathbf{x}}\frac{1}{2}\left[A_e(t,z)e^{i(\omega' t-k'z)} + \text{c.c.}\right], \tag{2}$$

where $A_e(t,z)$ is a slowly varying envelope.

For the description of a FEL operating in the crystal, where the transverse degrees of freedom of the particles are quantized, we will begin from the second quantized Hamiltonian, which in the Rotating Frame Approximation can be reduced to the form

$$\widehat{H} = \sum_{p_z}\left[\mathcal{E}_0(p_z)\widehat{a}^+_{0,p_z}\widehat{a}_{0,p_z} + \mathcal{E}_1(p_z)\widehat{a}^+_{1,p_z}\widehat{a}_{1,p_z}\right] + \widehat{H}_{int} \tag{3}$$

with the interaction Hamiltonian:

$$\widehat{H}_{int} = \sum_{p_z}\frac{\beta_\perp}{2c}\left[ieA_0e^{i\omega t}\widehat{a}^+_{0,p_z+\hbar k}\widehat{a}_{1,p_z} + ieA_ee^{i\omega' t}\widehat{a}^+_{0,p_z-\hbar k'}\widehat{a}_{1,p_z} + \text{h.c.}\right]. \tag{4}$$

The creation and annihilation operators $\widehat{a}^+_{\mu,p_z}(t)$ and $\widehat{a}_{\mu,p_z}(t)$, associated with positive energy $\mathcal{E}_\mu(p_z)$ solutions of the Dirac equation, satisfy the usual anticommutation rules at equal times. Here μ, p_z are the complete set of quantum numbers $\mu = \{p_y, n, \sigma\}$ for the planar channeling and $\mu = \{\mathfrak{m}, n, \sigma\}$ for the axial one, n is the main quantum number and $\mathfrak{m}$ is the magnetic quantum number, σ characterizes spin polarization and p_y, p_z are the components of particle momentum; ψ_{μ,p_z} are the normalized eigenvectors of channeled particle corresponding to the given set of quantum numbers. We will assume that probe and pump waves resonantly couple only two transverse levels, which will be labeled (0) and (1). It is also assumed that the particle beam is nonpolarized and the probability of transitions with the spin flip is negligible (this imposes a restriction on the wave frequency: $\hbar\omega' << \mathcal{E}_\mu(p_z)$).

Included in Eq. (4) $\beta_\perp$ is the transition matrix element for the transverse velocity operator:

$$\beta_\perp = \Omega_{nn'}x_{\mu\mu'}, \tag{5}$$

where $\Omega_{nn'} = \left(\mathcal{E}_{\perp n'} - \mathcal{E}_{\perp n}\right)/\hbar$ is the transition frequency between the initial and excited states of the transversal motion of the particle in the crystal channel. The resonant frequencies of the probe and pump waves for resonant coupling of the two transverse levels are defined from the conditions:

$$\omega = \frac{\Omega_{01}}{1 + n(\omega)\frac{\mathrm{v}}{c}}, \tag{6}$$

$$\omega' = \frac{\Omega_{01}}{1 - n\left(\omega'\right)\frac{\overline{\mathrm{v}}}{c}}. \tag{7}$$

Here $\overline{\mathrm{v}}$ is the electrons' mean longitudinal velocity in the beam and $n\left(\varpi\right)$ is the index of refraction of a crystal medium ($n\left(\omega'\right) \simeq 1$ for the frequency region under consideration).

The energy spectrum of the planar channeled electron in the potential well $U(x) = -U_0/\cosh^2(x/b)$ (U_0 -depth of the well, b characterizes crystal) has the form [6]:

$$\mathcal{E}_{\perp n} = -\frac{\hbar^2}{2b^2 m\gamma}\left[s - n\right]^2; \quad n = 0, 1, \ldots, [s], \tag{8}$$

where $s = -1/2 + \sqrt{1/4 + 2b^2 m\gamma U_0/\hbar^2}$ and $\gamma = 1/\sqrt{1 - \overline{\mathrm{v}}^2/c^2}$ is the Lorentz factor.

For the axial channeled electron with the potential of the atomic chain along the crystal axis $U(\rho) = -\alpha/\rho$ (α characterizes the concrete crystal, ρ -distance from the crystal axis) the energy spectrum reads [6]:

$$\varepsilon_{\perp n} = -\frac{m\gamma\alpha^2}{2\hbar^2}\frac{1}{\left(n + \frac{1}{2}\right)^2}; \quad n = 0, 1, 2, \ldots. \tag{9}$$

The selection rules for transitions are determined by the matrix element of dipole momentum and for the axial channeling are: $\Delta\mathfrak{m} = \pm 1$. For the planar channeling, $x_{\mu\mu'}$ differ from zero between the states having different parities. For the axial channeling there is degeneracy by the magnetic quantum number and in the case of a wave with linear polarization both of the states $\mathfrak{m} = \pm 1$ will have a contribution in the resonant interaction process. Because $\beta_\perp$ depends on $|\mathfrak{m}|$ for $\Delta\mathfrak{m} = \pm 1$ transitions, so the $\mathfrak{m} = \pm 1$ states are equally populated if the initial populations are also equal.

At the planar channeling for the $\mu_0 = \{0, 0\} \longrightarrow \mu = \{0, 1\}$ transition we have

$$\beta_\perp = \frac{\hbar}{2bm\gamma}\left(2s - 1\right)\left(\frac{s - 1}{2}\right)^{\frac{1}{2}}\frac{\Gamma^2\left(s - \frac{1}{2}\right)}{\Gamma^2\left(s\right)}, \tag{10}$$

where $\Gamma\left(s\right)$ is the Euler gamma function.

At the axial channeling for the transition $\mu_0 = \{0, 0\} \longrightarrow \mu = \{\pm 1, 1\}$ we have

$$\beta_\perp = \sqrt{2}\frac{\alpha}{\hbar}\sqrt{\frac{3}{32}}, \tag{11}$$

where the factor $\sqrt{2}$ is related to the degeneracy for axial channeling.

For the determination of the self-consistent field we need the evolution equation for the single-particle density matrix $\rho_{ij}(p_z, p'_z) =< \widehat{a}^+_{j,p'_z}\widehat{a}_{i,p_z} >$. From the Heisenberg equation we obtain the following equations for the populations of ground and excited states:

$$\frac{\partial\rho_{00}(p_z, p'_z, t)}{\partial t} + \frac{i}{\hbar}\left[\mathcal{E}_0(p_z) - \mathcal{E}_0(p'_z)\right]\rho_{00}(p_z, p'_z, t) = \frac{e}{2\hbar c}\beta_\perp\left[A_0\rho_{01}(p_z, p'_z - \hbar k, t)e^{-i\omega t}\right.$$
$$\left.+A_0\rho_{10}(p_z - \hbar k, p'_z, t)e^{i\omega t} + A_e^*\rho_{01}(p_z, p'_z + \hbar k', t)e^{-i\omega' t} + A_e\rho_{10}(p_z + \hbar k', p'_z, t)e^{i\omega' t}\right], \tag{12}$$

$$\frac{\partial\rho_{11}(p_z, p'_z, t)}{\partial t} + \frac{i}{\hbar}\left[\mathcal{E}_1(p_z) - \mathcal{E}_1(p'_z)\right]\rho_{11}(p_z, p'_z, t) = -\frac{e}{2\hbar c}\beta_\perp\left[A_0\rho_{10}(p_z, p'_z + \hbar k, t)e^{i\omega t}\right.$$
$$\left.+A_e\rho_{10}(p_z, p'_z - \hbar k', t)e^{i\omega' t} + A_0\rho_{01}(p_z + \hbar k, p'_z, t)e^{-i\omega t} + A_e^*\rho_{01}(p_z - \hbar k', p'_z, t)e^{-i\omega' t}\right], \tag{13}$$

and for the nondiagonal elements we have:

$$\frac{\partial\rho_{01}(p_z,p_z',t)}{\partial t}+\frac{i}{\hbar}\left[\mathcal{E}_0(p_z)-\mathcal{E}_1(p_z')\right]\rho_{01}(p_z,p_z',t)=-\frac{e}{2\hbar c}\beta_\perp\left[A_0\rho_{00}(p_z,p_z'+\hbar k)e^{i\omega t}\right.$$

$$\left.-A_0\rho_{11}(p_z-\hbar k,p_z')e^{i\omega t}+A_e\rho_{00}(p_z,p_z'-\hbar k')e^{i\omega' t}-A_e\rho_{11}(p_z+\hbar k',p_z')e^{i\omega' t}\right], \qquad (14)$$

$$\rho_{10}(p_z,p_z')=\rho_{01}^*(p_z',p_z). \qquad (15)$$

This set of equations should be supplemented by the Maxwell equation, which is reduced to

$$\frac{\partial A_e}{\partial t}+c\frac{\partial A_e}{\partial z}=-\varkappa A_e+\frac{2ce}{\hbar\omega'}\beta_\perp e^{-i\omega' t}\int\rho_{01}(p_z,p_z+\hbar k')dp_z. \qquad (16)$$

where $\varkappa$ is the linear damping coefficient. Equations (12)–(16) define the FEL dynamics with the pump EM wave when one or two transverse degrees of freedom of the particles are quantized.

We will assume that the pump laser field is not too strong (Rabi frequency is small compared with the resonance detuning) and, consequently, the population of transverse excited state remains small. The main terms responsible for the wave amplification in this case are $\rho_{00}(p_z,p_z+\hbar k'+\hbar k,t)$ and $\rho_{01}(p_z,p_z+\hbar k')$. Hence, from the set of Eqs. (12)–(15) in the first order by the fields, when

$$\rho_{ij}(p_z,p_z')=\rho_{ij}^{(0)}(p_z,p_z')+\rho_{ij}^{(1)}(p_z,p_z'),$$

and keeping only the resonant terms we obtain the self-consistent set of equations which determines the evolution and dynamics of the considered FEL:

$$\frac{\partial J(z,t,p_z)}{\partial t}+\overline{\mathrm{v}}\frac{\partial J(z,t,p_z)}{\partial z}-i\Delta_c\left(p_z\right)J(z,t,p_z)=\frac{e^4\beta_\perp^4A_0^2}{8\hbar^3\omega^2c^3\delta^2}A_eN\left(p_z\right), \qquad (17)$$

$$\frac{\partial A_e(z,t)}{\partial t}+c\frac{\partial A_e(z,t)}{\partial z}=-\varkappa' A_e(z,t)+\frac{4\pi c}{\omega'}\int J(p_z,z,t)dp_z, \qquad (18)$$

where $N\left(p_z\right)$ is defined via initial momentum distribution function $F_0\left(p_z\right)$:

$$N\left(p_z\right)=F_0\left(p_z\right)-F_0\left(p_z-\hbar k'-\hbar k\right) \qquad (19)$$

and represents population inversion in momentum space, and

$$J\equiv\frac{e\beta_\perp}{2\pi\hbar}\rho_{01}^{(1)}(p_z-\hbar k'-\hbar k,p_z-\hbar k).$$

For the initially cold electron beam with the density N_0 and a mean momentum $\overline{p}$:

$$F_0\left(p_z\right)=N_0\delta(p_z-\overline{p}). \qquad (20)$$

In Eqs. (17) and (18)

$$\Delta_c\left(p_z\right)=\frac{1}{\hbar}\left[\mathcal{E}_0(p_z)+\hbar\omega-\mathcal{E}_0(p_z-\hbar k'-\hbar k)-\hbar\omega'\right] \qquad (21)$$

is the resonance detuning for the Compton scattering, $\delta=\left|\omega+k\overline{\mathrm{v}}-\Omega_{01}\right|/\omega$ – is the relative resonant detuning of the pump wave coupling of the two transverse levels, and

$\varkappa'$ is the modified linear damping coefficient because of concurrent process of the wave absorption by electrons.

By the Fourier transformation for slowly varying envelopes of the probe wave and electric current density

$$A_e(z,t) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} A_\varpi(z)e^{i\varpi t}d\varpi, \quad J(z,t,p_z) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} J_\varpi(z,p_z)e^{i\varpi t}d\varpi, \tag{22}$$

Eqs. (17) and (18) are reduced to the equations:

$$\frac{\partial J_\varpi(z,p_z)}{\partial z} - i\Theta_\varpi\left(p_z\right)J_\varpi(z,p_z) = \frac{e^2\beta_\perp^4}{8\hbar c^3\overline{\mathrm{v}}}\left(\frac{mc^2}{\hbar\omega}\right)^2\frac{\xi_0^2}{\delta^2}A_\varpi(z)N\left(p_z\right), \tag{23}$$

$$\frac{\partial A_\varpi(z)}{\partial z} + \left(i\frac{\varpi}{c} + \frac{\varkappa'}{c}\right)A_\varpi(z) = \frac{4\pi}{\omega'}\int J_\varpi(z,p_z)dp_z, \tag{24}$$

where $\Theta_\varpi\left(p_z\right) = \left(\Delta_c\left(p_z\right) - \varpi\right)/\overline{\mathrm{v}}$ and $\xi_0 = eA_0/mc^2$ is the relativistic invariant parameter of the pump wave intensity. The solution of Eq. (23) can be written as:

$$J_\varpi(z,p_z) = J_\varpi(0,p_z)e^{i\Theta_\varpi(p_z)z} + \frac{e^2\beta_\perp^4}{8\hbar c^3\overline{\mathrm{v}}}\left(\frac{mc^2}{\hbar\omega}\right)^2\frac{\xi_0^2}{\delta^2}\int\limits_0^z e^{i\Theta_\varpi(p_z)(z-z')}A_\varpi(z')N\left(p_z\right)dz'. \tag{25}$$

Here it is assumed that

$$J_\varpi(0,p_z) = \overline{J}_\varpi\delta(p_z - \overline{p}), \tag{26}$$

where $\overline{J}_\varpi$ characterizes the shot noise in the electron beam or modulation depth for the initially modulated beam. Substituting Eq. (25) into Eq. (24), for the initial cold electron beam (20) we obtain an integro-differential equation for the phase transformed amplitude $\widetilde{A}_\varpi(z) = A_\varpi(z)e^{-i\Theta_\varpi(\overline{p})z}$ of the amplifying wave field:

$$\frac{\partial\widetilde{A}_\varpi(z)}{\partial z} - \left(i\left(1-\frac{\overline{\mathrm{v}}}{c}\right)\frac{\varpi}{\overline{\mathrm{v}}} - \frac{\varkappa'}{c}\right)\widetilde{A}_\varpi(z) = \frac{4\pi}{\omega'}\overline{J}_\varpi$$
$$+ i\frac{2\pi e^2\beta_\perp^4}{\hbar c^3\overline{\mathrm{v}}^2\gamma}\frac{mc^2}{\hbar\omega}\frac{\xi_0^2}{\delta^2}N_0\int\limits_0^z\left(z-z'\right)\widetilde{A}_\varpi(z')dz', \tag{27}$$

Without an initial seed the solution of Eq.(27) is given as:

$$\widetilde{A}_\varpi(z) = -i\frac{2}{\omega'}\oint\frac{\overline{J}_\varpi\eta e^{\eta z}}{\left(\eta-\eta_1\right)\left(\eta-\eta_2\right)\left(\eta-\eta_3\right)}d\eta, \tag{28}$$

where $\eta_{1,2,3}$ are the solutions of the characteristic equation

$$\eta^3 - \left(i\left(1-\frac{\overline{\mathrm{v}}}{c}\right)\frac{\varpi}{\overline{\mathrm{v}}} - \frac{\varkappa'}{c}\right)\frac{\varpi}{\overline{\mathrm{v}}}\eta^2 = i\frac{2\pi e^2\beta_\perp^4}{\hbar c^3\overline{\mathrm{v}}^2\gamma}\frac{mc^2}{\hbar\omega}\frac{\xi_0^2}{\delta^2}N_0. \tag{29}$$

The average spectral power is defined as:

$$\frac{dP_\varpi(x)}{d\varpi} = \frac{S_e\omega'^2}{8\pi^2 c\tau_e}\left\langle\left|\widetilde{A}_{\varpi-\omega'}(x)\right|^2\right\rangle, \tag{30}$$

where S_e and τ_e are the electron beam cross-sectional area and pulse duration, respectively. Hence, from Eqs. (28)–(30) for the average spectral intensity we have:

$$\frac{dP_{\varpi}(x)}{d\varpi} = \frac{S_e\left\langle\left|\overline{J}_{\varpi-\omega'}\right|^2\right\rangle}{6c\tau_e G^2}\exp\left[-\frac{(\varpi-\omega')^2}{2\Delta^2(x)}\right]e^{2Gx}, \tag{31}$$

where

$$G = \frac{\sqrt{3}}{2}\left[\frac{2\pi e^2\beta_{\perp}^4}{\hbar c^3\overline{\mathrm{v}}^2\gamma}\frac{mc^2}{\hbar\omega}\frac{\xi_0^2}{\delta^2}N_0\right]^{1/3} \tag{32}$$

is the exponential growth rate in high gain regime [6]. In Eq. (31) it was assumed that $G >> \varkappa'/c$ (high gain regime). The spectral width in the SASE regime is defined as follows:

$$\Delta(x) = \sqrt{\frac{3G}{x}}\frac{\overline{\mathrm{v}}}{1-\frac{\overline{\mathrm{v}}}{c}}. \tag{33}$$

The shot noise power in the SASE regime can be calculated as in case of Compton FEL [13]. In considering case of X-ray laser on the channeled electrons, we will have:

$$\left\langle\left|\overline{J}_{\varpi}\right|^2\right\rangle = \frac{e^2N_0\beta_{\perp}^4}{4c^3S_e}\left(\frac{mc^2}{\hbar\omega}\right)^2\frac{\xi_0^2}{\delta^2}\tau_e, \tag{34}$$

and with the help of Eq. (31) for the average spectral intensity of the semiclassical X-ray SASE amplifier on the channeled electron beam we obtain the following final formula:

$$\frac{dP_{\varpi}(x)}{d\varpi} = \frac{e^2N_0}{24G^2}\frac{\beta_{\perp}^4}{c^4}\left(\frac{mc^2}{\hbar\omega}\right)^2\frac{\xi_0^2}{\delta^2}\exp\left[-\frac{(\varpi-\omega')^2}{2\Delta^2(x)}\right]e^{2Gx}. \tag{35}$$

Comparing Eq. (35) with the SASE power of Compton FEL it is easy to see that for the same pump wave and electron beam parameters in case of the channeled electron beam the start-up intensity and the gain are enhanced by factor $\eta = (\beta_{\perp}^2\mathcal{E}_0/c^2\delta\hbar\omega)^{2/3} >> 1$.

For the fundamental transitions of electrons in the monocrystals at the planar and axial channeling $\beta_{\perp}/c \sim 10^{-3}$ and the resonance can be achieved by optical pump lasers: $\hbar\omega \simeq \hbar\Omega/2 \sim 2$ eV. In this case the wave lengths of amplifying radiation for mildly relativistic electron beams ($\gamma \sim 10 \div 30$) are: $\lambda \sim 10 \div 1\ A$. The enhancement factor η for resonance detuning $\delta \sim 10^{-2}$ (which is defined by the dechanneling effects) will be 100. In this aspect the scheme of X-ray laser on the ion beams will have rather high yields due to the incomparably small resonance width.

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