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Conventional and Non-Conventional Fishbone Instabilities Driven by Circulating Energetic Ions

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Outline

- **Double-kink fishbone instability caused by circulating energetic ions**

  1. **High frequency ($\omega >> \omega_{dia}$) EPM in the symmetric case ($|s_1|=|s_2|$)**
  2. **"Doublet" instability in the non-symmetric case ($|s_1| \neq |s_2|$)**

- **Quasi-interchange fishbone mode induced by circulating energetic ions in low-shear tokamaks**

  1. **Motivation**
  2. **$m = n = 1$ fishbone in plasmas with $1 - q_0 \sim \varepsilon_1$**
  3. **"Infernal" fishbones with arbitrary (m, n)**

- **Summary**

Double-kink fishbone instability

"Top-hat" eigenfunction for the *(m, n)* radial displacement amplitude

$$\Downarrow$$

Generic dispersion relation

$$\underbrace{\delta I}_{\substack{\text{Inertial}\\ \text{layers}}} + \underbrace{\delta W_c}_{\text{MHD}} + \underbrace{\delta W_h}_{\substack{\text{Energetic}\\ \text{ions}}} = 0$$

$$\delta I = -2\pi^2 \frac{R_0}{m^2} n_0 m_i \int_{layers} r^3 dr \left(\frac{d\xi}{dr}\right)^2 (\omega^2 - k_\parallel^2 V_A^2) dr =$$

$$-\frac{i\omega}{2V_A} \frac{B^2}{|m| q_s} (r_{s1}^2 |s_s| + r_{s2}^2 |s_2|) \xi_0^2$$

$$\Downarrow$$

$$i\frac{\omega}{\omega_A} + \lambda_c + \lambda_h = 0$$

$$\omega_A = |m|(|s_1| + |s_2|)\frac{V_A}{q_s R_0}$$

$\lambda_c \rightarrow$ **Gimblett *et al.*, PoP <u>3</u> (1996) 3369**

$$\lambda_h = \lambda_{hf} + \lambda_{hk}$$

$$\lambda_{hf} = -\frac{\pi^2 m_\alpha q_s}{\xi_0^2 (|s_1| + |s_2|)^2 r_{\min}^2 B_0^2} \sum_\sigma \int dP_\phi \int v^5 dv \int d\Lambda \frac{\partial F_\alpha}{\partial P_\phi} \tau_b \left\langle \xi_{0r}^2 r \cos\theta \right\rangle$$

$$\lambda_{hk} = \frac{2\pi^2 R_0 m_\alpha q_s^2}{\omega_{c\alpha} r_{\min}^2 \xi_0^2 B_0^2 (|s_1| + |s_2|)^2} \sum_\sigma \int v^3 dv \int dP_\phi \int d\Lambda \times$$

$$\tau_b \frac{\partial F_\alpha}{\partial \mathrm{E}} \frac{\omega - n\omega_{*\alpha}}{\omega - k_\parallel v_\parallel} \left| \left\langle \left(\frac{v_\perp^2}{2} + v_\parallel^2\right) \vec{\xi} \bullet \vec{\kappa} \exp[i(\omega - k_\parallel v_\parallel)t] \right\rangle \right|^2$$

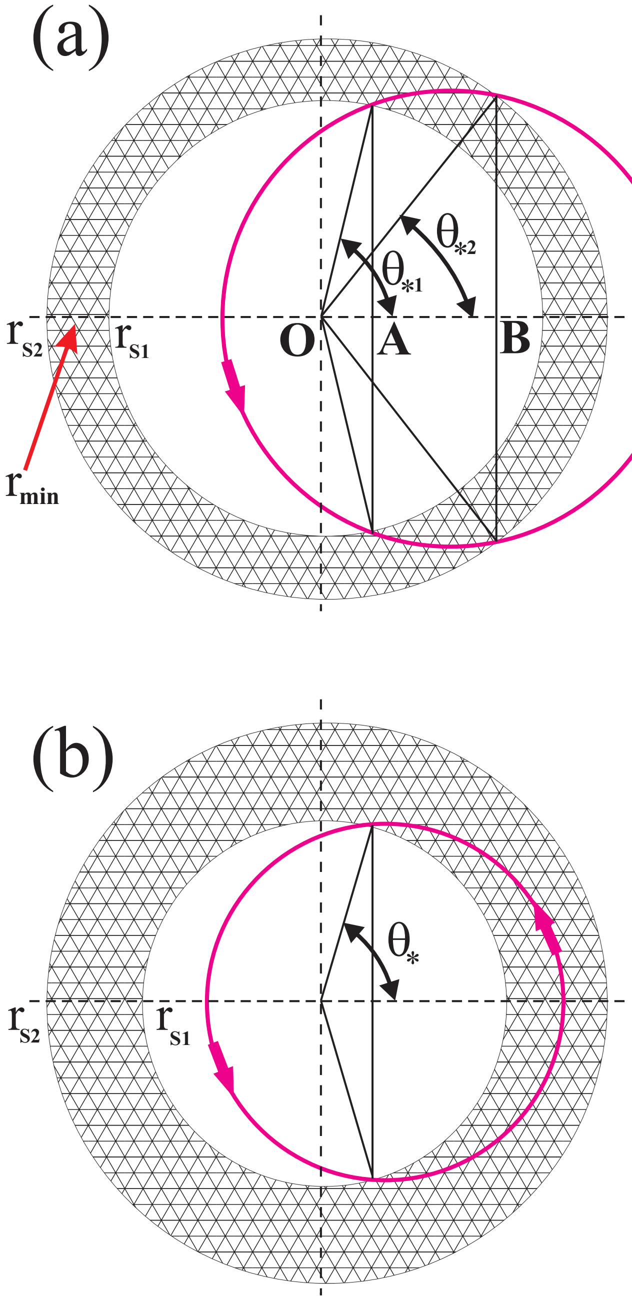

FIG. 1. A particle orbit crossing the region where a double kink mode is localized (shaded region): a, the orbit width exceeds the mode width; b, the mode width exceeds the orbit width. Notations: r_{s1} and r_{s2} are two rational surfaces with the same $q(r)$, r_{min} is the radius where $q = q_{min}$, OA and OB are the cosines of the angles θ_{*1} and θ_{*2} at which a particle crosses the edges of the mode localization region.

A. The case of large orbit width

Short summary: EPM is absent; only diamagnetic fishbone mode is possible with strongly reduced growth rate (in comparison with internal kink case)

B. The case of small orbit width

$$\Downarrow$$

Similar to internal kink case
[R. Betti, Plasma Phys. Control. Fusion 35 (1993) 941]

$$\Downarrow$$

$$\lambda_k = \sum_{i=1,2} \lambda_{ki} = \sum_{i=1,2} \frac{1}{3} \frac{R_0}{r_{si}} \frac{q_s^2}{(|s_1|+|s_2|)^2} \left[-\frac{\Delta r_{b\alpha}}{|s|} \frac{d\beta_\alpha}{dr} \right]_{r_{si}} F\left(\frac{\omega}{\omega_{si}} \right)$$

$$F(x) \equiv \frac{1}{\pi} \left\{ 10x - 8x^{3/2} \left[\tan^{-1} \frac{1}{\sqrt{x}} + \tanh^{-1} \frac{1}{\sqrt{x}} \right] + (1+3x^2) \ln \frac{1+x}{x-1} \right\}$$

$$\omega_{si} = \frac{|s_i| v_{\|\alpha}^2}{\omega_{c\alpha} R_0 r_{si}} \quad (\sim \omega_D \quad for \quad s \sim \frac{q}{2})$$

B1. High frequency EPM in the symmetric case ($|s_1|=|s_2|$)

$$0 = D(\Omega) = -i\Omega - \tilde{\lambda}_c - \pi_\alpha F(\Omega)$$

$$\Omega = \frac{\omega}{\omega_s} \quad , \quad \pi_\alpha = -\frac{1}{3} \frac{m(m/n)^2}{|s|^3} \frac{V_A}{v_\alpha} R_0 \left(\left. \frac{d\beta_\alpha}{dr} \right|_{r_{s1}} + \left. \frac{d\beta_\alpha}{dr} \right|_{r_{s2}} \right)$$

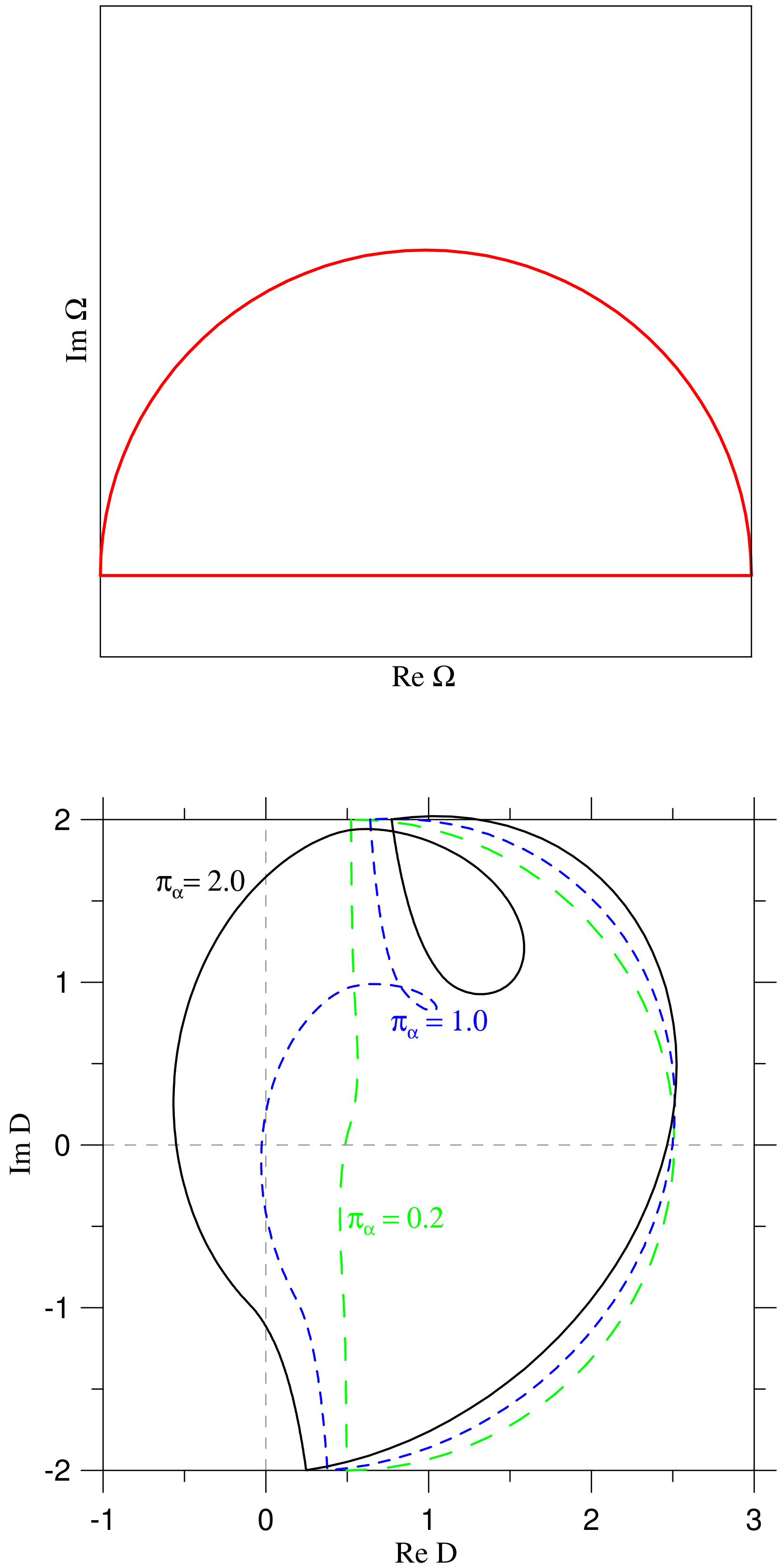

FIG. 2. Nyquist contour in the plane $(\mathrm{Re}\,\Omega, \mathrm{Im}\,\Omega)$ and its map in the plane $(\mathrm{Re}\,D, \mathrm{Im}\,D)$ when $s_1 = s_2$, $\lambda_c = -0.5$ for various π_α. Notations: $\Omega = \omega/\omega_{s1}$, $\omega_{s1} = |s_1|v^2_{\|\alpha}/(\omega_c R_0 r_s)$, $\pi_\alpha \propto -d\beta_\alpha/dr$

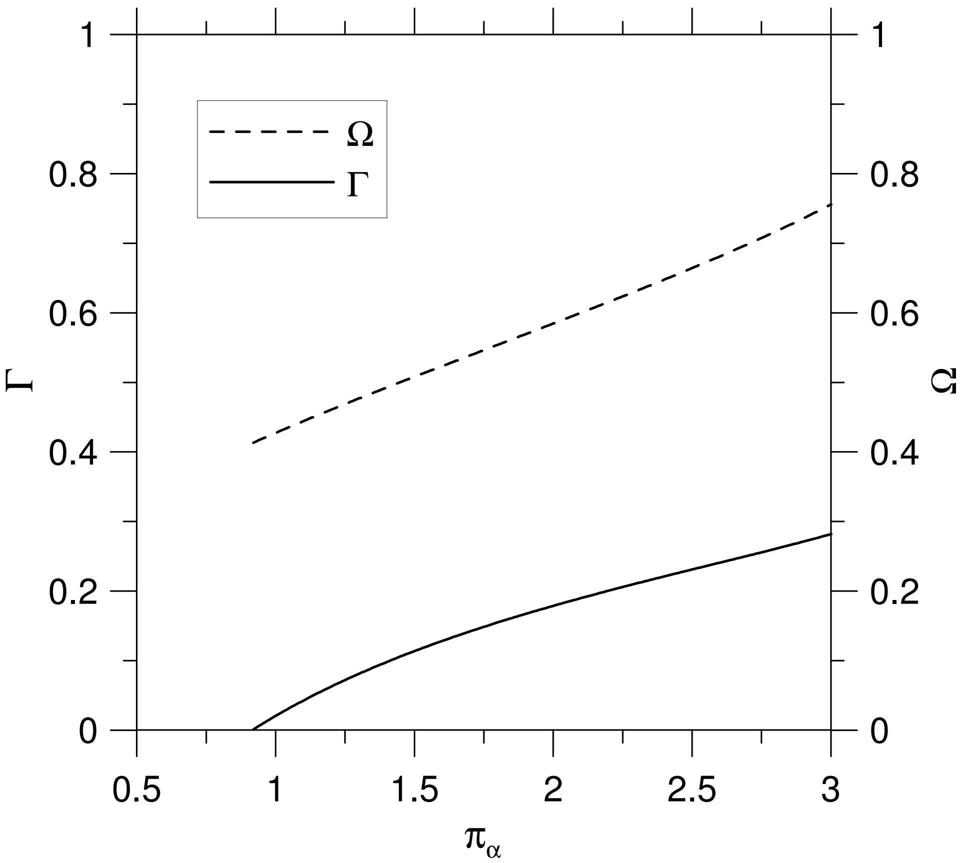

FIG. 3. Normalized growth rate, $\Gamma = \gamma/\omega_{s1}$, and the mode frequency, $\Omega = \omega/\omega_{s1}$, of the EPM fishbone instability in a plasma with a monotonic $\beta_\alpha(r)$ and $\widetilde{\lambda}_c = -0.1$.

B2. "Doublet" instability in the non-symmetric case ($|s_1| \neq |s_2|$)

$$0 = D(\Omega) = -i\Omega - \tilde{\lambda}_c - \pi_{\alpha 1} F(\Omega) - \pi_{\alpha 2} F\left(\left|\frac{s_1}{s_2}\right| \Omega\right)$$

$$\pi_{\alpha i} = -\frac{8}{3} \frac{m(m/n)^2}{(|s_1| + |s_2|)^2} \left(\frac{V_A}{v_\alpha} \frac{R_0}{s} \frac{d\beta_\alpha}{dr} \right)_{r_{si}}$$

Two-frequency instability in ASDEX-U
[S. Guenter *et al.,* Nucl. Fusion 43 (2003) 161]

- ***m = n = 1*** **mode with** $f_1 \approx 15\ kHz$ **and** $f_2 \approx 20\ kHz$
- **off-axis tangential NBI [** $\beta_\alpha(r)$ **non-monotonic]**
- **two** ***q = 1*** **surfaces due to off-axis NBI CD**
- **doublet frequencies comparable with the fishbone frequency during the radial injection (consistent with** $\omega_s \sim \omega_D$ **)**

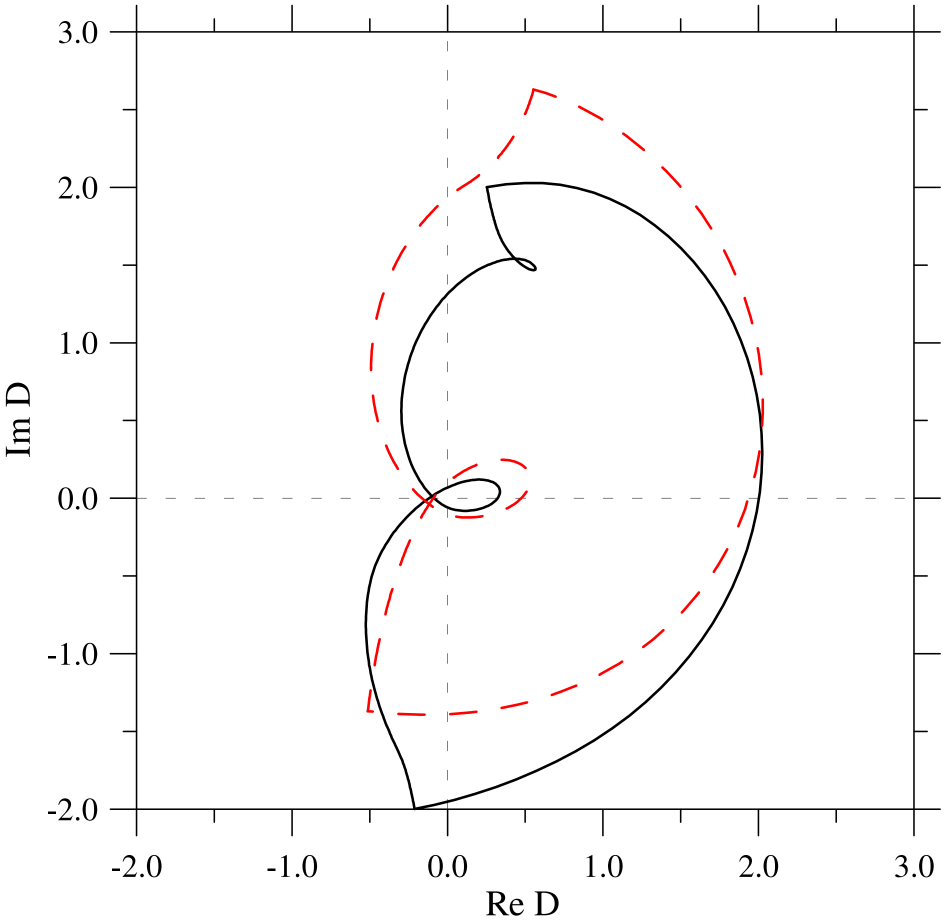

FIG. 4. Map of a Nyquist contour in the case of "doublet" instability for $\widetilde{\lambda}_c = -0.01$: solid line, $s_1/s_2 = 0.6$, $\pi_{\alpha 1} = -2.5$, $\pi_{\alpha 2} = 2.4$; dotted line, $s_1/s_2 = 0.2$, $\pi_{\alpha 1} = -1.5$, $\pi_{\alpha 2} = 1.35$.

Quasi-interchange fishbone mode induced by circulating energetic ions in low-shear tokamaks

Motivation

- **"Hybrid" regime with $q \approx const \approx 1$ in the central core has been proposed recently as a third operational scenario for ITER**

- **Flat $q(r)$ with $m - nq \sim \varepsilon$ in the central core are typical for high β discharges in ST**

- **Strong NBI (and/or α-heating for ITER)**

$$\Downarrow$$

Kinetic stability in the presence of energetic ions

<u>**$m = n = 1$ fishbone in plasmas with $1 - q_0 \sim \varepsilon$**</u>

MHD counterpart: quasi-interchange mode [Wesson (1986)]

- **Eigenfunction of "cellular" character (in contrast with rigid kink for $1 - q_0 >> \varepsilon$)**

$$\Downarrow$$

- **Finite average power transfer at the fundamental resonance $\omega = k_{||} v_{||}$ for all particles deposited in the shear-free core**

$$\Downarrow$$

- **Possibility of the EPM with $\omega \sim (1 - q_0) v_\alpha / R_0 << v_\alpha / R_0$**

Dispersion relation for the QI fishbone mode

$$\mathrm{E} = \frac{R_0}{\pi^2 B_0^2}(\delta W_{MHD} + \delta W_k) - \frac{\omega^2}{\omega_A^2} N$$

$$N = \frac{1}{2\pi^2 R_0}\int d^3 r \, |\vec{\xi}_\perp|^2$$

$$\delta W_k \equiv \frac{1}{2}\int \vec{\xi}_\perp^* \bullet \nabla \delta \Pi_\alpha^k d^3 r = -\frac{\pi^2 m_\alpha}{\omega_{c\alpha}} \sum_\sigma \int v^3 dv \int dP_\phi \int d\Lambda \times$$

$$\tau_b \frac{\partial F_\alpha}{\partial \mathrm{E}} \frac{\omega - \omega_{*\alpha}}{\omega - k_\| v_\|} \left| \left\langle \left(\frac{v_\perp^2}{2} + v_\|^2 \right) \vec{\xi}_\perp \bullet \vec{\kappa} \exp[i(\omega - k_\| v_\|)t] \right\rangle \right|^2$$

$$\vec{\xi}_\perp \bullet \vec{\kappa} = -\frac{1}{R_0}\xi_1\{r[\theta(t)]\}\cos[\theta(t)]\exp\{i[\theta(t) - \phi(t) - \omega t]\}$$

$$r[\theta(t)] = \bar{r} - \Delta_\alpha \cos[\theta(t)]; \quad \Delta_\alpha = \frac{q(\bar{r})}{v_\| \omega_{c\alpha}} \left(\frac{v_\perp^2}{2} + v_\|^2 \right)$$

$$\theta(t) = \frac{v_\|}{q(\bar{r}) R_0} t; \quad \phi(t) = \frac{v_\|}{R_0} t$$

$$F_\alpha = \frac{\sqrt{2} m_\alpha}{\pi \mathrm{E}_\alpha} p_\alpha(\bar{r}) H(\mathrm{E}_\alpha - \mathrm{E}) \mathrm{E}^{-3/2} \delta(\Lambda)$$

$$\Downarrow$$

$$\frac{R_0}{\pi^2 B_0^2}\delta W_k = -\frac{2}{\pi^2}\rho_\alpha^3 R_0 F\left(\frac{\omega}{k_{0\|} v_\alpha}\right) \int_0^a dr \left| \frac{d\xi_1}{dr} \right|^2 \frac{d\beta_\alpha}{dr}$$

$$F(\Omega) \equiv \frac{1}{5} + \frac{\Omega}{4} + \frac{\Omega^2}{3} + \frac{\Omega^3}{2} + \Omega^4 + \Omega^5 \ln\left(1 - \frac{1}{\Omega}\right)$$

Model fast ion distribution

$$\beta_\alpha(r) = \beta_{\alpha 0}\left[1-\left(\frac{r}{r_0}\right)^4\right]$$

$$\Downarrow$$

Eigenmode equations

$$\frac{d}{dr}\left\{\left[(\mu-1)^2 + l_\alpha(\omega,r) - \frac{\omega^2}{\omega_A^2}\right] r^3 \frac{d\xi_1}{dr}\right\} - G\{\xi_1\} = C\{\xi_2\}$$

$$\frac{d}{dr}\left[\left(\mu-\frac{1}{2}\right)^2 r^3 \frac{d\xi_2}{dr}\right] - 3\left(\mu-\frac{1}{2}\right)^2 r\xi_2 = C^+\{\xi_1\}$$

[*G* ~ *O*(*ε*^{2}), *C* ~ *O*(*ε*)] → Waelbroeck & Hazeltine (1988)

$$\int_0^a dr f(r) C\{g(r)\} = \int_0^a dr g(r) C^+\{f(r)\}$$

$$l_\alpha(\omega,r) \equiv \frac{8}{\pi^2}\frac{\rho_\alpha^3 R_0}{r_0^4} F\left(\frac{\omega}{k_{0\parallel} v_\alpha}\right)\beta_{\alpha 0} H(r_0 - r)$$

In the shear-free core [*μ ~1 +O(ε)*]

$$\frac{d}{dr}\left\{\varepsilon^{-2}\left[(\mu-1)^2 + l_\alpha(\omega,r) - \frac{\omega^2}{\omega_A^2}\right] r^3 \frac{d\xi_1}{dr}\right\} - 4\left(\frac{r}{4}\frac{d\beta_p}{dr} + \beta_p\right)^2 r^3 \xi_1 = \left(\frac{r}{4}\frac{d\beta_p}{dr} + \beta_p\right)\frac{d}{dr}(r^3 \hat{\xi}_2)$$

$$\frac{d}{dr}\left(r^3 \frac{d\hat{\xi}_2}{dr}\right) - 3r\hat{\xi}_2 = -4r^3 \frac{d}{dr}\left[\left(\frac{r}{4}\frac{d\beta_p}{dr} + \beta_p\right)\xi_1\right]$$

$$\xi_2 \equiv \varepsilon\hat{\xi}_2, \quad \varepsilon \equiv \frac{a}{R_0}$$

$$\beta_p(r) = -\frac{8\pi R_0^2}{r^4 B_0^2}\int_0^r \hat{r}^2 \frac{dp_c}{d\hat{r}} d\hat{r}$$

General solution for ξ_2

$$\hat{\xi}_2 = r^{-3}\int_0^r \hat{r}^4 \beta_p(\hat{r}) \frac{d\xi_1}{d\hat{r}} d\hat{r} + [e - \beta_p(r)\xi_1(r)]r$$

$$\Downarrow$$

$$\frac{d\xi_1}{dr} = \frac{\varepsilon^2 er\beta_p}{(\mu-1)^2 + l_\alpha(\omega,r) - \omega^2/\omega_A^2}$$

In the sheared region $|\mu - 1| \sim 1 \Rightarrow \xi_1 \sim \varepsilon^2$

$$\Downarrow$$

$C^+\{\xi_1\}$ can be neglected

$$\Downarrow$$

$$\frac{d}{dr}\left[\left(\mu - \frac{1}{2}\right)^2 r^3 \frac{d\hat{\xi}_2}{dr}\right] - 3\left(\mu - \frac{1}{2}\right)^2 r\hat{\xi}_2 = 0$$

Asymptotic solution in the shear-free region

$$\hat{\xi}_2 \propto \frac{r}{r_2} + \sigma\left(\frac{r}{r_2}\right)^{-3}, \quad \mu(r_2) = \frac{1}{2}$$

$$\Downarrow$$

Dispersion relation

$$\sigma = \left(\frac{r_2}{a}\right)^2 \int_0^a \frac{[\varepsilon\beta_p(r)]^2}{(\mu-1)^2 + l_\alpha(\omega,r) - \omega^2/\omega_A^2}\left(\frac{r}{r_2}\right)^5 \frac{dr}{r_2} \approx$$

$$\left(\frac{r_2}{a}\right)^2 \frac{\varepsilon^2}{(\mu_0 - 1)^2 + l_\alpha(\omega) - \omega^2/\omega_A^2}\int_0^{r_0} \beta_p^2(r)\left(\frac{r}{r_2}\right)^5 \frac{dr}{r_2} + \sigma_{res}(\omega)$$

$$\sigma_{res}(\omega) = \left(\frac{r_2}{a}\right)^2 \varepsilon^2 \beta_p^2(r_A)\left(\frac{r_A}{r_2}\right)^5 \underbrace{\lim}_{\eta\to 0}\int_{r_A-0}^{r_A+0} \frac{dr/r_2}{(\mu-1)^2 - (\omega+i\eta)^2/\omega_A^2} =$$

$$i\pi\left(\frac{r_2}{a}\right)^2 [\varepsilon\beta_p(r_A)]^2 \frac{(r_A/r_2)^5}{r_2\,|(\partial/\partial r)(\mu-1)^2|_{r=r_A}|} \equiv i\sigma_1(\omega)$$

Final form of the dispersion relation

$$\left\{(\mu_0-1)^2\left[1-\left(\frac{v_\alpha}{V_A}\right)^2\Omega^2\right]+\hat{\beta}_\alpha F(\Omega)\right\}[\sigma-i\sigma_1(\Omega)]=$$

$$\left(\frac{r_2}{a}\right)^2\int_0^{r_0}[\varepsilon\,\beta_p(r)]^2\left(\frac{r}{r_2}\right)^5\frac{dr}{r_2}$$

$$\Omega\equiv\frac{\omega}{k_{0\|}v_\alpha}\;;\quad \hat{\beta}_\alpha\equiv\frac{8}{\pi^2}\frac{\rho_\alpha^3R_0}{r_0^4}\beta_{\alpha0}\;;\quad \rho_\alpha\equiv\frac{v_\alpha}{\omega_{c\alpha}}$$

Infernal fishbone modes with arbitrary *(m,n)*

Eigenmode equations in the shear-free core with $|q_0 - m/n| \sim \varepsilon$

$$\frac{d}{dr}\left\{\left[\left(\frac{\mu}{n}-\frac{1}{m}\right)^2+\frac{l_\alpha}{(mn)^2}-\left(\frac{\omega}{\omega_A mn}\right)^2\right]r^3\frac{d\xi_m}{dr}\right\}-$$

$$(m^2-1)\left[\left(\frac{\mu}{n}-\frac{1}{m}\right)^2-\left(\frac{\omega}{\omega_A mn}\right)^2\right]r\xi_m-\frac{\varepsilon^2}{2m^2}\left(r\frac{d\beta_p}{dr}+4\beta_p\right)^2r^3\xi_m+$$

$$\underbrace{\frac{\varepsilon^2}{m^2}\left(1-\frac{n^2}{m^2}\right)\left(r\frac{d\beta_p}{dr}+4\beta_p\right)r^3\xi_m}_{\textit{average magnetic well}}=$$

$$\frac{\varepsilon^2 n}{2m^2(m+1)}r^{1+m}\left(r\frac{d\beta_p}{dr}+4\beta_p\right)\frac{d}{dr}(r^{2+m}\hat{\xi}_{m+1})$$

$$\frac{d}{dr}\left(r^3\frac{d\hat{\xi}_{m+1}}{dr}\right)-[(m+1)^2-1]r\hat{\xi}_{m+1}=$$

$$-\frac{m+1}{2n}r^{2+m}\left[\left(r\frac{d\beta_p}{dr}+4\beta_p\right)r^{1+m}\xi_m\right]$$

General solution for ξ_{m+1}

$$n\hat{\xi}_{m+1}=-\frac{1}{2}(1+m)r^{-(2+m)}\int_0^r\left(\hat{r}\frac{d\beta_p}{dr}+4\beta_p\right)\hat{r}^{2+m}\xi_m d\hat{r}+er^m$$

$$\Downarrow$$

$$\frac{d}{dr}\left\{\left[\left(\frac{\mu}{n}-\frac{1}{m}\right)^2+\frac{l_\alpha}{(mn)^2}-\left(\frac{\omega}{\omega_A mn}\right)^2\right]r^3\frac{d\xi_m}{dr}\right\}-$$

$$\underbrace{(m^2-1)\left[\left(\frac{\mu}{n}-\frac{1}{m}\right)^2-\left(\frac{\omega}{\omega_A mn}\right)^2\right]r\xi_m}_{(1)}-\underbrace{\frac{\varepsilon^2}{m^2}\left(1-\frac{n^2}{m^2}\right)\frac{d}{dr}(r^4\beta_p)\xi_m}_{(2)}=$$

$$\frac{\varepsilon^2}{m^2}e\frac{d}{dr}(r^4\beta_p)r^{m-1}$$

Model pressure profile

$$p_c=p_0\left[1-\left(\frac{r}{a}\right)^{2\nu}\right]\Rightarrow\beta_p=\hat{\beta}_p\left(\frac{r}{a}\right)^{2\nu-2},\quad \hat{\beta}_p\sim 1$$

$$\Downarrow$$

(2)/(1) ~ $(r_0/a)^{2\nu} << 1$

Solution for ξ_m

$$\xi_m=\frac{2\varepsilon^2 e\hat{\beta}_p(\nu+1)(r^{2\nu}-1)r^{m-1}}{[(2\nu+m)^2-1]l_\alpha(\omega,r)/n^2+4\nu(\nu+m)[(m/nq-1)^2-(\omega/\omega_A n)^2]}$$

$$\hat{\xi}_{m+1}\propto\left(\frac{r}{r_{m+1}}\right)^m+\sigma_m\left(\frac{r}{r_{m+1}}\right)^{-(2+m)},\quad \mu(r_{m+1})=\frac{n}{m+1}$$

Asymptotic matching

$$\sigma_m=\frac{1+m}{n(\nu+m)}\left(\frac{r_{m+1}}{a}\right)^{-2(m+1)}\left(\frac{r_0}{a}\right)^{2(\nu+m)}\times$$

$$\frac{\varepsilon^2\hat{\beta}_p^2(\nu+1)^2}{[(2\nu+m)^2-1]l_\alpha(\omega)/n^2+4\nu(\nu+m)[(m/nq_0-1)^2-(\omega/\omega_A n)^2]}+\sigma_{res}$$

Model μ – profile

$$\mu=\frac{n}{m+1}+\left(\mu_0-\frac{n}{m+1}\right)\left[1-\left(\frac{r}{r_{m+1}}\right)^{2\lambda}\right]$$

$$\Downarrow$$

$$\sigma_m\approx\frac{m}{m+2}\left(1-\frac{m+1}{\lambda}\right),\quad \lambda\geq m+2$$

Continuum damping

$$\sigma_{res}(\omega) = i\pi \frac{m+1}{8\lambda n} \frac{\varepsilon^2 \hat{\beta}_p^2 (\nu+1)^2}{\nu(\nu+m)} \left(\frac{r_{m+1}}{a}\right)^{2(\nu-1)} \left[\frac{(m+1)\omega}{n\omega_A}\right]^{\frac{\nu+m}{\lambda}-2} \equiv i\sigma_{1m}$$

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m=2 fishbones in NSTX (Darrow *et al.*, poster EX/P2-01 in the 19th IAEA Fusion Energy Conference)

Parameters

80 keV D* co-NBI, *m = 2/ n = 1, q_0 ≈ 1.7, R_0/a = 1.5, v_α/V_A ≈ 2, r_2/a ≈ 0.6

$$(\nu = 6,\ \lambda = 4) \Rightarrow [\sigma_2 = 1/8,\ \sigma_{12}(\omega) = const,\ r_3/a \approx 0.85,\ r_0/a \approx r_2/a]$$

Marginal MHD stability without fast ions ($l_\alpha = \omega = 0$)

$$4n\nu\sigma_m \left(\frac{m}{nq_0} - 1\right)^2 = \frac{(1+m)(\nu+1)^2}{(\nu+m)^2} \left(\frac{r_{m+1}}{a}\right)^{-2(m+1)} \left(\frac{r_0}{a}\right)^{2(\nu+m)} \varepsilon^2 \hat{\beta}_p^2$$

$$\hat{\beta}_p = \frac{\beta_0}{\varepsilon^2}\left(\frac{m}{n}\right)^2 \frac{\nu}{\nu+1}$$

$$\Downarrow$$

$$\beta_0^{m\arg} \approx 0.35 \Rightarrow \sigma_{12} \approx 0.55$$

At the margin of fishbone stability (*Im Ω = 0*)

$$\hat{\beta}_\alpha^{crit} = \frac{192}{195} \frac{4\Omega^2 (2\mu_0 - 1)^2}{\operatorname{Re} F(\Omega) + (\sigma_{12}/\sigma_2) \operatorname{Im} F(\Omega)}$$

$$\operatorname{Im} F(\Omega) \left[4\Omega^2 \left(\frac{\sigma_{12}}{\sigma_2} + \frac{\sigma_2}{\sigma_{12}}\right) - \frac{\sigma_{12}}{\sigma_2}\right] = \operatorname{Re} F(\Omega)$$

$$\Downarrow$$

$$\Omega \approx 0.6 \Rightarrow \hat{\beta}_\alpha^{crit} \approx 2.5\times 10^{-2}$$

$$r_0 \approx 40\,cm,\ R_0 \approx 100\,cm,\ \rho_\alpha \approx 20\,cm,\ \langle\beta_\alpha\rangle = \frac{2}{3}\left(\frac{r_0}{a}\right)^2 \beta_{\alpha 0}$$

$$\Downarrow$$

$$\langle\beta_\alpha\rangle \approx 2.4\%,\ f \approx 46\,kHz$$

Experiment

$$\langle\beta_\alpha\rangle \approx 2\%,\ f \approx 45\,kHz$$

Summary

- **High frequency ($\omega >> \omega_{dia}$) fishbones can be destabilized by circulating energetic ions for both internal kink mode and double kink mode**

- **For reversed shear with $| s_1 | \neq | s_2 |$, and off-axis deposition of energetic ions, "doublet" instability is possible (consistent with observation in ASDEX-U)**

- **In plasmas with shear-free core and $1 - q_0 \sim \varepsilon_1$, quasi-interchange fishbone mode can be destabilized with frequency $\omega \sim k_{0\,\|} v_\alpha << v_\alpha / qR$ (particularly relevant for ST)**

- **Infernal fishbones with arbitrary (m, n) and similar properties are also possible in weak-shear plasmas**

- **Reasonable quantitative agreement with NSTX observations**

- **Work on high frequency "infernal" fishbones, with $\omega \sim (k_{0\|} + S/q_0 R) v_\alpha$ and $S = \pm 1$, is in progress**